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Final Exam Solutions

EE 105 **Spring 2007**
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1. (a) (B) holes
   (b) (A) electrons
   (c) (A) electrons
   (d) (B) holes
   (e) $V_{FB} = \phi_p - \phi_{p+} = (60m\mathrm{V}) \log \frac{N_a}{n_i} + 550m\mathrm{V} = -300m\mathrm{V} + 550m\mathrm{V} = \boxed{250m\mathrm{V}}$
   (f)

$$V_{ox} = \frac{1}{C_{ox}}\sqrt{2q\epsilon_s N_a\left(-2\phi_p\right)} \tag{1}$$
$$= \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2q\epsilon_s N_a\left(-2\phi_p\right)} \tag{2}$$
$$= \frac{10n\mathrm{m}}{3.9\epsilon_0}\sqrt{2q\epsilon_s 10^{15}\left(600m\mathrm{V}\right)} \tag{3}$$
$$= \boxed{40.88m\mathrm{V}} \tag{4}$$

   (g) $R = \frac{V_T}{10m\mathrm{A}} = \boxed{2.6\Omega}$
   (h) Recall the equation for the junction capacitance of a PN junction:

$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_{bi}}}} \tag{5}$$

So the largest capacitance will occur with the smallest reverse bias V_D. Thus, the ratio is as follows:

$$\frac{C_{j,max}}{C_{j,min}} = \frac{C_j(-1)}{C_j(-10)} \tag{6}$$
$$= \frac{C_{j0}}{\sqrt{1 + \frac{1}{\phi_{bi}}}}\frac{\sqrt{1 + \frac{10}{\phi_{bi}}}}{C_{j0}} \tag{7}$$
$$= \frac{\sqrt{1 + \frac{10}{\phi_{bi}}}}{\sqrt{1 + \frac{1}{\phi_{bi}}}} \tag{8}$$
$$= \sqrt{\frac{1 + \frac{10}{720m\mathrm{V}}}{1 + \frac{1}{720m\mathrm{V}}}} \tag{9}$$
$$= \boxed{2.497} \tag{10}$$

2. (a)

$$V_X = V_{CC} - V_{EB,3} \tag{11}$$
$$= 3 - V_T \ln \frac{I_{REF}}{I_{S,3}} \tag{12}$$
$$= \boxed{2.341\text{V}} \tag{13}$$

(b) Since the emitter area of Q_2 is 10 times more than that of Q_3, the collector current of Q_2 is also 10 times larger. This means the collector current of Q_1 is also 10 times larger, so $I_{C,1} = 1m\text{A}$.

$$g_{m,1} = \frac{I_C}{V_T} = \boxed{38.46m\text{S}} \tag{14}$$
$$r_{\pi,1} = \frac{\beta}{g_{m,1}} = \boxed{2.6k\Omega} \tag{15}$$
$$r_{o,1} = \frac{V_A}{I_C} = \boxed{50k\Omega} \tag{16}$$

(c) Noting that $r_{o2} = r_{o1}$, we have:

$$A_v = -\frac{g_{m,1}\left(r_{o,2}||R_L\right)}{1 + R_E\left(g_{m,1} + \frac{1}{r_{\pi,1}}\right) + \frac{R_S}{r_{\pi,1}}} \tag{17}$$
$$= -\frac{g_{m,1}\left(r_{o,1}||R_L\right)}{1 + R_E\left(g_{m,1} + \frac{1}{r_{\pi,1}}\right) + \frac{R_S}{r_{\pi,1}}} \tag{18}$$
$$= \boxed{-31.87} \tag{19}$$

(d) $R_{in} = R_S + r_{\pi,1} + (1+\beta)\,R_E = \boxed{104.6k\Omega}$

(e)

$$I_{REF} = I_{C,3} + I_{B,3} + I_{B,2} \tag{20}$$
$$= \left(1 + \frac{1}{\beta}\right) I_{C,3} + \frac{1}{\beta} I_{C,2} \tag{21}$$
$$= \left(1 + \frac{1}{\beta} + \frac{10}{\beta}\right) I_{C,3} \tag{22}$$
$$I_{C,3} = 90.09\mu\text{A} \tag{23}$$
$$I_{C,1} = I_{C,2} = 10 I_{C,3} = \boxed{900.9\mu\text{A}} \tag{24}$$
$$g'_{m,1} = 34.65m\text{S} \tag{25}$$
$$r'_{\pi,1} = 2.886k\Omega \tag{26}$$
$$r'_{o,1} = 55.5k\Omega \tag{27}$$
$$A'_v = \boxed{-34.03} \tag{28}$$
$$\text{Error} = \frac{A'_v - A_v}{A'_v} \tag{29}$$
$$= 0.06351 \tag{30}$$
$$= \boxed{6.351\%} \tag{31}$$

