UNIVERSITY OF CALIFORNIA AT BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

Discussion Notes #7

EE 105 **Spring 2007**
Prof. Wu

1 Frequency Analysis of a Common Source Amplifier

1.1 Motivation

So far, when discussing gain, we've been talking about DC gain. That means if we slowly apply a small signal v_{in}, we know how to get v_{out} based on the equations we've derived so far, such as $\frac{v_{out}}{v_{in}} = -g_m\left(r_o||r_D\right)$ for a common source amplifier. However, when designing amplifiers, we almost always are dealing with signals with high frequencies.

Consider a typical audio amplifier. The human ear has a range of hearing from about 20Hz to 20kHz. When designing an audio amplifier, we'd like the gain to be constant across this range of frequencies. Otherwise, when listening to a piece of music, we'd have some notes being amplified much more than others, leading to poor sounding music. If all notes are amplified equally (approximately), then our source material will come out as it originally was, just much louder than before, which is exactly what we want from an audio amplifier (though we may use an equalizer to adjust the exact frequency response of our amplification system).

Thus, we have to worry about the poles and zeroes in an amplifier to ensure the frequency response meets the specifications we need for whatever the amplifier is being built for, such as audio signals.

1.2 Derivation

Consider the small signal model of a common source amplifier shown in figure 11.19(c) of Razavi (the one on the right—the left one is for the bipolar equivalent). Although the derivation on page 519 uses the generic terms in Figure 11.19(d), I'm going to use the MOS-specific terms in this document since at this point, we're only concerned with MOS transistors (and it would also be pointless to re-copy the derivation from page 519 if I didn't change something). Also note that we could replace R_L with $(r_o||R_L)$ at any point without changing the validity of the solution—we're leaving out r_o for brevity (you could also assume that $r_o >> R_L$ to justify this choice as well).

Let's start by writing KCL at the gate and at the output node. This gives the following equations:

$$\frac{V_G - V_{out}}{Z_{GD}} + \frac{V_G}{Z_{GS}} + \frac{V_G - V_{in}}{R_S} = 0 \tag{1}$$

$$\frac{V_{out} - V_G}{Z_{GD}} + \frac{V_{out}}{Z_{DB}} + \frac{V_{out}}{R_L} + g_m V_G = 0 \tag{2}$$

Note that $V_{GS} = V_G$ since the source is grounded. We can simplify these equations by noting that $Z_C = \frac{1}{sC}$, where s represents the complex number $s = \sigma + j\omega$ (if you want to think of it in terms of what you learned in EE40, just treat s as if $s = j\omega$). If we plug this into our equations, we get:

$$\left(V_G - V_{out}\right) sC_{GD} + V_G sC_{GS} + \frac{V_G - V_{in}}{R_S} = 0 \tag{3}$$

$$\left(V_{out} - V_G\right) sC_{GD} + V_{out} sC_{DB} + \frac{V_{out}}{R_L} + g_m V_G = 0 \tag{4}$$

We now have a system of two equations with two unknowns: V_G and $\frac{V_{out}}{V_{in}}$. Let's solve this system. We'll start by solving Equation (4) for V_G:

$$V_{out}\left[s\left(C_{GD}+C_{DB}\right)+\frac{1}{R_L}\right]=V_G\left(sC_{GD}-g_m\right) \tag{5}$$

$$\Rightarrow V_G=V_{out}\frac{s\left(C_{GD}+C_{DB}\right)+\frac{1}{R_L}}{sC_{GD}-g_m} \tag{6}$$

Let's also separate V_G to one side of Equation (3):

$$V_G\left[s\left(C_{GD}+C_{GS}\right)+\frac{1}{R_S}\right]=V_{out}s\left(C_{GD}+C_{GS}\right)+\frac{V_{in}}{R_S} \tag{7}$$

Plugging the result from Equation (6) into Equation (7) gives:

$$V_{out}\frac{s\left(C_{GD}+C_{DB}\right)+\frac{1}{R_L}}{sC_{GD}-g_m}\left[s\left(C_{GD}+C_{GS}\right)+\frac{1}{R_S}\right]=V_{out}s\left(C_{GD}+C_{GS}\right)+\frac{V_{in}}{R_S} \tag{8}$$

Dividing both sides by V_{in} gives:

$$\frac{V_{out}}{V_{in}}\frac{s\left(C_{GD}+C_{DB}\right)+\frac{1}{R_L}}{sC_{GD}-g_m}\left[s\left(C_{GD}+C_{GS}\right)+\frac{1}{R_S}\right]=\frac{V_{out}}{V_{in}}s\left(C_{GD}+C_{GS}\right)+\frac{1}{R_S} \tag{9}$$

$$\frac{V_{out}}{V_{in}}\left[\frac{s\left(C_{GD}+C_{DB}\right)+\frac{1}{R_L}}{sC_{GD}-g_m}\left[s\left(C_{GD}+C_{GS}\right)+\frac{1}{R_S}\right]-s\left(C_{GD}+C_{GS}\right)\right]=\frac{1}{R_S} \tag{10}$$

