Discussion Notes #9

EE 105 **Spring 2007**
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1 BJT Amplifiers

Recall from Chapter 7 our discussion of MOSFET amplifiers: common-source, common-gate, and common-drain/source-follower. We can build analogous amplifiers using BJTs as well, namely the common-emitter, common-base, and common-collector/emitter-follower amplifiers. Although the MOS and BJT amplifier topologies share many similarities, there are some important differences. This discussion will go over the BJT amplifier topologies and their properties, emphasizing how they differ from the familiar MOS amplifiers.

1.1 Common-Emitter Amplifier

Consider a common-emitter configuration with a base resistor R_B, collector resistor R_C, and emitter resistor R_E. Let's look the gain, input resistance, and output resistance of this amplifier, ignoring the Early Effect (including the Early Effect results in some horrendous algebra, which is why I'm leaving it out). We'll compare the results to the properties of the MOS common-source amplifier.

1.1.1 Gain

Razavi notes that there are two ways to find the gain in this instance: using KCL on the small signal circuit, or noting that $A_v = \frac{v_{out}}{v_{in}} = \frac{v_B}{v_{in}}\frac{v_{out}}{v_B}$ and finding each stage of the gain independently. We'll use the latter method since it is easier.

First, note from Figure 5.43 that the inclusion of R_B simply adds a resistor in series with r_π and $(\beta+1)\,R_E$ (the original justification for the degenerated resistor having this value is on page 198—basically, the current flowing through R_E is $i_B + g_m v_\pi = i_B\,(1+g_m r_\pi) = (1+\beta)\,i_B$). Thus, using Figure 5.43, we can see that

$$\frac{v_B}{v_{in}} = \frac{r_\pi + (1+\beta)\,R_E}{r_\pi + (1+\beta)\,R_E + R_B} \tag{1}$$

Now we can focus on computing $\frac{v_{out}}{v_B}$ (see page 196 of Razavi). Looking at Figure 5.36, we can write KCL at each node (I'm replacing v_{in} with v_B since we're including the base resistor in our calculation):

$$g_m v_\pi + \frac{v_{out}}{R_C} = 0 \tag{2}$$

$$-\frac{v_\pi}{r_\pi} + \frac{v_B - v_\pi}{R_E} - g_m v_\pi = 0 \tag{3}$$

Rearranging (3) gives

$$\frac{v_B}{R_E} = v_\pi\left(\frac{1}{r_\pi} + \frac{1}{R_E} + g_m\right) \tag{4}$$

$$v_\pi = \frac{v_{in}}{1 + \frac{R_E}{r_\pi} + g_m R_E} \tag{5}$$

Plugging (5) into (2) gives

$$g_m \frac{v_B}{1 + \frac{R_E}{r_\pi} + g_m R_E} + \frac{v_{out}}{R_C} = 0 \tag{6}$$

$$\frac{v_{out}}{v_B} = -\frac{g_m R_C}{1 + R_E \left(\frac{1}{r_\pi} + g_m \right)} \tag{7}$$

Now we can multiply (1) and (7) to get the total gain (see page 202 for further reduction of this expression)

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{r_\pi + (1+\beta) R_E}{r_\pi + (1+\beta) R_E + R_B} \cdot \frac{g_m R_C}{1 + R_E \left(\frac{1}{r_\pi} + g_m \right)} \tag{8}$$

$$= -\frac{g_m R_C}{1 + R_E \left(g_m + \frac{1}{r_\pi} \right) + \frac{R_B}{r_\pi}} \tag{9}$$

I want to compare this to our calculations for the MOS common source amplifier. Recall that for a degenerated CS stage, our gain was (ignoring channel length modulation)

$$A_v = -\frac{g_m R_D}{1 + g_m R_S} \tag{10}$$

Note the differences between (9) and (10). The most glaring difference is that a resistor at the gate of a MOSFET does not affect the gain at all. We don't have that voltage divider expression you see in (8) for the MOS transistor. Also, note that even if we set $R_B = 0$, we still have some differences in the gain expression. Note how r_π, the input resistance of the BJT, attenuates the gain in (8) by a small amount. Since a MOSFET has infinite resistance at the gate (i.e. $r_\pi \to \infty$ for a MOSFET), we don't see that effect. Other than these two differences, though, the expressions are largely the same.