3. (a)

$$V_Y = V_T \ln \frac{I_{REF}}{I_{S,3}} = \boxed{0.6585\text{V}} \tag{32}$$

$$V_X = V_Y + V_{GS,7} \tag{33}$$

$$= V_Y + \sqrt{\frac{2I_{REF}}{(W/L)_7\, \mu_n C_{ox}}} + V_{Tn} \tag{34}$$

$$= \boxed{1.512\text{V}} \tag{35}$$

(b)

$$\frac{A_{E,2}}{A_{E,3}} = \boxed{4} \tag{36}$$

$$A_{E,2} = 4A_{E,3} \tag{37}$$

$$= \boxed{16\mu m^2} \tag{38}$$

(c)

$$R_{out} = R_{up} || R_{down} \tag{39}$$

$$R_{up} = g_{m,4} r_{o,4} r_{o,1} + r_{o,1} + r_{o,4} \tag{40}$$

$$R_{down} = g_{m,5} r_{o,5} r_{o,2} + r_{o,2} + r_{o,5} \tag{41}$$

$$g_{m,4} = \sqrt{2\,(W/L)_4\, \mu_p C_{ox} I_D} \tag{42}$$

$$= \sqrt{2\,(W/L)_4\, \mu_p C_{ox} 4I_{REF}} \tag{43}$$

$$= 3.2m\text{S} \tag{44}$$

$$g_{m,5} = \sqrt{2\,(W/L)_5\, \mu_n C_{ox} 4I_{REF}} \tag{45}$$

$$= 2.263m\text{S} \tag{46}$$

$$r_{o,4} = r_{o,5} = \frac{1}{\lambda I_D} \tag{47}$$

$$= \frac{1}{\lambda 4I_{REF}} \tag{48}$$

$$= 50k\Omega \tag{49}$$

$$r_{o,1} = r_{o,2} = \frac{V_A}{I_C} \tag{50}$$

$$= \frac{V_A}{4I_{REF}} \tag{51}$$

$$= 25k\Omega \tag{52}$$

$$R_{out} = \boxed{1.695M\Omega} \tag{53}$$

4. (a)

$$\frac{1}{2}\left(\frac{W}{L}\right)_1 \mu_p C_{ox}\left(V_{SG,1}+V_{Tp}\right)^2\left(1+\lambda V_{SD,1}\right)=\frac{1}{2}\left(\frac{W}{L}\right)_2 \mu_n C_{ox}\left(V_{GS,2}-V_{Tn}\right)^2\left(1+\lambda V_{DS,2}\right) \tag{54}$$

$$V_{GS,2} = V_B \tag{55}$$

$$V_{SG,1} = V_{DD} - V_B \tag{56}$$

$$V_{DS,2} = V_{out} = 1\text{V} \tag{57}$$

$$V_{SD,1} = V_{DD} - V_{out} = 1\text{V} \tag{58}$$

$$\left(1.5 - V_B\right)^2(1+\lambda) = 2\left(V_B - 0.5\right)^2(1+\lambda) \tag{59}$$

$$\left(1.5 - V_B\right)^2 = 2\left(V_B - 0.5\right)^2 \tag{60}$$

$$1.5 - V_B = \pm\sqrt{2}\left(V_B - 0.5\right) \tag{61}$$

$$1.5 - V_B = \pm\sqrt{2}V_B \mp 0.5\sqrt{2} \tag{62}$$

$$1.5 \pm 0.5\sqrt{2} = x\left(1 \pm \sqrt{2}\right) \tag{63}$$

$$V_B = \frac{1.5 \pm 0.5\sqrt{2}}{1 \pm \sqrt{2}} \tag{64}$$

$$= 0.9142\text{V or } -1.914\text{V} \tag{65}$$

$$V_B = \boxed{0.9142\text{V}} \tag{66}$$

(b) This is was solved in the homework, but even if you didn't remember you could easily re-derive it from the small-signal model.

$$A_v = -\left(g_{m,1} + g_{m,2}\right)\left(r_{o,1}||r_{o,2}||R_L\right) \tag{67}$$

$$g_{m,1} = (W/L)_1\, \mu_p C_{ox}\left(V_{SG,1} + V_{Tp}\right)\left(1 + \lambda V_{SD,2}\right) = 3.075m\text{S} \tag{68}$$

$$g_{m,2} = (W/L)_2\, \mu_n C_{ox}\left(V_{GS,1} - V_{Tn}\right)\left(1 + \lambda V_{DS,1}\right) = 4.349m\text{S} \tag{69}$$

$$I_D = \frac{1}{2}\left(\frac{W}{L}\right)_2 \mu_n C_{ox}\left(V_{GS,1} - V_{Tn}\right)^2\left(1 + \lambda V_{DS,1}\right) = 900.8\mu\text{A} \tag{70}$$

$$r_{o,1} = \frac{1}{\lambda I_D} = 22.20k\Omega \tag{71}$$

$$r_{o,2} = 22.20k\Omega \tag{72}$$

$$A_v = \boxed{-6.811} \tag{73}$$

(c)

