Lecture 11

OUTLINE

• Cascode Stage

Reading: Chapter 9.1
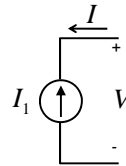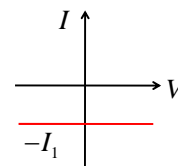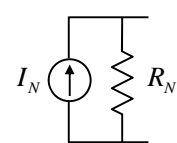
Ideal Current Source

Circuit Symbol   **I-V Characteristic**   **Equivalent Circuit**

• An ideal current source has infinite output impedance.

How can we increase the output impedance of a BJT that is used as a current source?
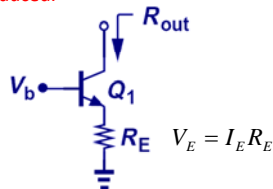
Boosting the Output Impedance

• Recall that emitter degeneration boosts the impedance seen looking into the collector.
   – This improves the gain of the CE or CB amplifier. However, headroom is reduced.

$$V_E = I_E R_E$$
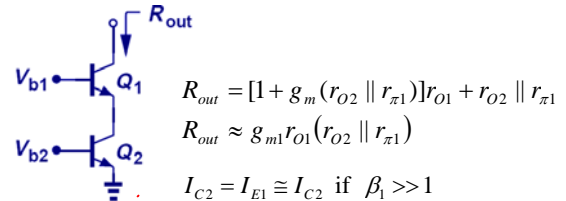
$$R_{out} = [1 + g_m(R_E \| r_\pi)]r_O + R_E \| r_\pi$$

Cascode Stage

• In order to relax the trade-off between output impedance and voltage headroom, we can use a transistor instead of a degeneration resistor:

$$R_{out} = [1 + g_m(r_{O2} \| r_{\pi1})]r_{O1} + r_{O2} \| r_{\pi1}$$

$$R_{out} \approx g_{m1}r_{O1}(r_{O2} \| r_{\pi1})$$
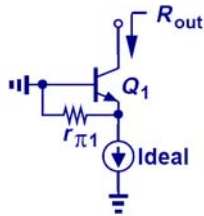
$$I_{C2} = I_{E1} \cong I_{C2} \text{ if } \beta_1 >> 1$$

• V_{CE} for Q_2 can be as low as ~ 0.4V ("soft saturation")

Maximum Bipolar Cascode Output Impedance

$$R_{out,\max} \approx g_{m1} r_{01} r_{\pi 1}$$
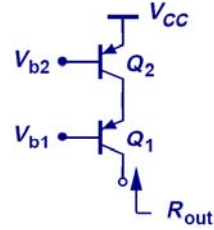$$R_{out,\max} \approx \beta_1 r_{01}$$

- The maximum output impedance of a bipolar cascode is bounded by the ever-present r_π between emitter and ground of Q_1.

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PNP Cascode Stage

$$R_{out} = [1 + g_m(r_{02} \| r_{\pi 1})]r_{01} + r_{02} \| r_{\pi 1}$$
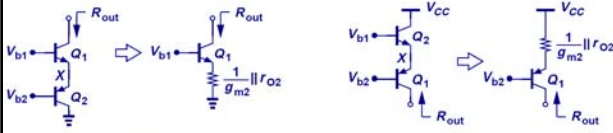$$R_{out} \approx g_{m1} r_{01}(r_{02} \| r_{\pi 1})$$

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False Cascodes

$$R_{out} = \left[1 + g_{m1}\left(\frac{1}{g_{m2}} \| r_{O2} \| r_{\pi 1}\right)\right]r_{O1} + \frac{1}{g_{m2}} \| r_{O2} \| r_{\pi 1}$$
$$R_{out} \approx \left(1 + \frac{g_{m1}}{g_{m2}}\right)r_{O1} + \frac{1}{g_{m2}} \approx 2r_{O1}$$

- When the emitter of Q_1 is connected to the emitter of Q_2, it's no longer a cascode since Q_2 becomes a diode-connected device instead of a current source.
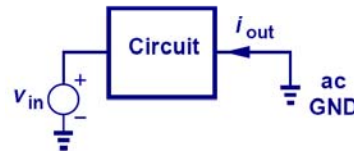
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Short-Circuit Transconductance

- The **short-circuit transconductance** of a circuit is a measure of its strength in converting an input voltage signal into an output current signal.

