Lecture 13

OUTLINE

- Frequency Response
  - General considerations
  - High-frequency BJT model
  - Miller's Theorem
  - Frequency response of CE stage

Reading: Chapter 11.1-11.3

Review: Sinusoidal Analysis

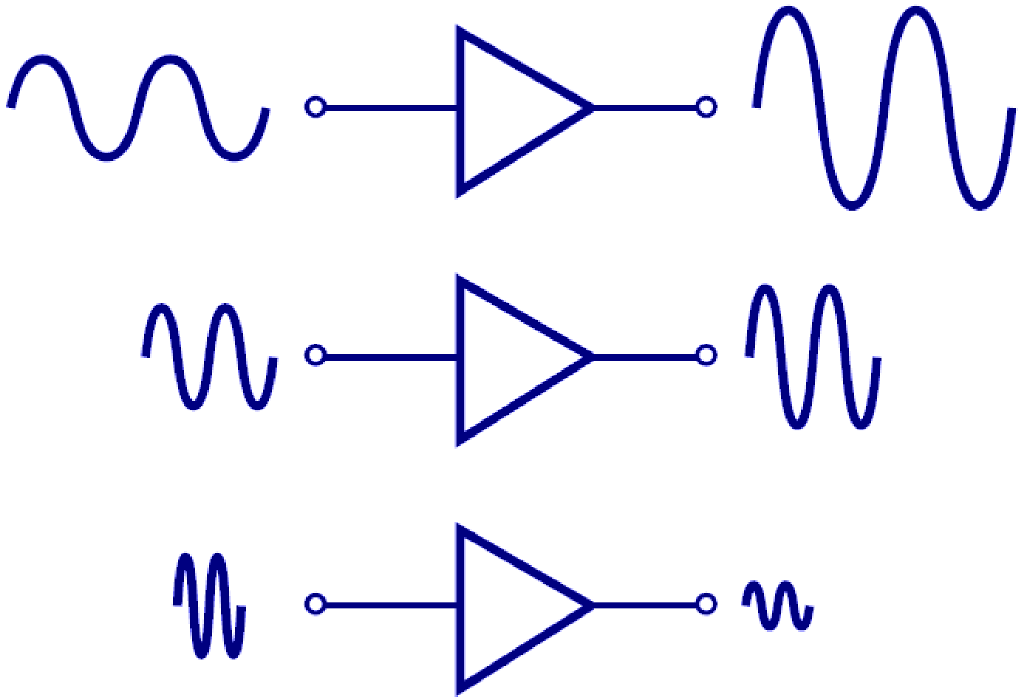
- Any voltage or current in a linear circuit with a sinusoidal source is a sinusoid of the same frequency (ω).
  - We only need to keep track of the amplitude and phase, when determining the response of a linear circuit to a sinusoidal source.

- Any time-varying signal can be expressed as a sum of sinusoids of various frequencies (and phases).

→ Applying the principle of superposition:
  - The current or voltage response in a linear circuit due to a time-varying input signal can be calculated as the sum of the sinusoidal responses for each sinusoidal component of the input signal.

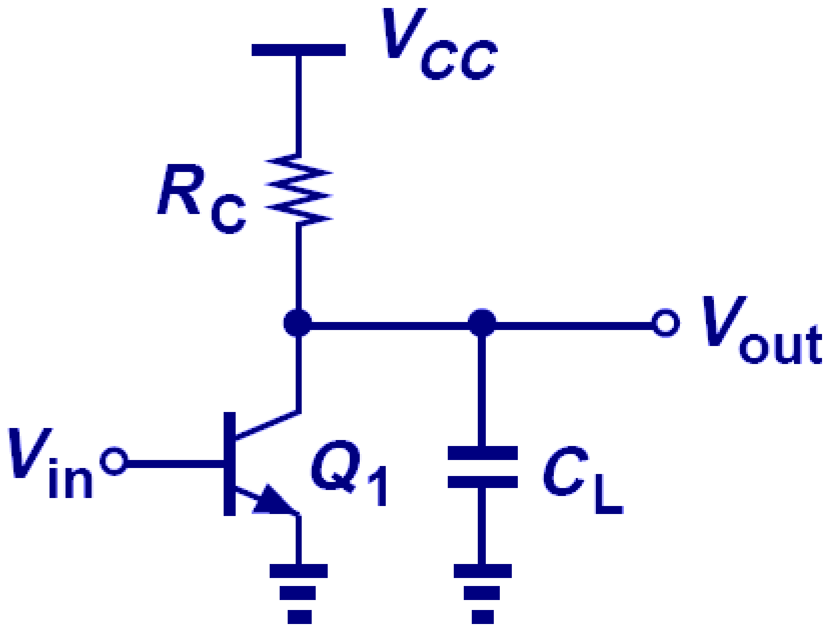
High Frequency "Roll-Off" in A_v

- Typically, an amplifier is designed to work over a limited range of frequencies.
  - At "high" frequencies, the gain of an amplifier decreases.