$$\omega_{p,in} = \frac{1}{R_S\left[2C_{GS} + 2C_{GD}\left(1 - A_v\right)\right]} \tag{74}$$

$$= \boxed{8.467\text{Grad/s}} \tag{75}$$

$$\omega_{p,out} = \frac{1}{\left(r_{o,1}||r_{o,2}||R_L\right)\left[2C_{DB} + 2C_{GD}\left(1 - 1/A_v\right)\right]} \tag{76}$$

$$= \boxed{66.19\text{Grad/s}} \tag{77}$$

$$\omega_{3dB} = min\left(\omega_{p,in}, \omega_{p,out}\right) \tag{78}$$

$$= 8.467\text{Grad/s} \tag{79}$$

$$= \boxed{1.348\text{GHz}} \tag{80}$$

5. (a) We ignore channel length modulation for parts of this problem not involving small-signal parameters.

$$I_{D,1} = 2I_{BIAS} = \boxed{200\mu\text{A}} \tag{81}$$
$$I_{D,3} = 10I_{BIAS} = \boxed{1m\text{A}} \tag{82}$$

(b)

$$\begin{aligned} A_v &= \left[-g_{m,1}\left(r_{o,1}||r_{o,2}\right)\right]\left[-g_{m,3}\left(r_{o,3}||r_{o,4}\right)\right] && (83)\\ &= g_{m,1}g_{m,3}\left(r_{o,1}||r_{o,2}\right)\left(r_{o,3}||r_{o,4}\right) && (84)\\ g_{m,1} &= \sqrt{2\left(W/L\right)_1 \mu_p C_{ox} I_{D,1}} = 632.5\mu\text{S} && (85)\\ g_{m,3} &= \sqrt{2\left(W/L\right)_3 \mu_n C_{ox} I_{D,3}} = 4.472m\text{S} && (86)\\ r_{o,1} &= \frac{1}{\lambda I_{D,1}} = 100k\Omega && (87)\\ r_{o,2} &= r_{o,1} = 100k\Omega && (88)\\ r_{o,3} &= 20k\Omega && (89)\\ r_{o,4} &= r_{o,3} = 20k\Omega && (90)\\ A_v &= \boxed{1,414} && (91)\\ R_{out} &= r_{o,3}||r_{o,4} && (92)\\ &= \boxed{10k\Omega} && (93) \end{aligned}$$

(c) For large R_L (ten times $r_{o,3}$ and $r_{o,4}$), a negligible amount of current will flow through the load. We can do our normal analysis using V_{dsat}:

$$V_{out,min} = V_{dsat,3} = \sqrt{\frac{2I_{D,3}}{\left(W/L\right)_3 \mu_n C_{ox}}} = \boxed{447.2m\text{V}} \tag{94}$$

$$V_{out,max} = V_{DD} - V_{dsat,4} = 3 - \sqrt{\frac{2I_{D,4}}{\left(W/L\right)_4 \mu_p C_{ox}}} = \boxed{2.368\text{V}} \tag{95}$$

(d) For small R_L (100 times smaller than $r_{o,3}$ and $r_{o,4}$), we have to worry about being able to supply enough current to the load if we swing too high or too low. We need to calculate the maximum and minimum currents we can supply. This is fixed by $V_{GS,3}$, since $V_{SG,4}$ is already fixed by the current mirror.

$$I_{D3,max} = \frac{1}{2}\left(\frac{W}{L}\right)_3 \mu_n C_{ox}\left(V_{GS3,max} - V_{Tn}\right)^2 \tag{96}$$

$$V_{GS3,max} = V_{DD} - V_{dsat,1} = 3 - \sqrt{\frac{2I_{D,1}}{(W/L)_1\, \mu_p C_{ox}}} = 2.368\text{V} \tag{97}$$

$$I_{D3,max} = 17.44m\text{A} \tag{98}$$

$$I_{D3,min} = \frac{1}{2}\left(\frac{W}{L}\right)_3 \mu_n C_{ox}\left(V_{GS3,min} - V_{Tn}\right)^2 \tag{99}$$

$$V_{GS3,min} = V_{dsat,2} = \sqrt{\frac{2I_{D,2}}{(W/L)_2\, \mu_n C_{ox}}} = 447.2m\text{V} < V_{Tn} \Rightarrow V_{GS3,min} = V_{Tn} = 0.5\text{V} \tag{100}$$

$$V_{GS3,min} = V_{Tn} = 0.5\text{V} \tag{101}$$

$$I_{D3,min} = 0 \tag{102}$$

$$I_{out,max} = I_{D,4} - I_{D3,min} = 1m\text{A} \tag{103}$$

$$I_{out,min} = I_{D,4} - I_{D3,max} = 1m\text{A} - 17.44m\text{A} = -16.44m\text{A} \tag{104}$$

$$V_{out,max} = I_{out,max} R_L = \boxed{0.1\text{V}} \tag{105}$$

$$V_{out,min} = I_{out,min} R_L = \boxed{-1.644\text{V}} \tag{106}$$

Note that these are AC values.