UNIVERSITY OF CALIFORNIA AT BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

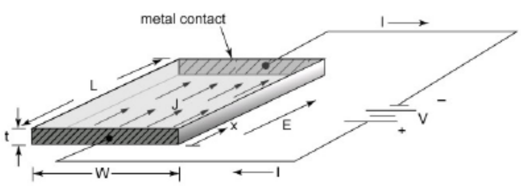
Discussion Notes #10

EE 105 **Spring 2007**
Prof. Wu

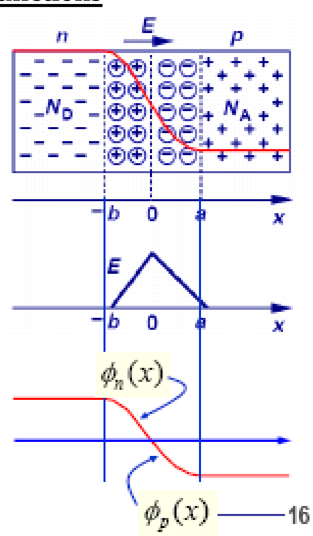
The following notes might be useful as a review for the final exam. This is not a comprehensive review, but it covers the key concepts that were discussed this semester.

Semiconductor Physics

- Intrinsic carrier concentration at 300 K: $n_i \sim 10^{10}$ cm^{-3}
- Mass action law (at thermal equilibrium): $n_0 p_0 = n_i^2$
- Doping
    - n-type: $n_0 = N_d - N_a$ and $p_0 = \frac{n_i^2}{N_d - N_a}$
    - p-type: $p_0 = N_a - N_d$ and $n_0 = \frac{n_i^2}{N_a - N_d}$
- Drift current: current due to movement of charged particles in an electric field
$$J_{drift} = J_{n,drift} + J_{p,drift} = q(\mu_n n_0 + \mu_p p_0)E = \sigma E$$
- Diffusion current: movement of charge carriers from high conc. to low conc.
$$J_{diff} = J_{n,diff} + J_{p,diff} = q\left(D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right)$$
- Einstein's relation: $\frac{D}{\mu} = \frac{kT}{q}$
- Resistivity
    - Conductivity: $\sigma = n_0 q \mu_n + p_0 q \mu_p$
    - Resistivity: $\rho = \frac{1}{\sigma} = \frac{1}{n_0 q \mu_n + p_0 q \mu_p}$
    - Resistance: $R = \rho \frac{L}{tW} = R_S \frac{L}{W}$, where R_S is the sheet resistance

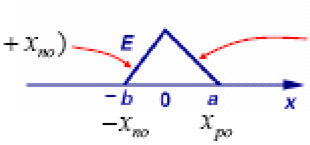


pn Junctions



- Depletion region:
  - Region of immobile ions at junction
  - Depletion region edge on n-side: $x_{n0} = \sqrt{\dfrac{2\varepsilon_s \phi_{bi}}{qN_d}\left(\dfrac{N_a}{N_a + N_d}\right)}$
  - Depletion region edge on p-side: $x_{p0} = \sqrt{\dfrac{2\varepsilon_s \phi_{bi}}{qN_a}\left(\dfrac{N_d}{N_a + N_d}\right)}$
  - Depletion width: $x_{d0} = x_{n0} + x_{p0} = \sqrt{\dfrac{2\varepsilon_s \phi_{bi}}{qN_d}\left(\dfrac{1}{N_a} + \dfrac{1}{N_d}\right)}$
- Charge density: $\rho_0(x) \cong \begin{cases} +qN_d & -x_{no} < x < 0 \\ -qN_a & 0 < x < x_{p0} \end{cases}$
- Electric field: solve using Gauss' Law: $\dfrac{dE_0}{dx} = \dfrac{\rho_0(x)}{\varepsilon_S}$

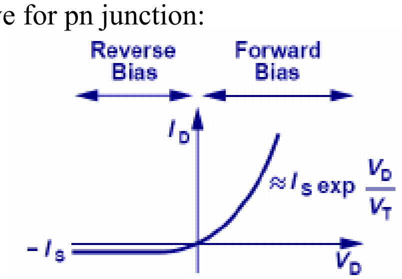
$E_0(x) = \dfrac{qN_d}{\varepsilon_s}(x + x_{no})$