A_v Roll-Off due to C_L

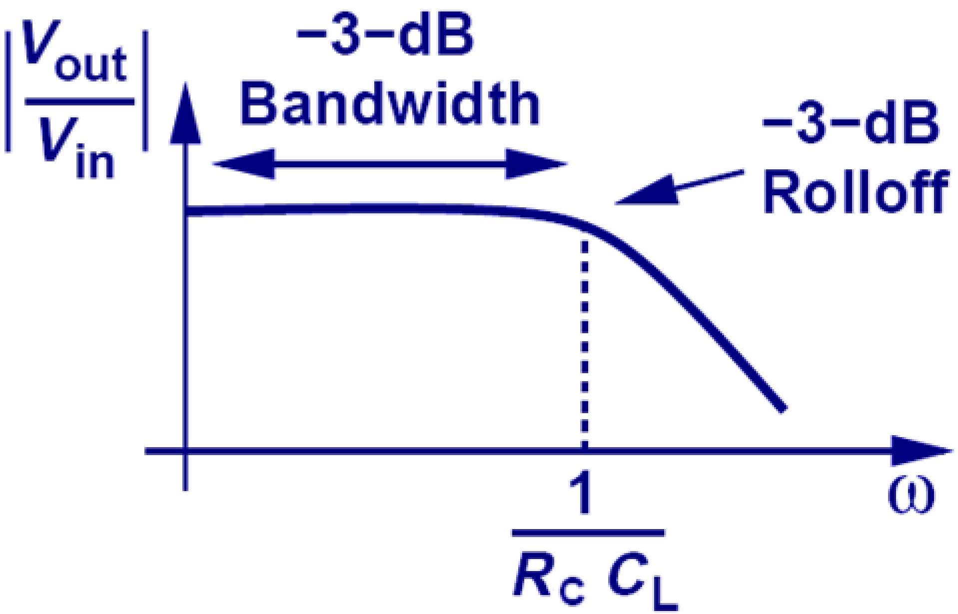
- A capacitive load (C_L) causes the gain to decrease at high frequencies.
  - The impedance of C_L decreases at high frequencies, so that it shunts some of the output current to ground.



$$A_v = -g_m\left(R_C \parallel \frac{1}{j\omega C_L}\right)$$

Frequency Response of the CE Stage

- At low frequency, the capacitor is effectively an open circuit, and A_v *vs.* ω is flat. At high frequencies, the impedance of the capacitor decreases and hence the gain decreases. The "breakpoint" frequency is $1/(R_C C_L)$.



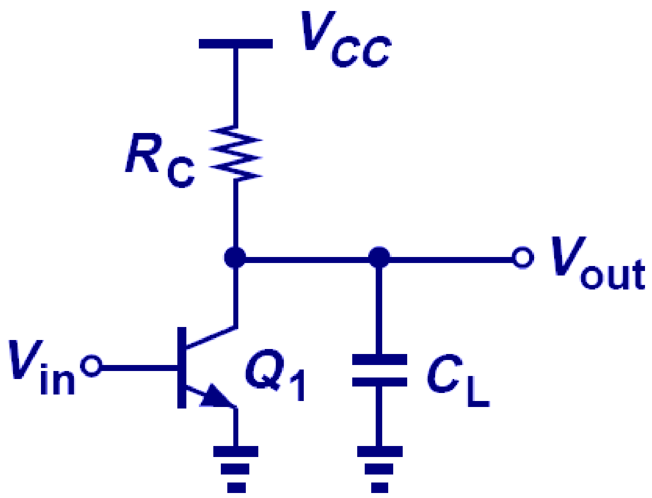
$$A_v = -g_m \frac{R_C \dfrac{1}{j\omega C_L}}{R_C + \dfrac{1}{j\omega C_L}}$$

$$= \frac{-g_m R_C}{1 + j\omega R_C C_L}$$

$$|A_v| = \frac{g_m R_C}{\sqrt{R_C^2 C_L^2 \omega^2 + 1}}$$

Amplifier Figure of Merit (FOM)

- The gain-bandwidth product is commonly used to benchmark amplifiers.
  - We wish to maximize both the gain and the bandwidth.
- Power consumption is also an important attribute.
  - We wish to minimize the power consumption.



$$\frac{\text{Gain} \times \text{Bandwidth}}{\text{Power Consumption}} = \frac{(g_m R_C)\left(\frac{1}{R_C C_L}\right)}{I_C V_{CC}}$$

$$= \frac{1}{V_T V_{CC} C_L}$$

Operation at low *T*, low V_{CC}, and with small C_L → superior FOM

Bode Plot

- The transfer function of a circuit can be written in the general form

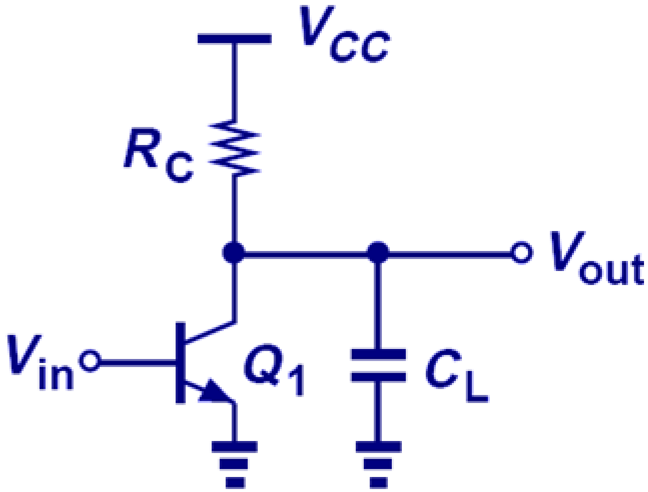
$$H(j\omega) = A_0 \frac{\left(1+\frac{j\omega}{\omega_{z1}}\right)\left(1+\frac{j\omega}{\omega_{z2}}\right)\cdots}{\left(1+\frac{j\omega}{\omega_{p1}}\right)\left(1+\frac{j\omega}{\omega_{p2}}\right)\cdots}$$

A_0 is the low-frequency gain
ω_{zj} are "zero" frequencies
ω_{pj} are "pole" frequencies