1.1.2 Input Resistance

Using Figure 5.43, we can see the input resistance for a common-emitter with a base resistor is just three series resistors, so

$$R_{in} = R_B + r_\pi + (1+\beta) R_E \tag{11}$$

Compare this to a common-source amplifier, which has $R_{in} = \infty$. Since a high input resistance is desirable for a voltage amplifier, we often use a base resistor on a BJT to improve the input resistance at the cost of gain. This is unnecessary with a common-source amplifier.

1.1.3 Output Resistance

The output resistance of the common-emitter amplifier is just $R_{out} = R_C$. To see this, consider Figure 5.40 on page 199 of Razavi. Let's add R_B in series with r_π just for completeness.

There are two possible cases to consider: $v_\pi > 0$, or $v_\pi < 0$. Let's consider first if $v_\pi > 0$. Since g_m must be positive, the current is flowing such that v_E must be positive. But that means v_π would have to be negative, contradicting our original assumption. Therefore v_π cannot be greater than 0.

Now let's assume that $v_\pi < 0$. That means the current is flowing such that $v_E < 0$. But if that's true, then v_π must be greater than zero, again contradicting our assumption. So if v_π can't be larger than zero and it can't be smaller than zero, then it must be zero (thinking through the case where $v_\pi = 0$ leads to no contradictions). That means all of the current from our test source flows through R_C, meaning the output resistance is R_C.

This is exactly analogous to the result for a common-source amplifier, which has $R_{out} = R_D$.

1.2 Common-Base Amplifier

Let's find the gain, input resistance, and output resistance of a common-base amplifier with base resistor R_B, collector resistor R_C, and emitter resistor R_E. We'll compare the results to the properties of a common-gate amplifier.

1.2.1 Gain

Figure 5.72 shows the small signal model for this circuit. Let's write KCL at each node:

$$g_m v_\pi + \frac{v_{out}}{R_C} = 0 \tag{12}$$

$$-g_m v_\pi + \frac{v_E}{r_\pi + R_B} + \frac{v_E - v_{in}}{R_E} = 0 \tag{13}$$

We can rearrange (12) to get

$$v_\pi = -\frac{v_{out}}{g_m R_C} \tag{14}$$

$$v_E = -\frac{v_\pi}{r_\pi}\left(r_\pi + R_B\right) \tag{15}$$

$$= \frac{v_{out}}{g_m r_\pi R_C}\left(r_\pi + R_B\right) \tag{16}$$

$$= \frac{v_{out}}{\beta R_C}\left(r_\pi + R_B\right) \tag{17}$$

We can plug (14) and (17) into (13) to get

$$g_m \frac{v_{out}}{g_m R_C} + \frac{\frac{v_{out}}{\beta R_C}\left(r_\pi + R_B\right)}{r_\pi + R_B} + \frac{\frac{v_{out}}{\beta R_C}\left(r_\pi + R_B\right) - v_{in}}{R_E} = 0 \tag{18}$$

$$\frac{v_{out}}{R_C} + \frac{v_{out}}{\beta R_C} + \frac{\frac{v_{out}}{\beta R_C}\left(r_\pi + R_B\right) - v_{in}}{R_E} = 0 \tag{19}$$

$$v_{out}\left[\frac{1}{R_C} + \frac{1}{\beta R_C} + \frac{r_\pi + R_B}{\beta R_E R_C}\right] = v_{in}\frac{1}{R_E} \tag{20}$$

$$v_{out}\left[\frac{\left(1+\beta\right)R_E + r_\pi + R_B}{\beta R_E R_C}\right] = v_{in}\frac{1}{R_E} \tag{21}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{\beta R_C}{\left(1+\beta\right)R_E + r_\pi + R_B} \tag{22}$$

Let's divide top and bottom by r_π to get an expression similar to (9) from our analysis of the CE amplifier:

$$A_v = \frac{v_{out}}{v_{in}} = \frac{g_m R_C}{1 + R_E\left(g_m + \frac{1}{r_\pi}\right) + \frac{R_B}{r_\pi}} \tag{23}$$

Note that this is exactly (9) except without the minus sign. Now recall the gain of a common-gate stage:

$$A_v = \frac{g_m R_D}{1 + g_m R_S} \tag{24}$$

The analogy is clear: the common-gate gain is also just the negative of the common-source gain. Therefore, the same discussion about the impact of R_B applies here as well.