$E_0(x) = \dfrac{qN_a}{\varepsilon_s}(x_{p0} - x)$

- Potential profile:
  - Potential on n-side (outside depletion region): $\phi_n = V_{th} \ln \dfrac{N_d}{n_i}$, where $V_{th} = \dfrac{kT}{q}$
  - Potential on p-side (outside depletion region): $\phi_p = V_{th} \ln \dfrac{n_i}{N_a}$
  - Built-in potential: $\phi_{bi} = \phi_n - \phi_p$

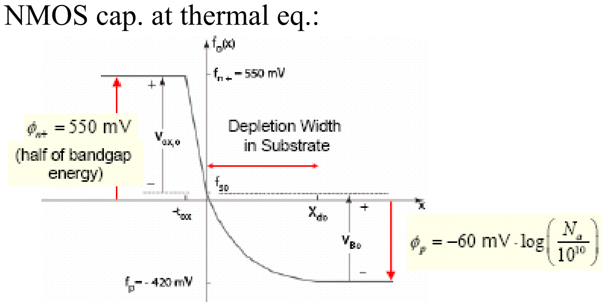
- Reverse bias: depletion region width ↑, electric field in junction ↑
  - Depletion width: $X_d(V_D) = X_{d0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$, where V_D is the reverse bias voltage
  - Junction capacitance: $C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_{bi}}}}$, where $C_{j0} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}}\left(\frac{N_a N_d}{N_a + N_d}\right)}$
- IV curve for pn junction:



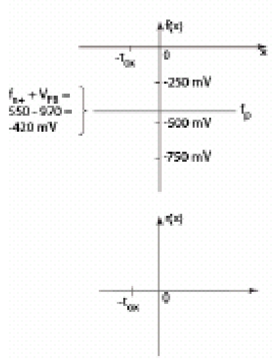
Note: breakdown occurs under a large reverse bias

Device Physics

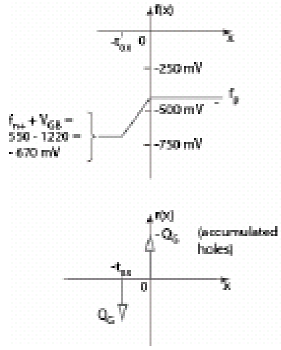
- MOS capacitor
  - NMOS cap. at thermal eq.:



  - NMOS cap. at flatband (no net charge):

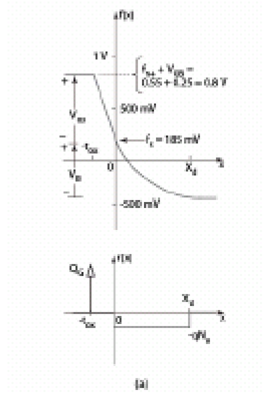


- NMOS cap. in accumulation ($V_{GB} < V_{FB}$):



$$Q_G = C_{ox} \cdot (V_{GB} - V_{FB})$$
$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \quad [\mathrm{F/cm^2}]$$

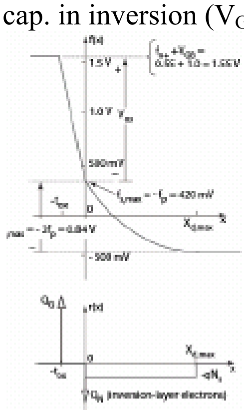
- NMOS cap. in depletion ($V_{FB}<V_{GB}<V_{Tn}$):



$$V_{GB} + \phi_{n+} - \phi_p = V_{ox} + V_B$$
$$= \frac{qN_a x_d}{C_{ox}} + \frac{qN_a x_d^2}{2\varepsilon_s}$$
$$= V_{GB} - V_{FB}$$

$$x_d = t_{ox}\frac{\varepsilon_s}{\varepsilon_{ox}}\left(\sqrt{1+\frac{2C_{ox}^2(V_{GB}-V_{FB})}{q\varepsilon N_a}}-1\right)$$

- NMOS cap. in inversion ($V_{GB}>V_{Tn}$):

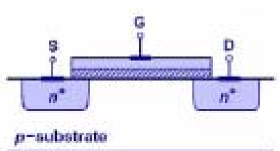


$$n_s = n_i e^{\frac{q\phi_s}{kT}} = N_a$$
$$\phi_s = -\phi_p$$

$$V_{Tn} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}}\sqrt{2q\varepsilon_s N_a(-2\phi_p)}$$