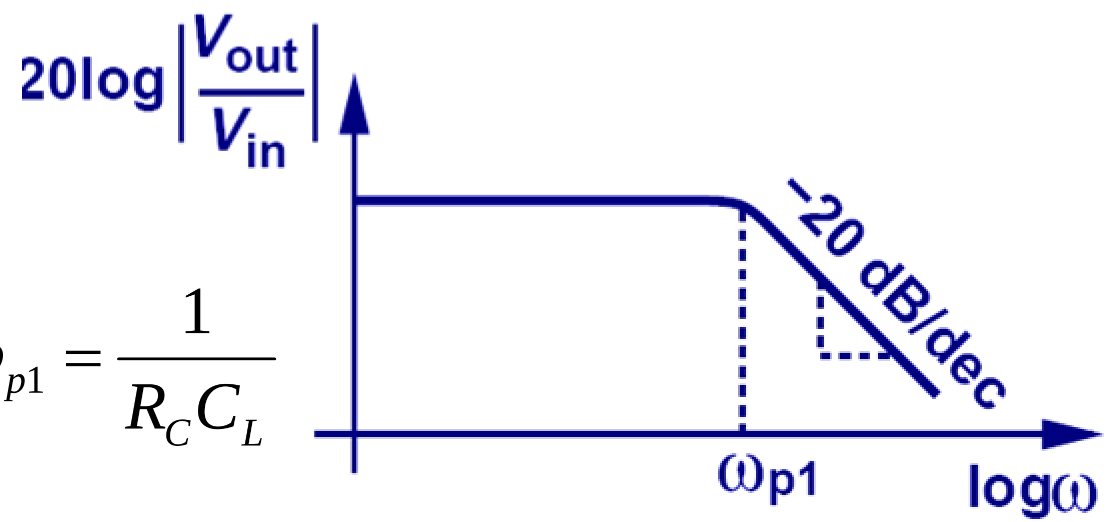
- Rules for generating a Bode magnitude *vs*. frequency plot:
  - As ω passes each **zero** frequency, the **slope of |H($j\omega$)| increases** by 20dB/dec.
  - As ω passes each **pole** frequency, the **slope of |H($j\omega$)| decreases** by 20dB/dec.

Bode Plot Example

- This circuit has only one pole at $\omega_{p1}=1/(R_C C_L)$; the slope of $|A_v|$ decreases from 0 to -20dB/dec at ω_{p1}.

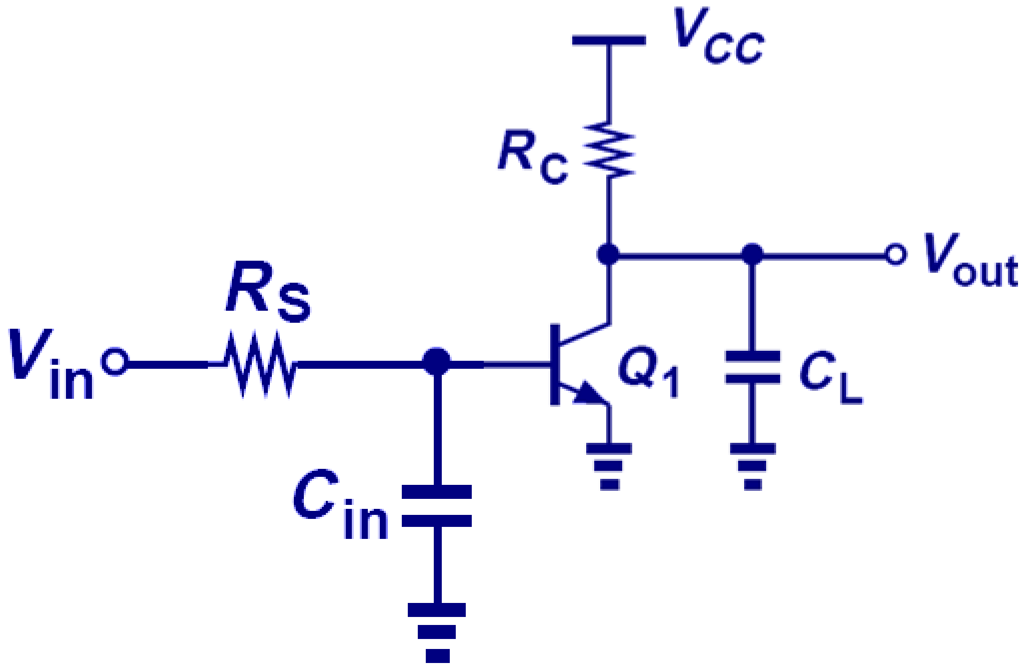


$$\omega_{p1} = \frac{1}{R_C C_L}$$



- In general, if **node *j* in the signal path** has a small-signal resistance of $\boldsymbol{R_j}$ to ground and a capacitance $\boldsymbol{C_j}$ to ground, then it contributes a **pole at frequency** $\boldsymbol{(R_j C_j)^{-1}}$