1.2.2 Input Resistance

We can compute the input resistance from Figure 5.74 (note that we can simply add R_E in series with the result computed from Figure 5.74, since it excludes R_E). Let's begin with KCL:

$$\frac{v_\pi}{r_\pi} + i_X + g_m v_\pi = 0 \tag{25}$$

We can also relate v_π and v_X since R_B and r_π form a voltage divider:

$$v_\pi = -\frac{r_\pi}{r_\pi + R_B} v_X \tag{26}$$

Plugging (26) into (25) gives

$$-\frac{\frac{r_\pi}{r_\pi+R_B} v_X}{r_\pi} + i_X + g_m \frac{r_\pi}{r_\pi + R_B} v_X = 0 \tag{27}$$

$$-\frac{v_X}{r_\pi + R_B} + i_X + g_m \frac{r_\pi}{r_\pi + R_B} v_X = 0 \tag{28}$$

$$v_X \left[\frac{1 + g_m r_\pi}{r_\pi + R_B}\right] = -i_X \tag{29}$$

$$R_{in} = \frac{v_X}{i_X} = \frac{r_\pi + R_B}{1 + g_m r_\pi} \tag{30}$$

Compare this to the input resistance for a common-gate stage: $R_{in} = \frac{1}{g_m}$. If we let $r_\pi \to \infty$, then the common-base result reduces to $\frac{1}{g_m}$. Since a MOSFET is similar to a BJT with r_π infinite, this makes intuitive sense. Note also that a resistor at the gate of a MOSFET would not affect the input resistance of a common-gate amplifier like the base resistance affects the input resistance here.

1.2.3 Output Resistance

The output resistance of the common-base stage is identical to that of the common-emitter stage: $R_{out} = R_C$. Note that when computing output resistance, we zero the input, so the resulting small signal circuits look identical. Recall that similarly, the common-gate and common-source stages also have identical output resistances.

1.3 Common-Collector Amplifier

For our analysis of the common-collector amplifier (or emitter-follower), we'll neglet R_B and R_C and include only R_E at the emitter.

1.3.1 Gain

Let's write KCL for Figure 5.85.

$$-g_m v_\pi - \frac{v_\pi}{r_\pi} + \frac{v_{out}}{R_E} = 0 \quad (31)$$

$$v_\pi = \frac{r_\pi}{1+\beta} \cdot \frac{v_{out}}{R_E} \quad (32)$$

$$v_{in} = v_{out} + v_\pi \quad (33)$$

$$= v_{out}\left[1 + \frac{r_\pi}{(1+\beta)\,R_E}\right] \quad (34)$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{(1+\beta)\,R_E}{r_\pi + (1+\beta)\,R_E} \quad (35)$$

$$\approx \frac{g_m R_E}{1 + g_m R_E} \quad (36)$$

As you can see, the gain for the emitter-follower is almost identical to that of the source-follower. Assuming $g_m R_E >> 1$, the gain is approximately unity.

1.3.2 Input Resistance

I'm going to skip this derivation (it's on page 233 of Razavi) and just cite the result: $R_{in} = r_\pi + (1+\beta)\,R_E$. Note that this is identical to the input resistance of the common-emitter amplifier we derived in (11) (minus R_B, which we excluded for this analysis).

1.3.3 Output Resistance

The output resistance is derived on page 235 of Razavi as $R_{out} = \left(\frac{R_B}{1+\beta} + \frac{1}{g_m}\right)||R_E$ (Razavi adds the base resistor in for this analysis). Note that if $r_\pi \to \infty$ (which sends β to ∞ as well), this reduces to $R_{out} = \frac{1}{g_m}||R_E$, which is identical to the result for a source-follower.