EE105 - Spring 2007 Microelectronic Devices and Circuits

Lecture 3
PN Junctions

Carrier Concentration and Potential

- At thermal equilibrium, there are no external bias and we expect both the electron and the hole current densities to be zero:

$$J_n = 0 = qn_0\mu_n E_0 + qD_n \frac{dn_o}{dx}$$

$$\frac{dn_o}{dx} = -\left(\frac{\mu_n}{D_n}\right) n_o E_0 = -\left(\frac{q}{kT}\right) n_o \left(-\frac{d\phi_0}{dx}\right)$$

$$d\phi_0 = \left(\frac{kT}{q}\right)\frac{dn_o}{n_0} = V_{th}\frac{dn_0}{n_0}$$

Carrier Concentration and Potential (2)

- If we integrate the above equation we have

$$\phi_0(x) - \phi_0(x_0) = V_{th} \ln \frac{n_0(x)}{n_0(x_0)}$$

- We define the potential reference at intrinsic Si:

$$\phi_0(x_0) = 0 \qquad n_0(x_0) = n_i$$

$$\phi_0(x) = V_{th} \ln \frac{n_0(x)}{n_i}$$

Carrier Concentration Versus Potential

- The carrier concentration is thus a function of potential

$$n_0(x) = n_i e^{\phi_0(x)/V_{th}}$$

- If we do a similar calculation for holes, we arrive at a similar equation

$$p_0(x) = n_i e^{-\phi_0(x)/V_{th}}$$

- Note that the law of mass action is upheld

$$n_0(x)p_0(x) = n_i^2 e^{-\phi_0(x)/V_{th}} e^{\phi_0(x)/V_{th}} = n_i^2$$

Doping Changes Potential

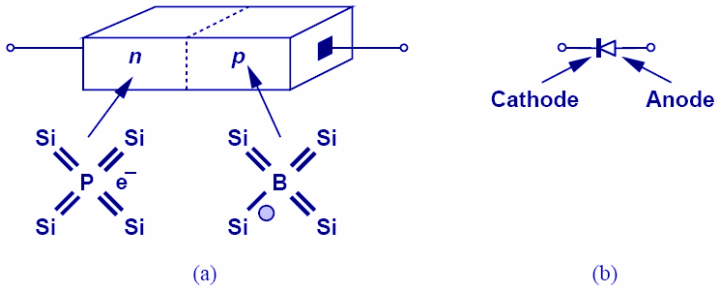
$$\phi_0(x) = V_{th} \ln \frac{n_0(x)}{n_i(x_0)} = 26\text{mV} \ \ln \frac{n_0(x)}{n_i(x_0)} \approx 26\text{mV} \cdot (\ln 10) \cdot \log \frac{n_0(x)}{10^{10}}$$

$$\phi_0(x) \approx 60\text{mV} \ \log \frac{n_0(x)}{10^{10}}$$

$$\phi_0(x) \approx -60\text{mV} \ \log \frac{p_0(x)}{10^{10}}$$

- Due to the log nature of the potential, the potential changes linearly for exponential increase in doping:
- Quick calculation aid:
  - For a p-type concentration of 10^{16} cm^{-3}, the potential is -360 mV
  - N-type materials have a positive potential with respect to intrinsic Si

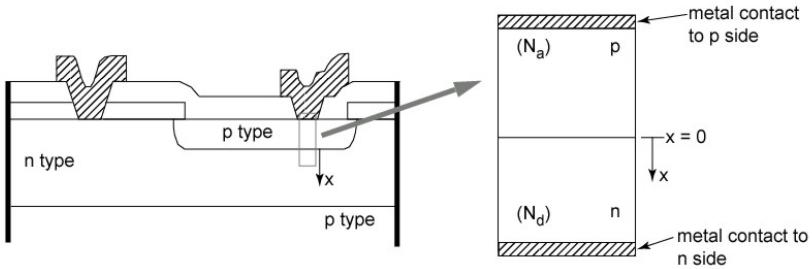
PN Junction (Diode)



- When N-type and P-type dopants are introduced side-by-side in a semiconductor, a PN junction or a diode is formed.