Pole Identification Example



$$\omega_{p1} = \frac{1}{R_S C_{in}}$$

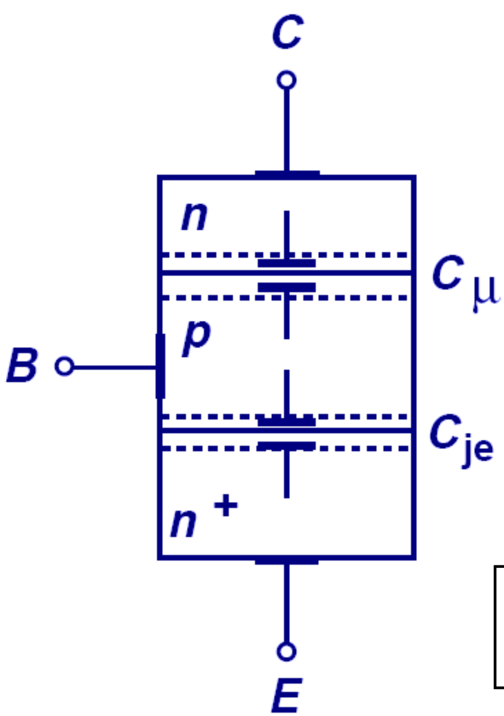
$$\omega_{p2} = \frac{1}{R_C C_L}$$

High-Frequency BJT Model

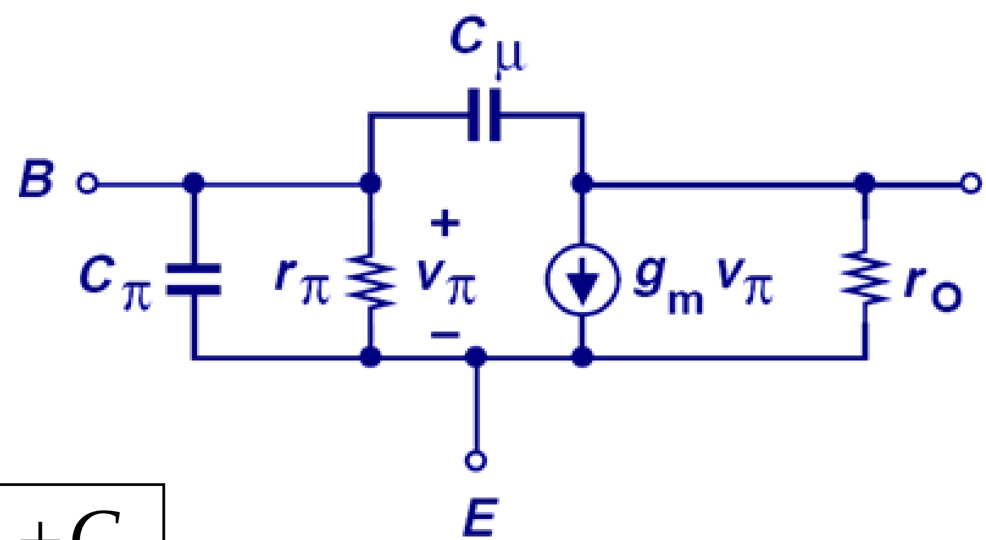
- The BJT inherently has junction capacitances which affect its performance at high frequencies.

  Collector junction: **depletion** capacitance, C_μ

  Emitter junction: **depletion** capacitance, C_{je}, and also **diffusion** capacitance, C_b.



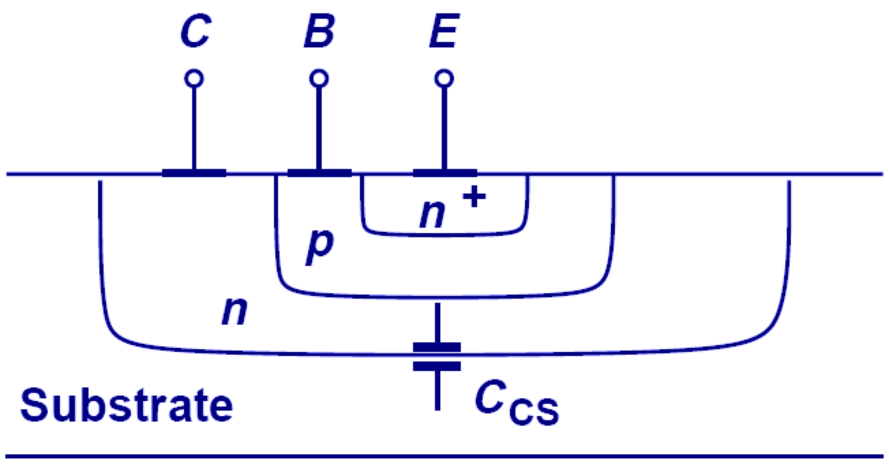
$$C_\pi \equiv C_b + C_{je}$$



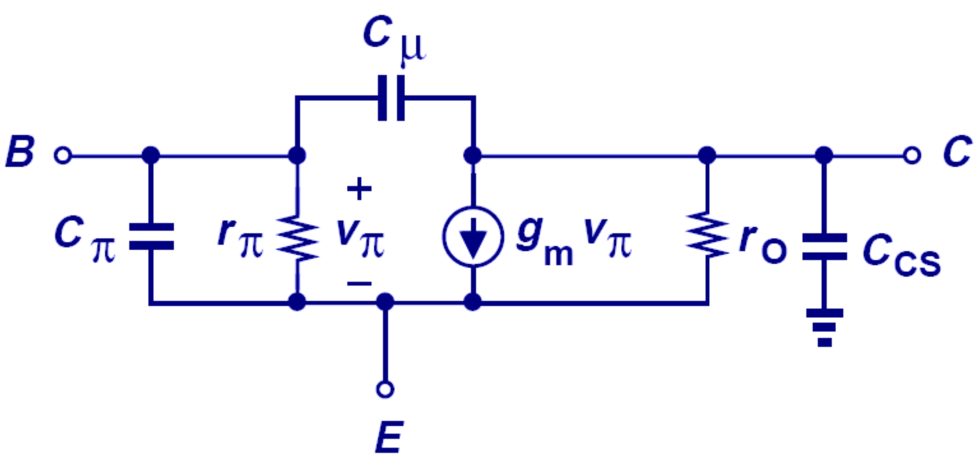
BJT High-Frequency Model (cont'd)

- In an integrated circuit, the BJTs are fabricated in the surface region of a Si wafer substrate; another junction exists between the collector and substrate, resulting in substrate junction capacitance, C_{CS}.