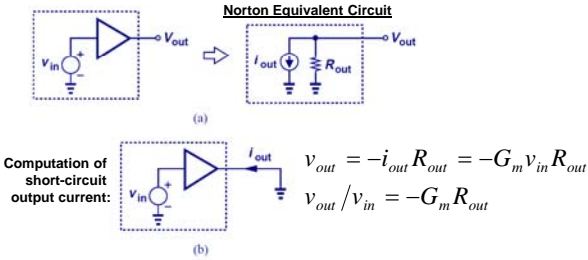$$G_m \equiv \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0}$$

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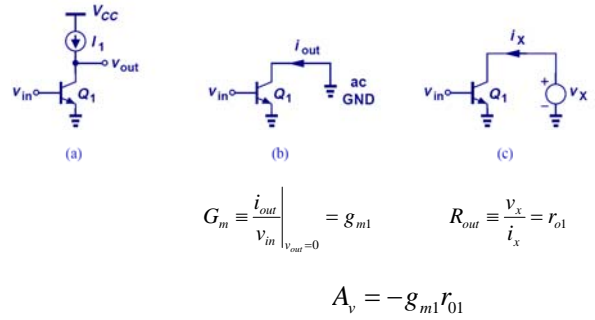
Voltage Gain of a Linear Circuit

- By representing a linear circuit with its Norton equivalent, the relationship between V_{out} and V_{in} can be expressed by the product of G_m and R_{out}.

Norton Equivalent Circuit

(a)

Computation of short-circuit output current:

$$v_{out} = -i_{out}R_{out} = -G_m v_{in} R_{out}$$
$$v_{out}/v_{in} = -G_m R_{out}$$

(b)

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Example: Voltage Gain

(a)   (b)   (c)

$$G_m \equiv \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0} = g_{m1} \qquad R_{out} \equiv \frac{v_x}{i_x} = r_{o1}$$

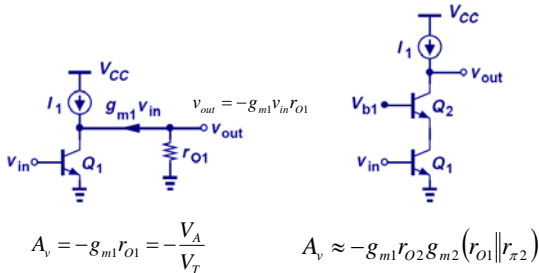$$A_v = -g_{m1}r_{01}$$

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Comparison of CE and Cascode Stages

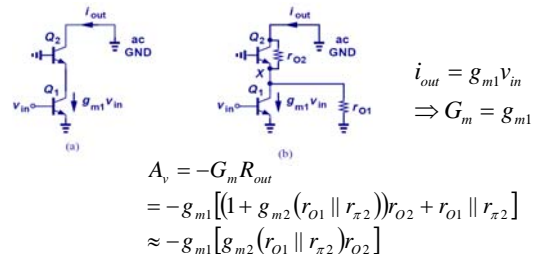- Since the output impedance of the cascode is higher than that of a CE stage, its voltage gain is also higher.

$$v_{out} = -g_{m1}v_{in}r_{O1}$$

$$A_v = -g_{m1}r_{O1} = -\frac{V_A}{V_T} \qquad A_v \approx -g_{m1}r_{O2}g_{m2}(r_{O1}\|r_{\pi2})$$

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Voltage Gain of Cascode Amplifier

- Since r_O is much larger than $1/g_m$, most of $I_{C,Q1}$ flows into diode-connected Q_2. Using R_{out} as before, A_V is easily calculated.

(a)   (b)

$$i_{out} = g_{m1}v_{in}$$
$$\Rightarrow G_m = g_{m1}$$

$$A_v = -G_m R_{out}$$
$$= -g_{m1}\big[(1 + g_{m2}(r_{O1}\|r_{\pi2}))r_{O2} + r_{O1}\|r_{\pi2}\big]$$
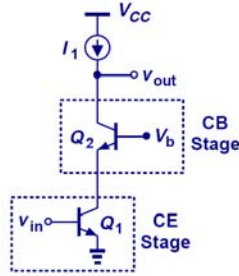$$\approx -g_{m1}\big[g_{m2}(r_{O1}\|r_{\pi2})r_{O2}\big]$$
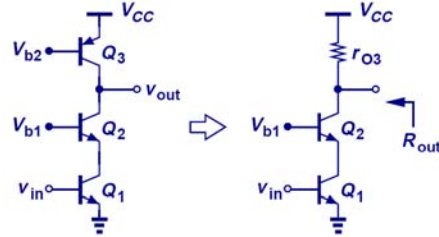
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Alternate View of Cascode Amplifier

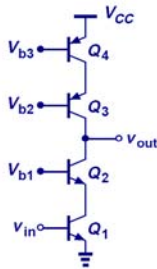- A bipolar cascode amplifier is also a CE stage in series with a CB stage.

Practical Cascode Stage

$$R_{out} \approx r_{O3} \| g_{m2} r_{O2} (r_{O1} \| r_{\pi 2})$$

- Since no current source can be ideal, the output impedance drops.

Improved Cascode Stage

$$R_{out} \approx \left[g_{m3} r_{O3} (r_{O4} \| r_{\pi 3}) \right] \| \left[g_{m2} r_{O2} (r_{O1} \| r_{\pi 2}) \right]$$
$$A_v = -g_{m1} R_{out}$$

- In order to preserve the high output impedance, a cascode PNP current source is used.