PN Junction: Overview

- Present in most IC structures

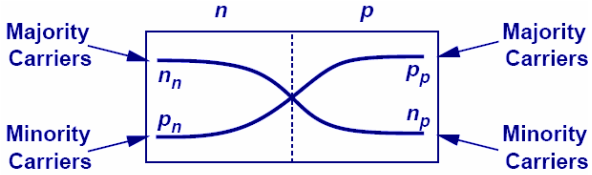


Diode's Three Operation Regions

PN Junction in Equilibrium		PN Junction Under Reverse Bias		PN Junction Under Forward Bias
• Depletion Region • Built−in Potential	⇨	• Junction Capacitance	⇨	• I/V Characteristics

- In order to understand the operation of a diode, it is necessary to study its three operation regions: equilibrium, reverse bias, and forward bias.

Current Flow Across Junction: Diffusion



n_n : Concentration of electrons on n side

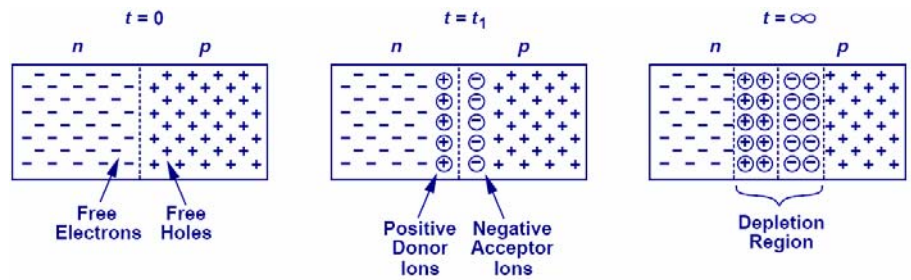
p_n : Concentration of holes on n side

p_p : Concentration of holes on p side

n_p : Concentration of electrons on p side

- Because each side of the junction contains an excess of holes or electrons compared to the other side, there exists a large concentration gradient. Therefore, a diffusion current flows across the junction from each side.

Depletion Region



- As free electrons and holes diffuse across the junction, a region of fixed ions is left behind. This region is known as the "depletion region."

Depletion Approximation

- Let's assume that the depletion region is completely free of electrons or holes (only immobile ions exist)
- Then the charge density is given by

$$\rho_0(x) \cong \begin{cases} +qN_d & -x_{no} < x < 0 \\ -qN_a & 0 < x < x_{p0} \end{cases}$$

- Electric field can be calculated by Gauss Law

$$\frac{dE_0}{dx} = \frac{\rho_0(x)}{\varepsilon_s}$$

$$E_0(x) = \int_{-x_{n0}}^{x} \frac{\rho_0(x')}{\varepsilon_s} dx' + \cancel{E_0(-x_{n0})}$$

There is no field outside depletion region

Depletion Approximation (2)

- Since charge density is a constant

$$E_0(x) = \int_{-x_{n0}}^{x} \frac{\rho_0(x')}{\varepsilon_s} dx' = \frac{qN_d}{\varepsilon_s}(x + x_{no})$$

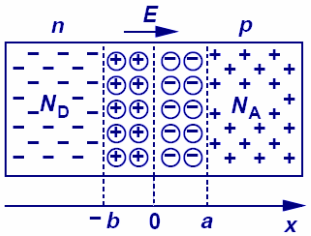
- If we start from the P-side we get the following result

$$\cancel{E_0(x_{n0})} = \int_{x}^{x_{p0}} \frac{\rho_0(x')}{\varepsilon_s} dx' + E_0(x) = \frac{-qN_a}{\varepsilon_s}(x_{p0} - x) + E_0(x)$$

Field zero outside depletion region

$$E_0(x) = \frac{qN_a}{\varepsilon_s}(x_{p0} - x)$$

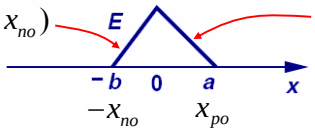
Electric Field Distribution



$$qN_a x_{po} = qN_d x_{no}$$

$$E_0(x) = \frac{qN_d}{\varepsilon_s}(x + x_{no})$$

$$E_0(x) = \frac{qN_a}{\varepsilon_s}(x_{p0} - x)$$

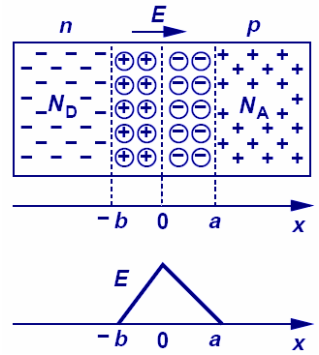


- E-Field is zero outside depletion region
- The depletion widths on N- and P-side could be asymmetric
- Higher doping → Narrower depletion width
- Peak E-Field occurs at junction