BJT cross-section

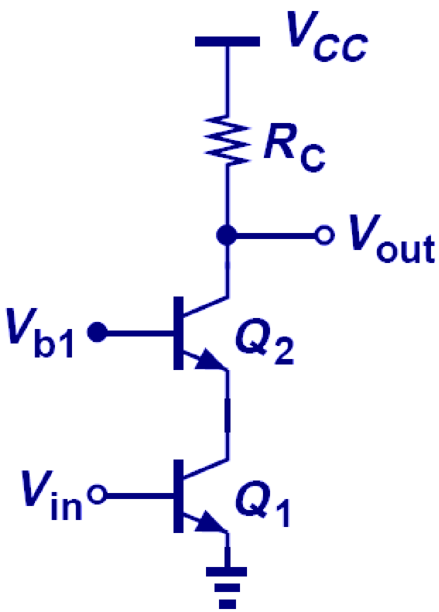


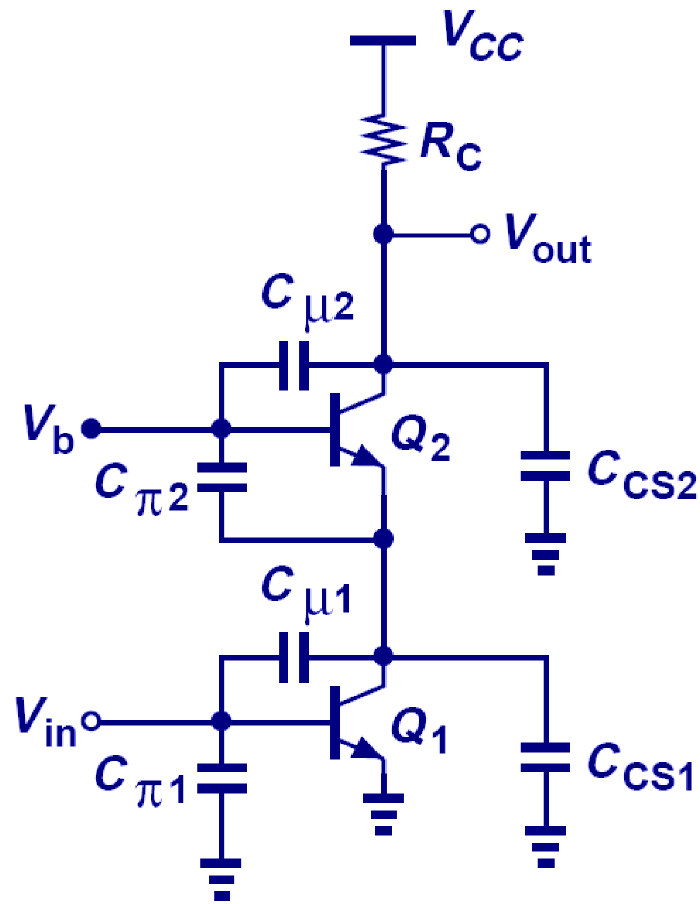
BJT small-signal model



Example: BJT Capacitances

- The various junction capacitances within each BJT are explicitly shown in the circuit diagram on the right.





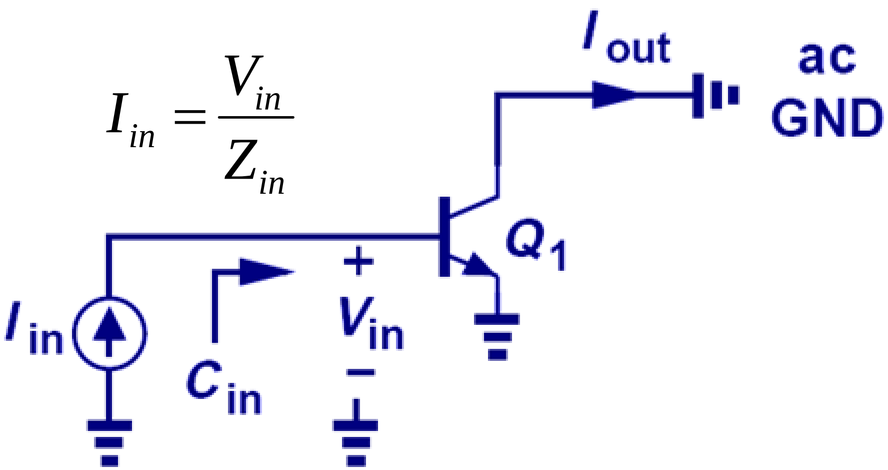
Transit Frequency, f_T

- The "transit" or "cut-off" frequency, f_T, is a measure of the intrinsic speed of a transistor, and is defined as the frequency where the current gain falls to 1.

Conceptual set-up to measure f_T

$$I_{out} = g_m V_{in}$$

$$I_{in} = \frac{V_{in}}{Z_{in}}$$



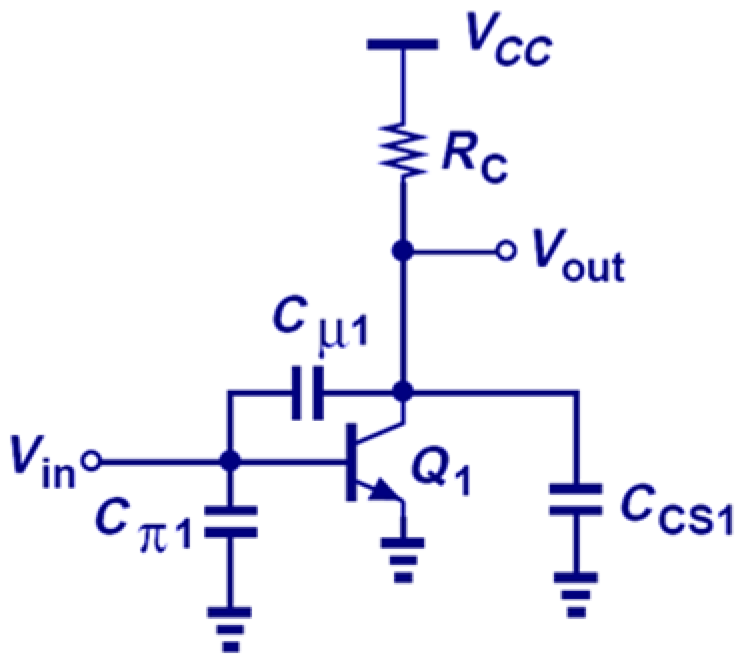
$$\left|\frac{I_{out}}{I_{in}}\right| = \left|g_m Z_{in}\right| = \left|g_m\left(\frac{1}{j\omega_T C_{in}}\right)\right| = 1$$