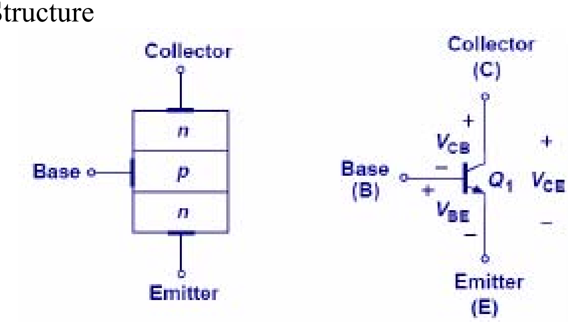
MOSFETs (nMOS)

Structure

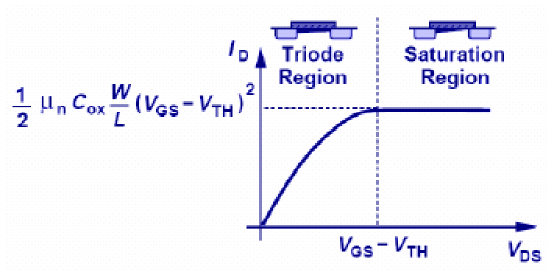


BJTs (npn)

Structure



MOSFET IV curve



MOSFET drain current (saturation):

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Tn})^2(1-\lambda V_{DS})$$

Transconductance and r_o:

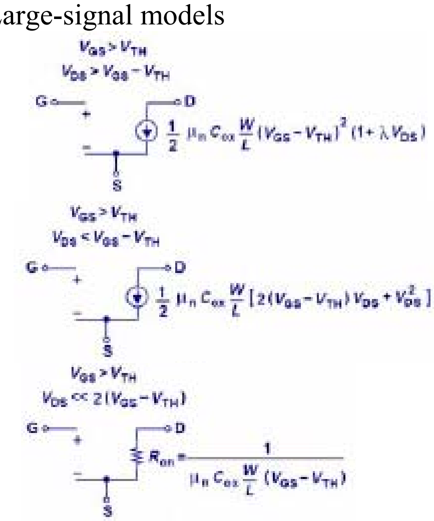
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Tn})$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

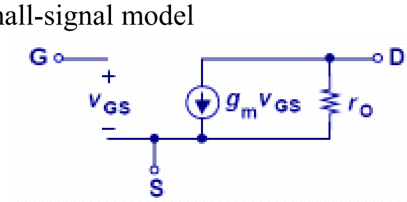
$$g_m = \frac{2I_D}{V_{GS}-V_{Tn}}$$

$$r_o \approx \frac{1}{\lambda I_D}$$

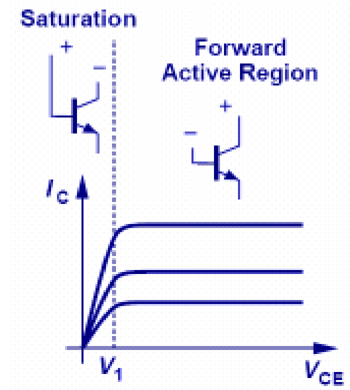
Large-signal models



Small-signal model



BJT IV curve



BJT currents

$$I_C = I_S \exp\frac{V_{BE}}{V_T}$$

$$I_B = \frac{1}{\beta}I_C$$

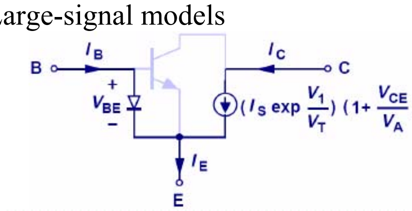
$$I_E = I_C + I_B$$

Transconductance and r_o:

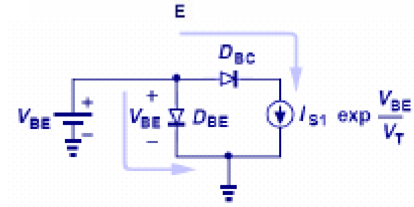
$$g_m = \frac{V_T}{I_C}$$

$$r_o \approx \frac{V_A}{I_C}$$

Large-signal models

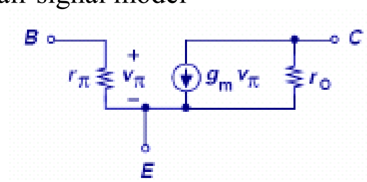


Forward-active



Saturation

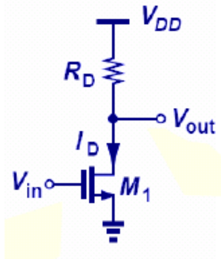
Small-signal model



Single-Stage Amplifiers

MOSFETs (nMOS)