Direction of Current Components Across Junction

$$J_n = qn\mu_n E_0 + qD_n \frac{dn}{dx}$$

$$J_p = qp\mu_p E_0 - qD_p \frac{dp}{dx}$$



- **Electron Drift Current** →
- **Electron Diffusion Current** ←
- **Hole Drift Current** →
- **Hole Diffusion Current** ←

Potential Across Junction

- The potential in the N-region is higher than P-region
- The potential has to smoothly transition form high to low in crossing the junction
- Physically, the potential difference is due to the charge transfer that occurs due to the concentration gradient
- We can integrate the field to get the potential:

$$E(x) = -\frac{d\phi(x)}{dx}$$

$$\phi(x) = \phi(-x_{no}) - \int_{-x_{n0}}^{x} \frac{qN_d}{\varepsilon_s}(x' + x_{no})dx'$$

$$\phi(x) = \phi_n - \frac{qN_a}{\varepsilon_s}\left(\frac{x'^2}{2} + x' x_{no}\right)\Bigg|_{-x_{n0}}^{x} = \phi_n - \frac{qN_a}{2\varepsilon_s}(x + x_{no})^2$$

Potential Across Junction

- Potential on N-side (parabolic)

$$\phi_n(x) = \phi_n - \frac{qN_d}{2\varepsilon_s}(x + x_{n0})^2$$

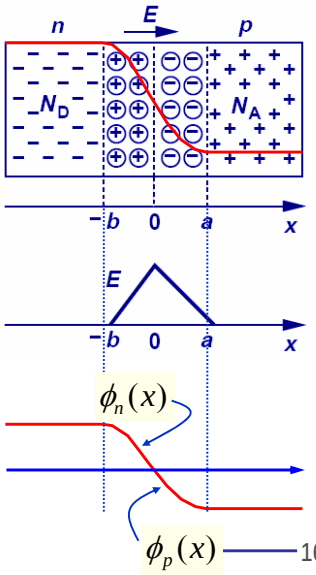
- Do integral on P-side

$$\phi_p(x) = \phi_p + \frac{qN_a}{2\varepsilon_s}(x - x_{p0})^2$$

- Potential must be continuous at interface

$$\phi_n(0) = \phi_n - \frac{qN_d}{2\varepsilon_s}x_{n0}^2$$

$$= \phi_p + \frac{qN_a}{2\varepsilon_s}x_{p0}^2 = \phi_p(0)$$



Solve for Depletion Widths

- We have two equations and two unknowns. We are finally in a position to solve for the depletion depths

$$\phi_n - \frac{qN_d}{2\varepsilon_s} x_{n0}^2 = \phi_p + \frac{qN_a}{2\varepsilon_s} x_{p0}^2 \qquad (1)$$

$$qN_a x_{po} = qN_d x_{no} \qquad (2)$$

$$x_{no} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} \qquad x_{po} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{qN_a}\left(\frac{N_d}{N_d + N_a}\right)}$$

$$\phi_{bi} \equiv \phi_n - \phi_p > 0$$

Total Depletion Width

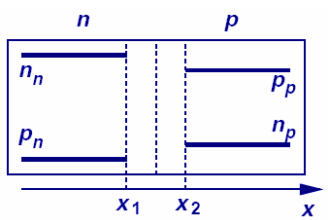
- Total depletion width

$$X_{d0} = x_{p0} + x_{n0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

- Due to high electric field, carriers move across region at saturation velocity
- Typical numbers:

$$X_{d0} = \sqrt{\frac{2\varepsilon_s \phi_{bi}}{q}\left(\frac{1}{10^{15}}\right)} \approx 1\mu \qquad E_{pn} \approx \frac{1\text{V}}{1\mu} = 10^4 \frac{\text{V}}{\text{cm}}$$

Current Flow Across Junction: Equilibrium



$$I_{drift,p} = I_{diff,p}$$

$$I_{drift,n} = I_{diff,n}$$

- At equilibrium, the drift current flowing in one direction cancels out the diffusion current flowing in the opposite direction, creating a net current of zero.
- The figure shows the charge profile of the PN junction.