$$\Rightarrow \omega_T = \frac{g_m}{C_{in}}$$

$$\boxed{2\pi f_T = \frac{g_m}{C_\pi}}$$

Dealing with a Floating Capacitance

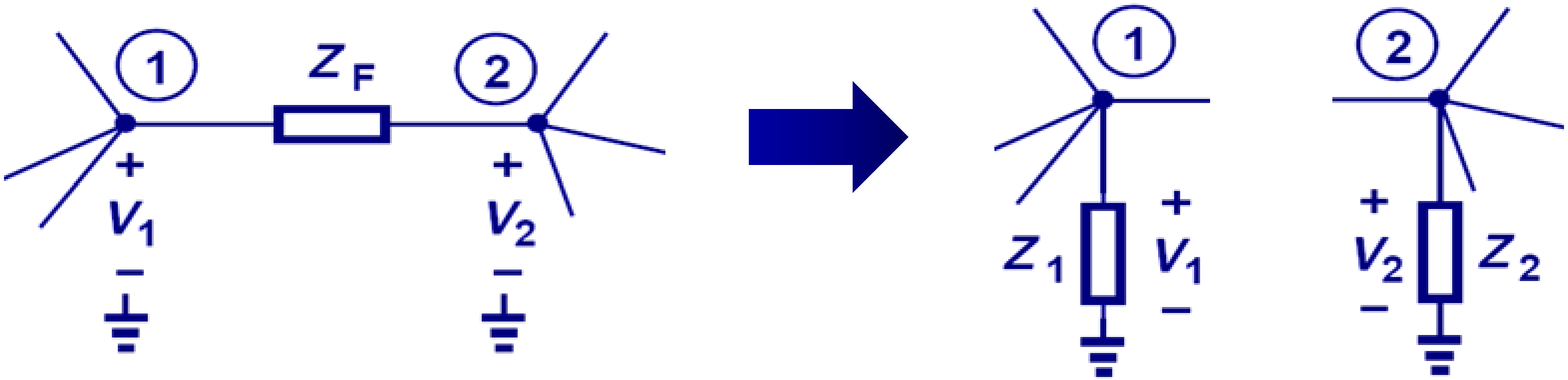
- Recall that a pole is computed by finding the resistance and capacitance between a node and GROUND.
- It is not straightforward to compute the pole due to $C_{\mu 1}$ in the circuit below, because neither of its terminals is grounded.



Miller's Theorem

- If A_v is the voltage gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2:

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1} \quad \Rightarrow \quad Z_1 = Z_F \frac{V_1}{V_1 - V_2} = Z_F \frac{1}{1 - A_v} = Z_1$$

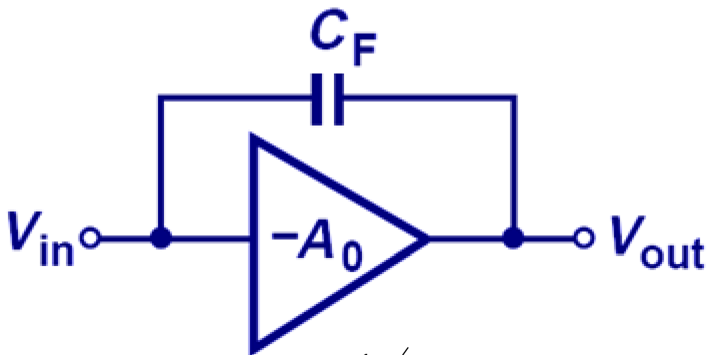


$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2} \quad \Rightarrow \quad Z_2 = -Z_F \frac{V_2}{V_1 - V_2} = Z_F \frac{1}{1 - \frac{1}{A_v}} = Z_2$$

Miller Multiplication

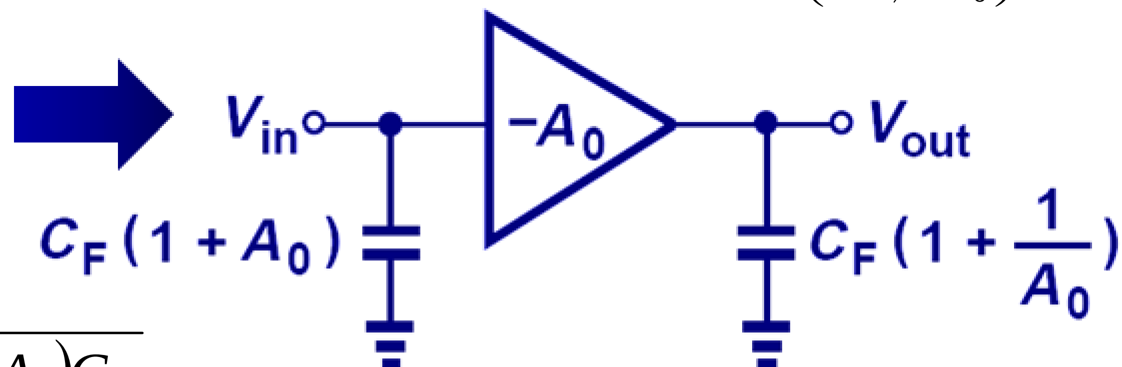
- Applying Miller's theorem, we can convert a floating capacitance between the input and output nodes of an amplifier into two grounded capacitances.
- **The capacitance at the input node is larger than the original floating capacitance.**

