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Discussion Notes #5

EE 105 **Spring 2007**
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Finding Rout by inspection. When analyzing circuits, you will often come about similar circuit configurations, such as when finding the output and input resistance. Instead of using KCL and KVL over and over again, it is much smarter to create a general formula that will work for most configurations.

R_d – Resistance looking into the drain

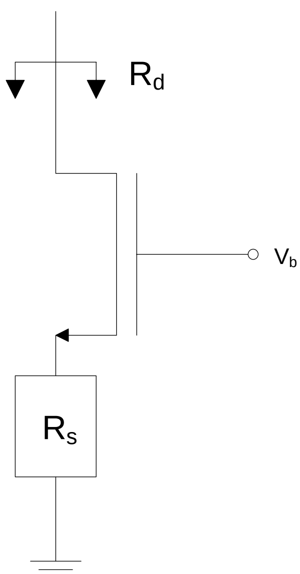


Figure 1: R_d is the resistance looking into the drain of a transistor with a source resistance. The gate of the transistor is grounded.

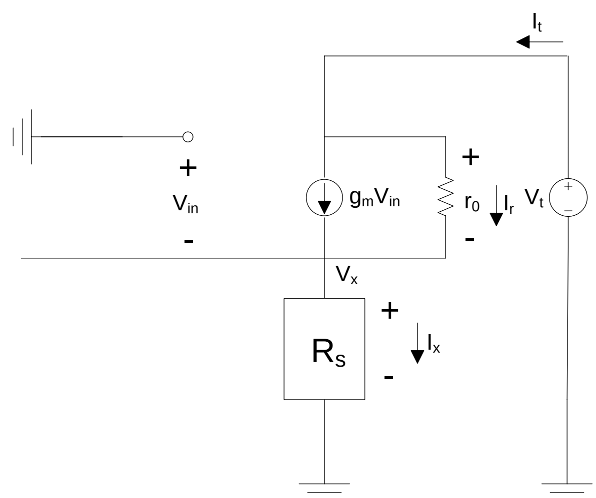


Figure 2: The small signal model of finding R_d, by applying a test voltage at the drain.

Figure 1 shows a typical problem of finding the resistance looking down into the drain of a transistor. The resistor Rs can represent any assortment of passive resistors. For example it could be one resistor, or several in a series and parallel combination, or it can be a chain of transistors. As long as there are no active sources in Rs, our following derivation will hold.

Often times when finding Rd, we deal with small signal analysis, in which the gate of the transistor is grounded. Also, recall that when finding Rd, you must open all sources, including the input. **Figure 2** shows the small signal equivalent of finding Rd, along with the applied test voltage.

We will now derive an equation for Rd, that will hold true given the following assumptions:

1) The transistor's gate is grounded (AC or DC grounded).
2) Source and Bulk are tied together
3) No Active sources
4) Lambda does not equal 0

Looking at figure 2, we can see that

$$\begin{aligned} &V_{in} = -V_x \\ &V_x = i_t R_s \\ &V_{in} = -i_t R_s \\ &and, \\ &I_x = i_t \end{aligned} \tag{1}$$

Applying KCL at node V_x,

$$\begin{aligned} &g_m V_{in} + I_r = i_t \\ &I_r = i_t - g_m V_{in} \\ &I_r = i_t + g_m i_t R_s \end{aligned} \tag{2}$$

Using KVL around the V_t loop and using equation (1) and (2)

$$\begin{aligned} &I_r r_o + R_s i_t = v_t \\ &r_o (i_t + g_m i_t R_s) + R_s i_t = v_t \\ &r_o i_t + g_m i_t R_s r_o + R_s i_t = v_t \end{aligned} \tag{3}$$

$$\boxed{R_d = r_o + g_m R_s r_o + R_s = \frac{v_t}{i_t}}$$

Equation (3) is a generic formula we can use when finding the resistance looking down into the drain of a transistor. Remember, Rs does not have to be a single resistor, it can be replaced with any passive equivalent resistance.