Common-source

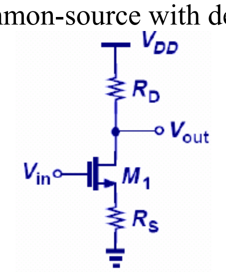


$$A_v = -g_m\left(R_D \parallel r_O\right)$$

$$R_{in} = \infty$$

$$R_{out} = R_D \parallel r_O$$

Common-source with degeneration:

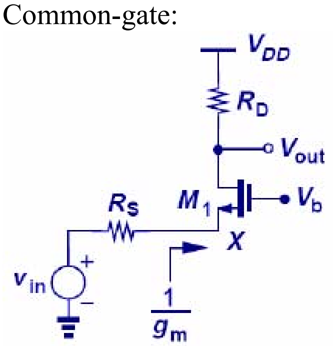


$$A_v = -\frac{R_D}{\frac{1}{g_m} + R_S}$$

$$R_{in} = \infty$$

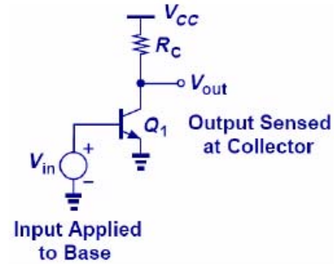
$$R_{out} = \left[(1+g_m r_O)R_S + r_O\right] \parallel R_D$$

Common-gate:



BJTs (npn)

Common-emitter

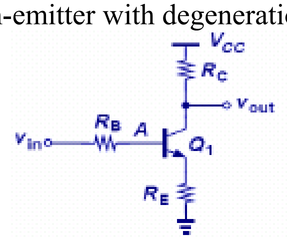


$$A_v = -g_m(R_C \parallel r_o)$$

$$R_{in} = r_\pi$$

$$R_{out} = R_C \parallel r_o$$

Common-emitter with degeneration:

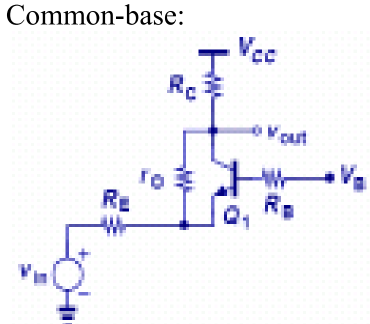


$$\frac{v_{out}}{v_{in}} = \frac{-\beta R_C}{r_\pi + (\beta+1)R_E + R_B}$$

$$R_{in} = r_\pi + (\beta+1)R_E + R_B$$

$$R_{out} = R_C \parallel r_o$$

Common-base:

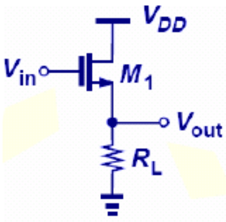


$$A_v = \frac{R_D}{\frac{1}{g_m} + R_S}$$

$$R_{in} = \frac{1}{g_m}$$

$$R_{out} = \left[\left(1 + g_m r_O\right) R_S + r_O\right] \| R_D$$

Common-drain (source follower):



$$A_V = \frac{r_O \| R_L}{\frac{1}{g_m} + r_O \| R_L}$$

$$R_{in} = \infty$$

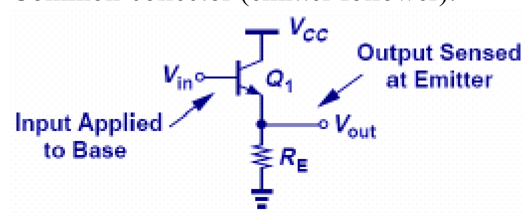
$$R_{out} = \frac{1}{g_m} \| r_O \| R_L$$

$$A_v = \frac{g_m R_C}{1 + R_E\left(g_m + \frac{1}{r_\pi}\right) + \frac{R_B}{r_\pi}}$$

$$R_{in} = \frac{r_\pi + R_B}{1 + g_m r_\pi} + R_E$$

$$R_{out} = R_C \| r_o$$

Common-collector (emitter follower):