Getting a common denominator on the left gives:

$$\frac{V_{out}}{V_{in}}\frac{1}{sC_{GD}-g_m}\left[\left(s\left(C_{GD}+C_{DB}\right)+\frac{1}{R_L}\right)\left(s\left(C_{GD}+C_{GS}\right)+\frac{1}{R_S}\right)-s^2\left(C_{GD}+C_{GS}\right)\left(C_{GD}-g_m\right)\right]=\frac{1}{R_S} \tag{11}$$

Now we are in a position to solve for V_{out}/V_{in}:

$$\frac{V_{out}}{V_{in}}=\frac{\left(sC_{GD}-g_m\right)R_L}{as^2+bs+1} \tag{12}$$

$$a=R_SR_L\left(C_{GS}C_{GD}+C_{DB}C_{GD}+C_{GS}C_{DB}\right) \tag{13}$$

$$b=\left(1+g_mR_L\right)C_{GD}R_S+R_SC_{GS}+R_L\left(C_{GD}+C_{DB}\right) \tag{14}$$

I've left out some messy algebra that Razavi skips over as well. It should be obvious how to get the solution above, though actually solving for a and b requires multiplying out the terms of Equation (11) and matching a with the s^2 coefficients and b with the s coefficients (after dividing by the constant term to ensure it is 1). At this point we only care about analyzing the result, so let's ignore any further algebra.

First, we can note there is a zero at $\omega_z=\frac{g_m}{C_{GD}}$. Razavi mentions that this is usually at a very high frequency, out of the range that would affect our frequency response. Consider if $g_m\sim10^{-3}$ and $C_{GD}\sim10^{-15}$, then $\omega_z\sim10^{12}$, or in the terahertz range, which is very high frequency.

1.3 Finding the Poles

Since the denominator of the transfer function is quadratic in s, we know we will have 2 poles. Let's call them ω_{p1} and ω_{p2}. We should be able to factor the denominator such that $as^2+bs+1 = \left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)$. Now consider if one pole dominates—that is, assume $\omega_{p1} << \omega_{p2}$. Then we can further simplify this expression to:

$$
\begin{aligned}
as^2+bs+1 &= \left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right) && (15)\\
&= \frac{s^2}{\omega_{p1}\omega_{p2}}+\frac{s}{\omega_{p1}}+\frac{s}{\omega_{p2}}+1 && (16)\\
&\approx \frac{s^2}{\omega_{p1}\omega_{p2}}+\frac{s}{\omega_{p1}}+1 && (17)\\
\Rightarrow b &\approx \frac{1}{\omega_{p1}} && (18)\\
a &\approx \frac{1}{\omega_{p1}\omega_{p2}} \approx \frac{b}{\omega_{p2}} && (19)
\end{aligned}
$$

Now we can solve for the dominant pole, ω_{p1}, and secondary pole, ω_{p2}:

$$
\begin{aligned}
\omega_{p1} &\approx \frac{1}{b} && (20)\\
&\approx \frac{1}{\left(1+g_mR_L\right)C_{GD}R_S+R_SC_{GS}+R_L\left(C_{GD}+C_{DB}\right)} && (21)\\
\omega_{p2} &\approx \frac{b}{a} && (22)\\
&\approx \frac{\left(1+g_mR_L\right)C_{GD}R_S+R_SC_{GS}+R_L\left(C_{GD}+C_{DB}\right)}{R_SR_L\left(C_{GS}C_{GD}+C_{DB}C_{GD}+C_{GS}C_{DB}\right)} && (23)
\end{aligned}
$$

2 Example 11.9

This example is already solved in the book, but it's useful to go over it anyway. We just need to find how C_{GD}, C_{GS}, C_{DB}, and R_L are changed by the new circuit topology. Drawing out all of the parasitic capacitances gives the circuit in Figure 11.21(b) (Razavi asks why C_{SB1}, C_{GS2}, and C_{SB2} don't come into play—the answer: the sources are all connected to the bodies, so C_{SB1} and C_{SB2} are gone; C_{GS2} is attached between two DC voltages V_b and V_{DD}, which both go to AC ground, eliminating that capacitor as well).

Looking at Figure 11.21(b), we can see how the capacitances are transformed. C_{GS} represented the capacitance from the input to ground, so that remains as C_{GS1}. C_{GD} represented the capacitance from input to output, which in this circuit is just C_{GD1}. C_{DB} was the capacitance from the output to ground, which now has 3 parallel capacitances, so C_{DB} is replaced by $C_{DB1}+C_{DB2}+C_{GD2}$.

We can summarize this as follows:

$$
\begin{aligned}
C_{GS} &\longrightarrow C_{GS1}\\
C_{GD} &\longrightarrow C_{GD1}\\
C_{DB} &\longrightarrow C_{DB1}+C_{DB2}+C_{GD2}
\end{aligned}
$$

We also note that in this case, $R_L = (r_{o1}||r_{o2})$. Thus we can find ω_{p1} and ω_{p2} by plugging these new values into Equations (21) and (23).