Rs – Resistance Looking into the source

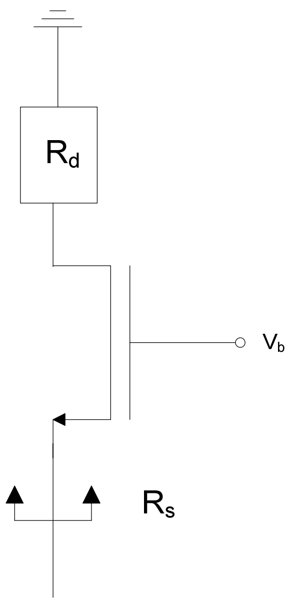


Figure 3: R_s is the resistance looking into the source of a transistor with a source resistance. The gate of the transistor is grounded.

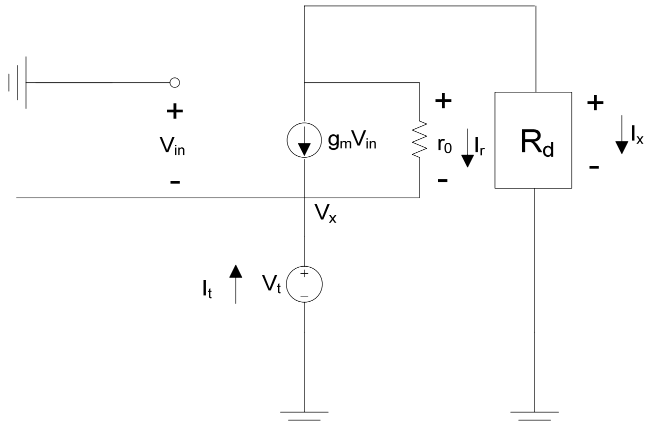


Figure 4: The small signal model of finding R_s, by applying a test voltage at the drain.

The steps for solving Rs are similar to those of finding Rd, but make sure to watch out for how you define the currents and voltages!

From Figure 4, we can see that,

$$\begin{aligned} V_{in} &= -v_t \\ i_t &= I_x \end{aligned} \tag{4}$$

Applying KCL at Vx and using equation (4),

$$\begin{aligned} g_m V_{in} + I_r &= -i_t \\ -g_m v_t + I_r &= -i_t \\ I_r &= g_m v_t - i_t \end{aligned} \tag{5}$$

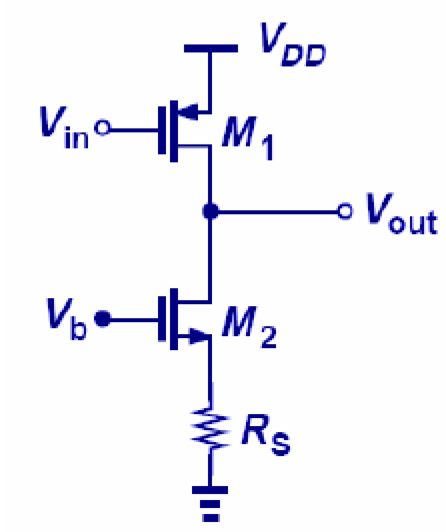
Applying KVL around the loop and using equation (5),

$$\begin{aligned} v_t &= -I_r r_o + R_d i_t \\ vt &= (g_m v_t - i_t) r_o + R_d i_t \end{aligned} \tag{6}$$

$$\boxed{R_s = \frac{v_t}{i_t} = \frac{R_d + r_o}{1 + g_m r_o}}$$

Example Problem

1) Find Rout



To find $R_{out,}$ we must zero out all sources, including the input; this will ground the gate of M_1, which allows us to use the equations derived above. Next, we notice that R_{out} is resistance looking up from V_{out}, R_{up}, in parallel with the resistance looking down, R_{down}. Using the equations we just derived:

$$R_{up} = R_d = r_{o1} + g_{m1} R_s r_{o1} + R_s = r_{o1} \tag{7}$$

Equation (7) proves our assumption that the equivalent resistance of M_1 is r_{o1}. Since V_b is small signal ground, we can still use the above equations for M_2,

$$R_{down} = R_d = r_{o2} + g_{m2} R_s r_{o2} + R_s \tag{8}$$

Thus the total output resistance is,

$$R_{up} \| R_{down} = r_{o2} + g_{m2} R_s r_{o2} + R_s \| r_{o1} \tag{9}$$