Built-in Potential

- The potential difference between the N-side and the P-side:

$$\phi_0(x) = V_{th} \ln \frac{n_0(x)}{n_i}$$

$$\phi_n = V_{th} \ln \frac{N_d}{n_i}, \qquad \phi_p = V_{th} \ln \frac{\left(\frac{n_i^2}{N_a}\right)}{n_i} = V_{th} \ln \frac{n_i}{N_a}$$

$$V_0 = \phi_{bi} = \phi_n - \phi_p = V_{th} \ln \frac{N_d N_a}{n_i^2}$$

$$\phi_{bi} = 60\text{mV} \cdot \log_{10} \frac{N_d N_a}{n_i^2}$$

Have We Invented a Battery?

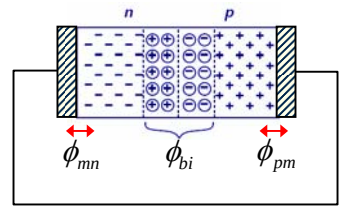
- Can we harness the PN junction and turn it into a battery?

$$\phi_{bi} \equiv \phi_n - \phi_p = V_{th}\left(\ln\frac{N_D}{n_i} + \ln\frac{N_A}{n_i}\right) = V_{th}\ln\frac{N_D N_A}{n_i^2}$$

- Numerical example:

$$\phi_{bi} = 26\text{mV}\ln\frac{N_D N_A}{n_i^2} = 60\text{mV}\times\log\frac{10^{15}10^{15}}{10^{20}} = 600\text{mV}$$

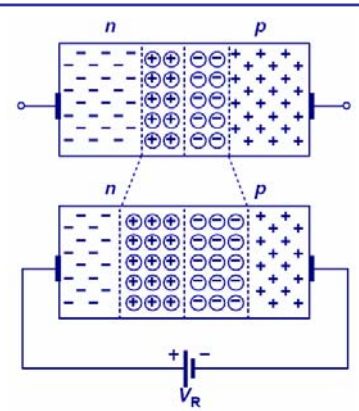
Contact Potential



$$0 = \phi_{bi} + \phi_{pm} + \phi_{mn}$$
$$\phi_{bi} = -(\phi_{pm} + \phi_{mn})$$

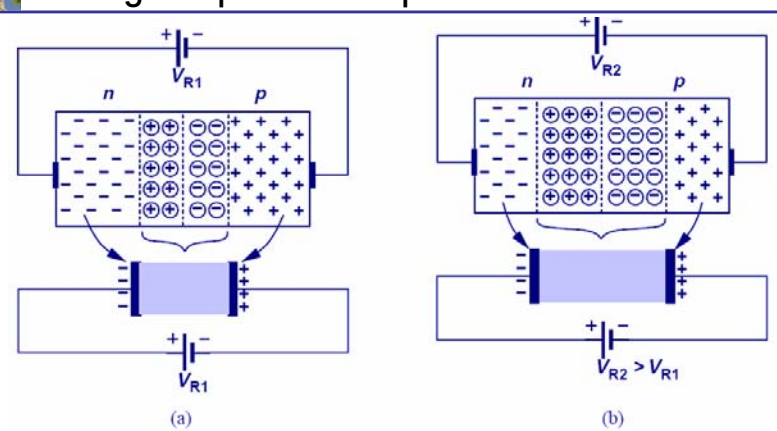
- The contact between a PN junction creates a potential difference
- When a metal-semiconductor junction is formed, a contact potential forms as well
- If we short a PN junction, the sum of the voltages around the loop must be zero:

Diode in Reverse Bias



- When the N-type region of a diode is connected to a higher potential than the P-type region, the diode is under reverse bias, which results in wider depletion region and larger built-in electric field across the junction.

Reverse-Biased Diode's Application: Voltage-Dependent Capacitor



- The PN junction can be viewed as a capacitor. By varying VR, the depletion width changes, changing its capacitance value; therefore, the PN junction is actually a voltage-dependent capacitor.