$$A_0 \equiv -A_v$$

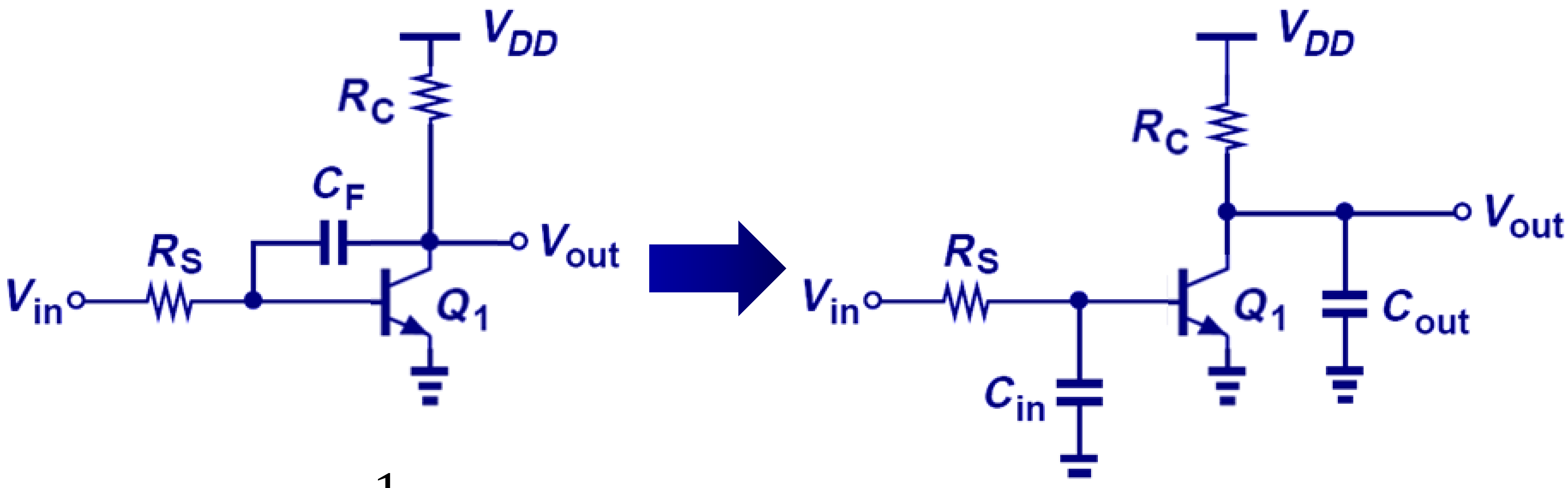


$$Z_1 = \frac{Z_F}{1-A_v} = \frac{1/j\omega C_F}{1+A_0} = \frac{1}{j\omega(1+A_0)C_F}$$

$$Z_2 = \frac{Z_F}{1-1/A_v} = \frac{1/j\omega C_F}{1+1/A_0} = \frac{1}{j\omega\left(1+1/A_0\right)C_F}$$



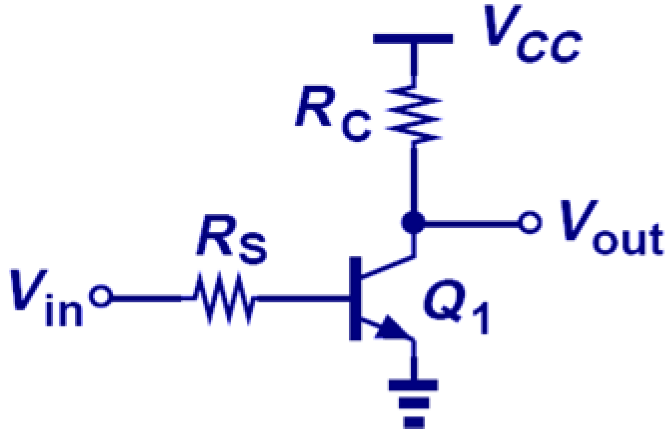
Application of Miller's Theorem

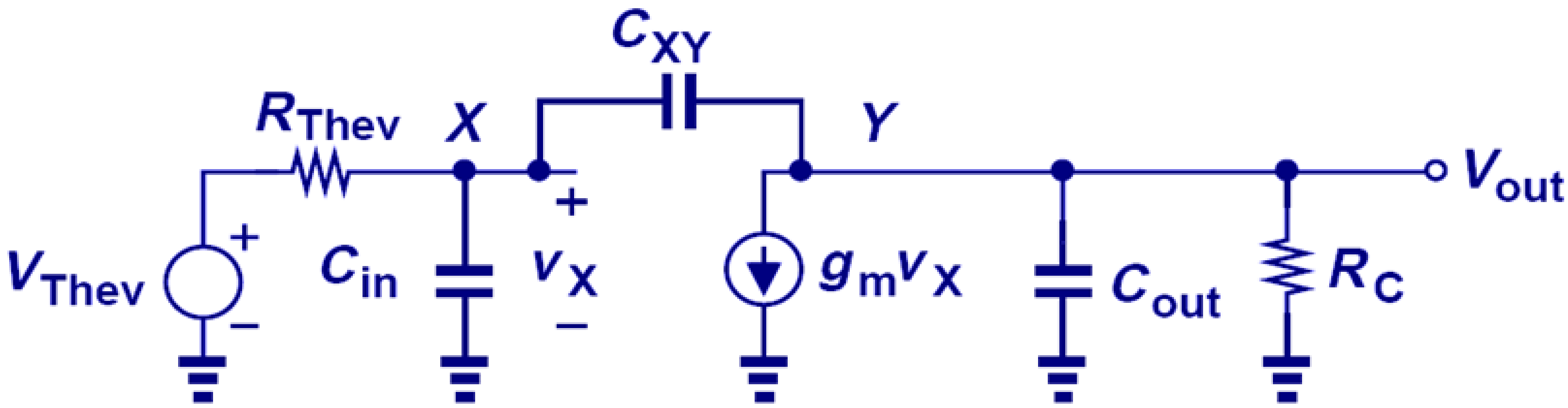


$$\omega_{p,in} = \frac{1}{R_S\left(1+g_m R_C\right)C_F} \Rightarrow \text{Dominant Pole since } \omega_{p,in} > \omega_{p,out}$$