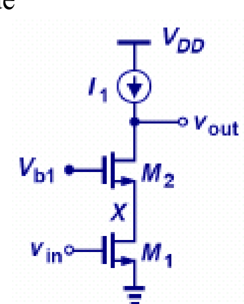
$$\frac{v_{out}}{v_{in}} = \frac{1}{1 + \frac{r_\pi}{\beta + 1} \cdot \frac{1}{R_E}} \approx \frac{R_E}{R_E + \frac{1}{g_m}}$$

$$R_{in} = r_\pi + (\beta + 1) R_E$$

$$R_{out} = R_E \| \frac{1}{g_m} \| r_o$$

Multistage Amplifiers

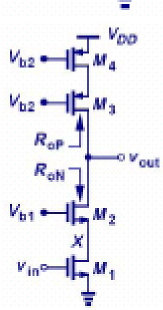
- Cascode



$$A_v = -G_m R_{out}$$

$$A_v \approx -g_{m1}\left[(1 + g_{m2} r_{O2}) r_{O1} + r_{O2}\right]$$

$$A_v \approx -g_{m1} r_{O1} g_{m2} r_{O2}$$

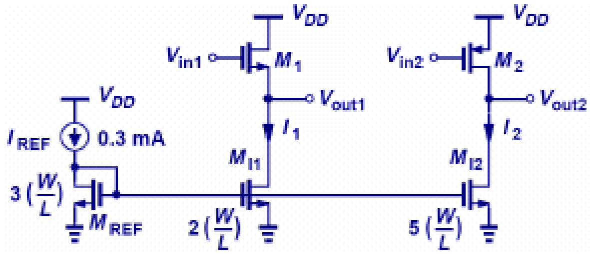


$$R_{on} \approx g_{m2} r_{O2} r_{O1}$$

$$R_{op} \approx g_{m3} r_{O3} r_{O4}$$

$$R_{out} = R_{on} \| R_{op}$$

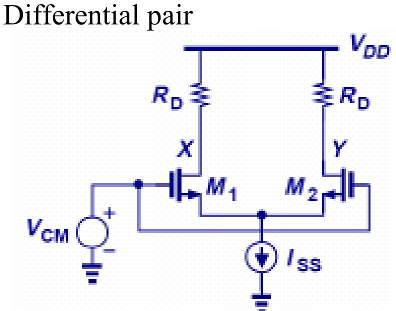
- Current mirror



Current mirrors formed by M_{REF} & M_{I1}, and by M_{REF} & M_{I2}

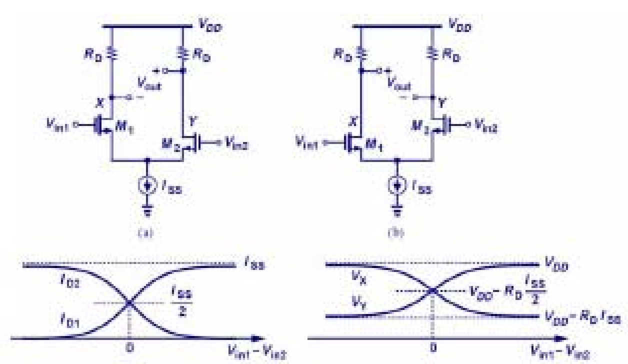
$$I_1 = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_{REF}} I_{REF} \qquad \text{and} \qquad I_2 = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_{REF}} I_{REF}$$

- Differential pair



$$V_X = V_Y = V_{DD} - R_D \frac{I_{SS}}{2}$$

Minimum common-mode output voltage: $V_{DD} - R_D \frac{I_{SS}}{2} > V_{CM} - V_{TH}$

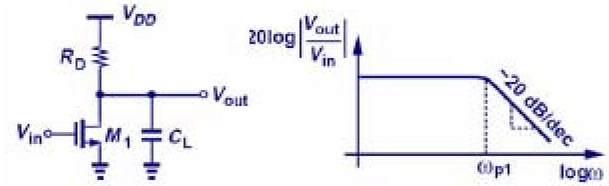


Frequency Response

- Bode plots
  Transfer function: $$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

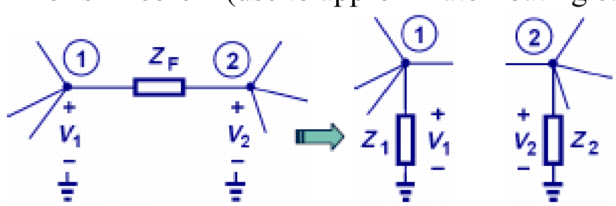
  Magnitude rises with slope of +20 dB/dec after a zero is reached
  Magnitude falls with slope of -20 dB/dec after a pole is reached

  Example Bode plot:



$$|\omega_{p1}| = \frac{1}{R_D C_L}$$

- Miller's Theorem (use to approximate floating capacitors):



$$Z_1 = \frac{Z_F}{1-A_v} \qquad Z_2 = \frac{Z_F}{1-1/A_v}$$

- Intrinsic MOSFET capacitances