Voltage Dependence of Depletion Width

- The expression of depletion width under reverse width is similar to that at thermal equilibrium except we replace the built-in potential with the effective reverse bias:

$$x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_d}\left(\frac{N_a}{N_a + N_d}\right)} = x_{n0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$x_p(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{qN_a}\left(\frac{N_d}{N_a + N_d}\right)} = x_{p0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

$$X_d(V_D) = x_p(V_D) + x_n(V_D) = \sqrt{\frac{2\varepsilon_s(\phi_{bi} - V_D)}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$$

$$X_d(V_D) = X_{d0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

Charge Versus Bias

- As we increase the reverse bias, the depletion region grows to accommodate more charge

$$Q_J(V_D) = -qN_a x_p(V_D) = -qN_a x_{p0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$$

- Charge is *not* a linear function of voltage
  → This is a nonlinear capacitor
- We can define a small-signal capacitance by breaking up the charge into a DC and an AC terms:

$$Q_J(V_D + v_D) = Q_J(V_D) + q(v_D)$$

Derivation of Small Signal Capacitance

$$Q_J(V_D + v_D) = Q_J(V_D) + \left.\frac{dQ_D}{dV}\right|_{V_D} v_D + \cdots$$

$$C_j = C_j(V_D) = \left.\frac{dQ_j}{dV}\right|_{V=V_D} = \left.\frac{d}{dV}\left(-qN_a x_{p0}\sqrt{1 - \frac{V}{\phi_{bi}}}\right)\right]_{V=V_R}$$

$$\boxed{C_j = \frac{qN_a x_{p0}}{2\phi_{bi}\sqrt{1 - \frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_{bi}}}}}$$

$$C_{j0} = \frac{qN_a x_{p0}}{2\phi_{bi}} = \frac{qN_a}{2\phi_{bi}}\sqrt{\left(\frac{2\varepsilon_s\phi_{bi}}{qN_a}\right)\left(\frac{N_d}{N_a + N_d}\right)} = \boxed{\sqrt{\frac{q\varepsilon_s}{2\phi_{bi}}\frac{N_a N_d}{N_a + N_d}}}$$

Physical Interpretation of Depletion Cap

$$C_{j0} = \sqrt{\frac{q\varepsilon_s}{2\phi_{bi}}\frac{N_a N_d}{N_a + N_d}}$$

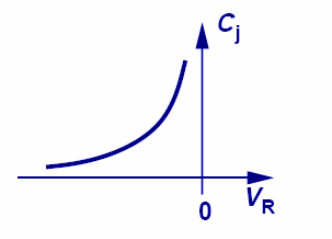
- Notice that the expression on the right-hand-side is just the depletion width in thermal equilibrium

$$C_{j0} = \varepsilon_s\sqrt{\frac{q}{2\varepsilon_s\phi_{bi}}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)^{-1}} = \frac{\varepsilon_s}{X_{d0}}$$

- This looks like a parallel plate capacitor!

$$C_j(V_D) = \frac{\varepsilon_s}{X_d(V_D)}$$

A Variable Capacitor (Varactor)

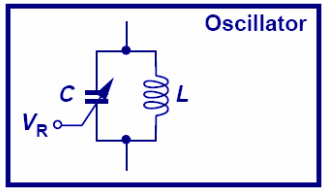


$$C_j = \frac{C_{j0}}{\sqrt{1+\frac{V_R}{V_0}}}$$

$$C_{j0} = \sqrt{\frac{\varepsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0}}$$

- The equations that describe the voltage-dependent capacitance are shown above.

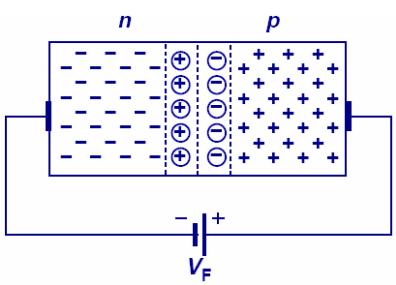
Voltage-Controlled Oscillator (VCO)



$$f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

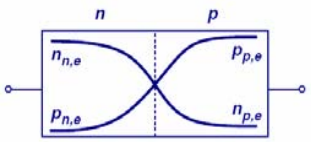
- A very important application of a reverse-biased PN junction is VCO, in which an LC tank is used in an oscillator. By changing VR, we can change C, which also changes the oscillation frequency.

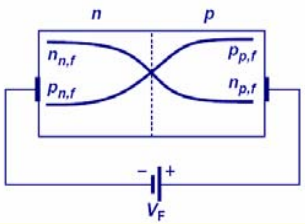
Forward-Biased Diode



- When the N-type region of a diode is at a lower potential than the P-type region, the diode is in forward bias.
- The depletion width is reduced and the built-in electric field decreased.