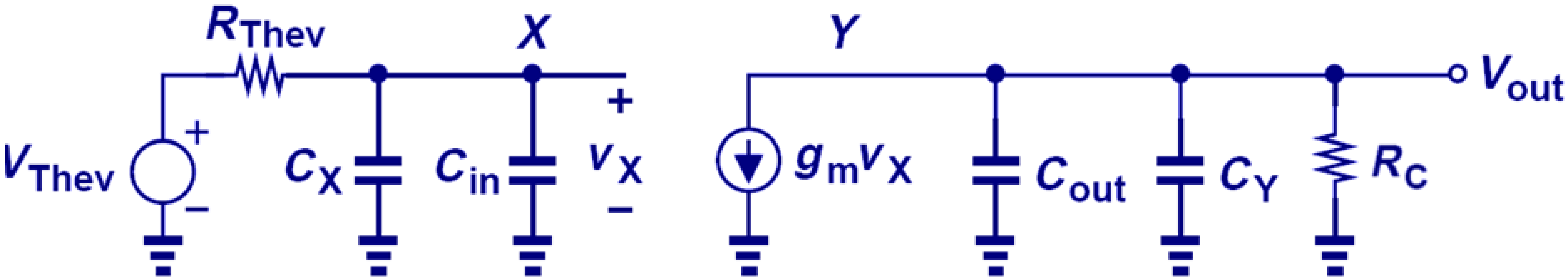
$$\omega_{p,out} = \frac{1}{R_C\left(1+\frac{1}{g_m R_C}\right)C_F}$$

Small-Signal Model for CE Stage





… Applying Miller's Theorem



$$V_{Thev} = V_{in} \frac{r_\pi}{r_\pi + R_S}$$

$$R_{Thev} = R_S \| r_\pi$$

$$C_X = C_\mu (1 + g_m R_C)$$

$$C_Y = C_\mu \left(1 + \frac{1}{g_m R_C}\right)$$

$$\omega_{p,in} = \frac{1}{R_{Thev}\left(C_{in} + \left(1 + g_m R_C\right) C_\mu\right)}$$

$\Rightarrow$ Dominant pole

$$\omega_{p,out} = \frac{1}{R_C\left(C_{out} + \left(1 + \frac{1}{g_m R_C}\right) C_\mu\right)}$$

Direct Analysis of CE Stage

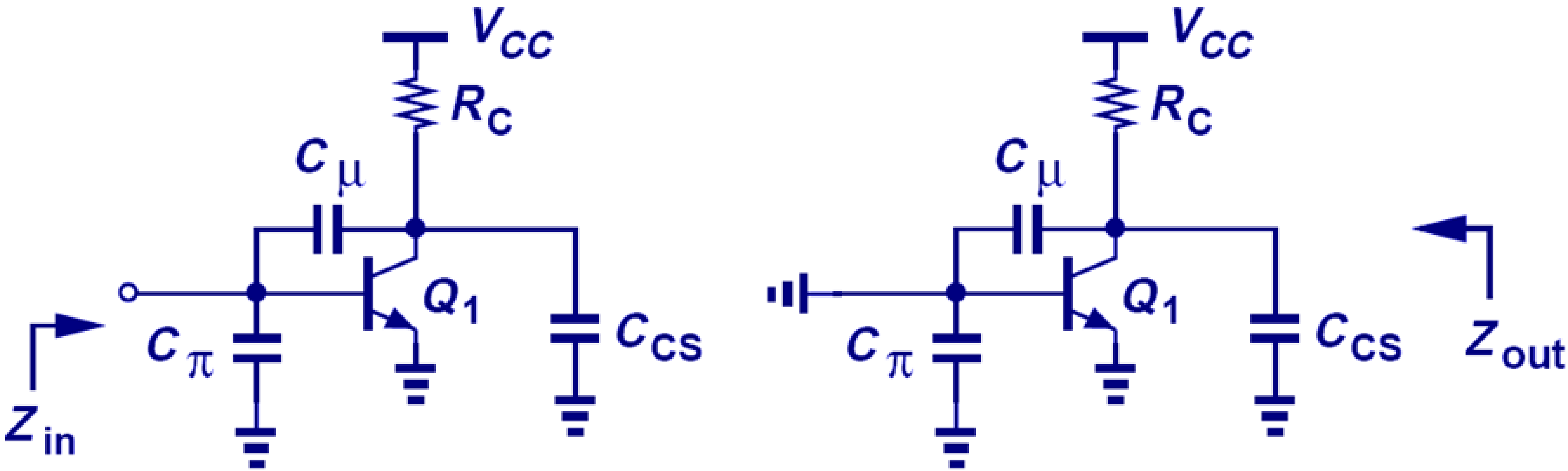
- Direct analysis yields slightly different pole locations and an extra zero:

$$\omega_z = \frac{g_m}{C_\mu}$$

$$\omega_{p1} = \frac{1}{(1+g_m R_C) C_\mu R_{Thev} + R_{Thev} C_{in} + R_C (C_\mu + C_{out})}$$

$$\omega_{p2} = \frac{(1+g_m R_C) C_\mu R_{Thev} + R_{Thev} C_{in} + R_C (C_\mu + C_{out})}{R_{Thev} R_C (C_{in} C_\mu + C_{out} C_\mu + C_{in} C_{out})}$$

I/O Impedances of CE Stage



$$Z_{in} \approx \frac{1}{j\omega\left[C_\pi + \left(1 + g_m\left(R_C \| r_o\right)\right)C_\mu\right]} \| r_\pi$$

$$Z_{out} = \frac{1}{j\omega\left[C_\mu + C_{CS}\right]} \| R_C \| r_o$$