Minority Carrier Profile in Forward Bias





$$p_{n,e} = \frac{p_{p,e}}{\exp\frac{V_0}{V_T}} = \frac{N_A}{\exp\frac{V_0}{V_T}}$$

$$p_{n,f} = \frac{p_{p,f}}{\exp\frac{V_0 - V_F}{V_T}} = \frac{N_A}{\exp\frac{V_0 - V_F}{V_T}} = p_{n,e} e^{\frac{V_F}{V_T}}$$

- Under forward bias, minority carriers in each region increase due to the lowering of built-in field/potential. Therefore, diffusion currents increase to supply these minority carriers.

Diffusion Current in Forward Bias

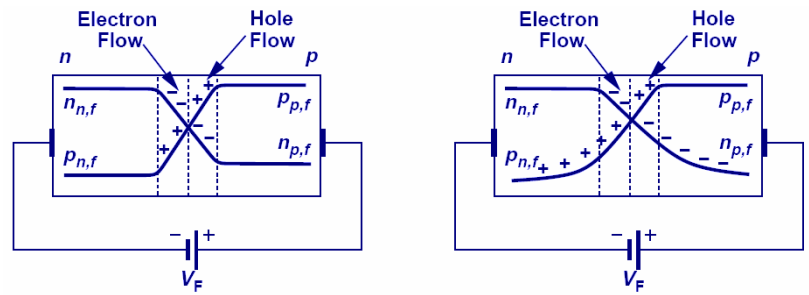
$$\Delta n_p \approx \frac{N_D}{\exp\frac{V_0}{V_T}}(\exp\frac{V_F}{V_T}-1) \qquad \Delta p_n \approx \frac{N_A}{\exp\frac{V_0}{V_T}}(\exp\frac{V_F}{V_T}-1)$$

$$I_{tot} \propto \frac{N_A}{\exp\frac{V_0}{V_T}}(\exp\frac{V_F}{V_T}-1)+\frac{N_D}{\exp\frac{V_0}{V_T}}(\exp\frac{V_F}{V_T}-1)$$

$$I_{tot} = I_s(\exp\frac{V_F}{V_T}-1) \qquad I_s = Aqn_i^2(\frac{D_n}{N_A L_n}+\frac{D_p}{N_D L_p})$$

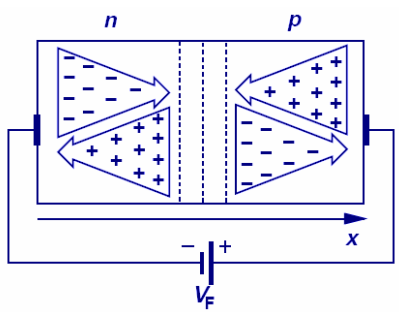
- Diffusion current will increase in order to supply the increase in minority carriers. The mathematics are shown above.

Minority Charge Gradient



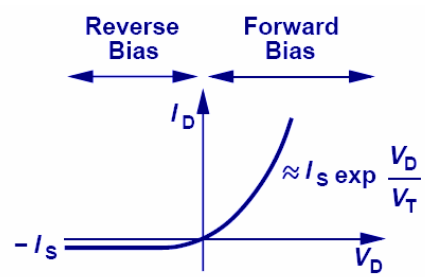
- Minority charge profile should not be constant along the x-axis; otherwise, there is no concentration gradient and no diffusion current.
- Recombination of the minority carriers with the majority carriers accounts for the dropping of minority carriers as they go deep into the P or N region.

Forward Bias Condition: Summary



- In forward bias, there are large diffusion currents of minority carriers through the junction. However, as we go deep into the P and N regions, recombination currents from the majority carriers dominate. These two currents add up to a constant value.

IV Characteristic of PN Junction



$$I_D = I_S(\exp\frac{V_D}{V_T}-1)$$

- The current and voltage relationship of a PN junction is exponential in forward bias region, and relatively constant in reverse bias region.

Diode Small-Signal Model

- The I-V relation of a diode can be linearized

$$I_D + i_D = I_S\left(e^{\frac{q(V_d+v_d)}{kT}} - 1\right) \approx I_S e^{\frac{qV_d}{kT}} e^{\frac{qv_d}{kT}}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

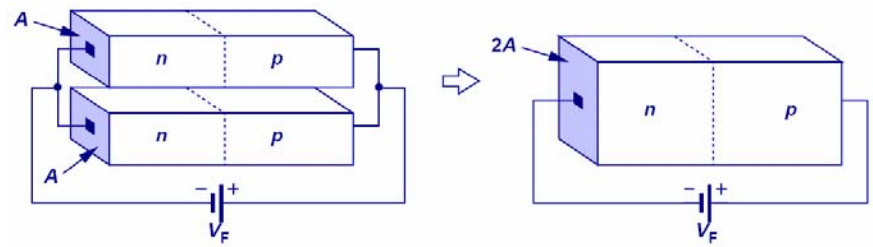
$$I_D + i_D \approx I_D\left(1 + \frac{q(V_d + v_d)}{kT} + \cdots\right)$$

$$i_D \approx I_D \frac{qv_d}{kT}$$

Small-Signal Resistance (or Dynamic Resistance)

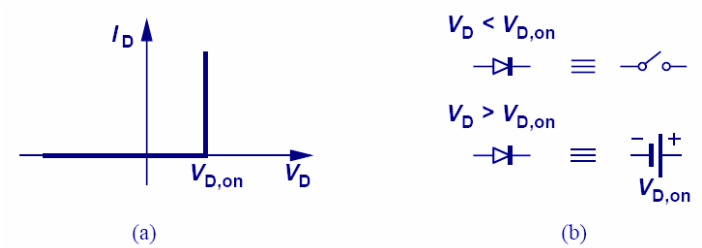
$$\boxed{r_d = \frac{v_d}{i_D} = \frac{kT}{q}\frac{1}{I_D} = \frac{V_T}{I_D}}$$

Parallel PN Junctions



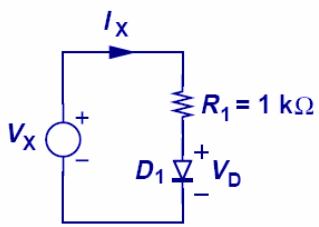
- Since junction currents are proportional to the junction's cross-section area. Two PN junctions put in parallel are effectively one PN junction with twice the cross-section area, and hence twice the current.

Constant-Voltage Diode Model



- Diode operates as an open circuit if VD< VD,on and a constant voltage source of VD,on if VD tends to exceed VD,on.

Example: Diode Calculations



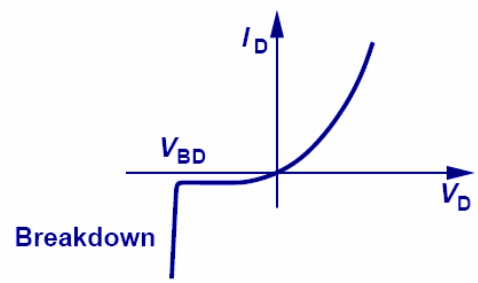
$$V_X = I_X R_1 + V_D = I_X R_1 + V_T \ln\frac{I_X}{I_S}$$

$$I_X = 2.2mA \quad for \quad V_X = 3V$$

$$I_X = 0.2mA \quad for \quad V_X = 1V$$

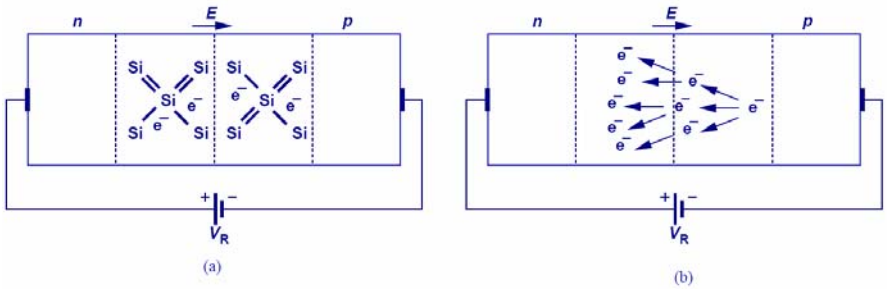
- This example shows the simplicity provided by a constant-voltage model over an exponential model.
- For an exponential model, iterative method is needed to solve for current, whereas constant-voltage model requires only linear equations.

Reverse Breakdown



- When a large reverse bias voltage is applied, breakdown occurs and an enormous current flows through the diode.

Zener vs. Avalanche Breakdown



- Zener breakdown is a result of the large electric field inside the depletion region that breaks electrons or holes off their covalent bonds.
- Avalanche breakdown is a result of electrons or holes colliding with the fixed ions inside the depletion region.