Lecture 14

OUTLINE

- Frequency Response (cont'd)
  - CB stage
  - Emitter follower
  - Cascode stage

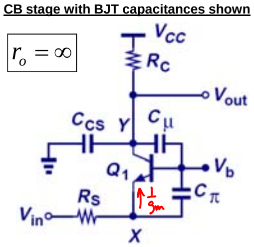
Reading: Chapter 11.4-11.6

CB Stage: Pole Frequencies

- Note that there is no capacitance between input & output nodes
  → No Miller multiplication effect!

CB stage with BJT capacitances shown

$r_o = \infty$



$$\omega_{p,Y} = \frac{1}{R_C C_Y}$$

$$C_Y = C_\mu + C_{CS}$$

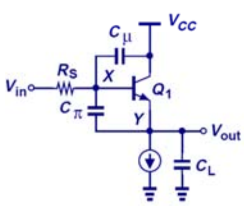
$$\omega_{p,X} = \frac{1}{\left(R_S \,\|\, \frac{1}{g_m}\right) C_X} > \omega_T$$

$$C_X = C_\pi$$

Emitter Follower

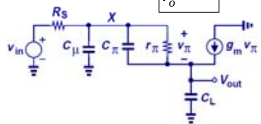
- Recall that the emitter follower provides high input impedance and low output impedance, and is used as a voltage buffer.

Follower stage with BJT capacitances shown
- C_L is the load capacitance



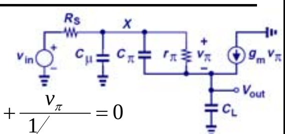
Circuit for small-signal analysis (A_v)

$r_o = \infty$



AC Analysis of Emitter Follower

$v_X = v_{out} + v_\pi$



- KCL at node *X*:

$$\frac{v_{out} + v_\pi - v_{in}}{R_S} + \frac{v_{out} + v_\pi}{1/j\omega C_\mu} + \frac{v_\pi}{r_\pi} + \frac{v_\pi}{1/j\omega C_\pi} = 0$$

- KCL at output node: $\frac{v_\pi}{r_\pi} + \frac{v_\pi}{1/j\omega C_\pi} + g_m v_\pi = \frac{v_{out}}{1/j\omega C_L}$

$$\Rightarrow \quad \frac{v_{out}}{v_{in}} \cong \frac{1 + \frac{C_\pi}{g_m}(j\omega)}{a(j\omega)^2 + b(j\omega) + 1}$$

$$a = \frac{R_S}{g_m}\left(C_\mu C_\pi + C_\mu C_L + C_\pi C_L\right)$$

$$b = R_S C_\mu + \frac{C_\pi}{g_m} + \left(1 + \frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}$$

Follower: Zero and Pole Frequencies

$$\frac{v_{out}}{v_{in}} \cong \frac{1 + \frac{C_\pi}{g_m}(j\omega)}{a(j\omega)^2 + b(j\omega) + 1}$$

$$a = \frac{R_S}{g_m}\left(C_\mu C_\pi + C_\mu C_L + C_\pi C_L\right)$$

$$b = R_S C_\mu + \frac{C_\pi}{g_m} + \left(1 + \frac{R_S}{r_\pi}\right)\frac{C_L}{g_m}$$

- The follower has one zero:

$$\omega_z = \frac{g_m}{C_\pi} = 2\pi f_T$$

- The follower has two poles at lower frequencies:

$$a(j\omega)^2 + b(j\omega) + 1 = \left(1 + \frac{j\omega}{\omega_{p1}}\right)\left(1 + \frac{j\omega}{\omega_{p2}}\right) = 1 + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)(j\omega) + \left(\frac{1}{\omega_{p1}\omega_{p2}}\right)(j\omega)^2$$

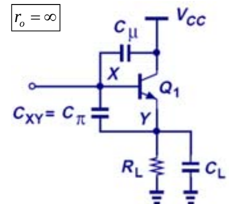
$$a = \frac{1}{\omega_{p1}\omega_{p2}}; \quad b = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}$$

Emitter Follower: Input Capacitance

- Recall that the voltage gain of an emitter follower is $A_v = \frac{R_L}{R_L + \frac{1}{g_m}}$

Follower stage with BJT capacitances shown

$r_o = \infty$



$$R_{in} = r_\pi + (\beta + 1)R_L$$

- C_{XY} can be decomposed into C_X and C_Y at the input and output nodes, respectively:

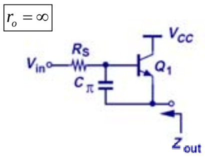
$$C_X = (1 - A_v)C_\pi = \frac{C_\pi}{1 + g_m R_L}$$

$$C_Y = \left(1 - \frac{1}{A_v}\right)C_\pi = \frac{-C_\pi}{g_m R_L}$$

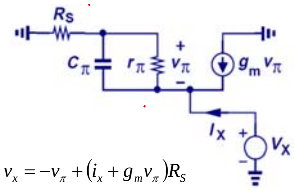
$$C_{in} = C_\mu + \frac{C_\pi}{1 + g_m R_L}$$

Emitter Follower: Output Impedance

$r_o = \infty$



Circuit for small-signal analysis (R_{out})



$$v_\pi = -(i_X + g_m v_\pi)\left(r_\pi \| \frac{1}{j\omega C_\pi}\right) \qquad v_x = -v_\pi + (i_x + g_m v_\pi)R_S$$

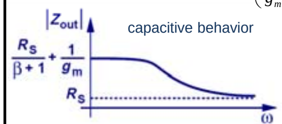
$$Z_{out} \equiv \frac{v_X}{i_X} = \frac{R_S r_\pi C_\pi (j\omega) + r_\pi + R_S}{r_\pi C_\pi (j\omega) + \beta + 1} = \frac{r_\pi + R_S}{\beta + 1} \cdot \frac{1 + \frac{j\omega}{(r_\pi + R_S)/R_S r_\pi C_\pi}}{1 + \frac{j\omega}{(\beta+1)/r_\pi C_\pi}} = \begin{cases} \frac{r_\pi + R_S}{1+\beta} & \text{for low } \omega \\ R_S & \text{for high } \omega \end{cases}$$

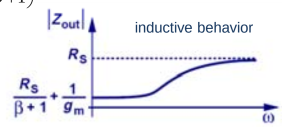
Emitter Follower as Active Inductor

$$Z_{out} \equiv \frac{v_X}{i_X} = \frac{R_S r_\pi C_\pi (j\omega) + r_\pi + R_S}{r_\pi C_\pi (j\omega) + \beta + 1} = \left(\frac{r_\pi + R_S}{\beta + 1}\right) \cdot \frac{1 + \frac{j\omega}{(r_\pi + R_S)/R_S r_\pi C_\pi}}{1 + \frac{j\omega}{(\beta+1)/r_\pi C_\pi}}$$

$$\swarrow \left(\frac{1}{g_m} + \frac{R_S}{\beta + 1}\right)$$

CASE 1: $R_S < 1/g_m$ **CASE 2**: $R_S > 1/g_m$





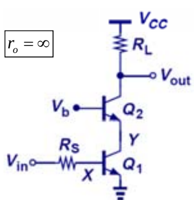
- A follower is typically used to lower the driving impedance

→ $R_S > 1/g_m$ so that the "active inductor" characteristic on the right is usually observed.

Cascode Stage

- Review:
  - A CE stage has large R_{in} but suffers from the Miller effect.
  - A CB stage is free from the Miller effect, but has small R_{in}.
- **A cascode stage provides high R_{in} with minimal Miller effect.**

$r_o = \infty$



$$A_{v,XY} \equiv \frac{v_X}{v_Y} = -g_{m1}\left(\frac{1}{g_{m2}}\right) \approx -1$$

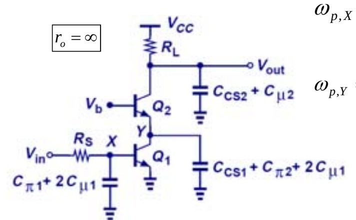
$$\Rightarrow C_X \approx 2C_{XY}$$

Cascode Stage: Pole Frequencies

Cascode stage with BJT capacitances shown

(Miller approximation applied)

$r_o = \infty$



$$\omega_{p,X} = \frac{1}{(R_S \| r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

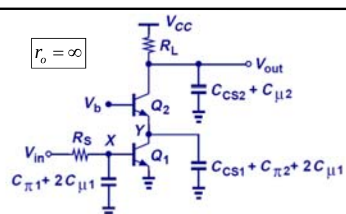
$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}(C_{CS1} + C_{\pi 2} + 2C_{\mu 1})}$$

Note that $\omega_{p,Y} \approx \frac{g_{m2}}{C_{\pi 2}} = 2\pi f_{T2}$

$$\omega_{p,out} = \frac{1}{R_L(C_{CS2} + C_{\mu 2})}$$

Cascode Stage: I/O Impedances

$r_o = \infty$



$$Z_{in} = r_{\pi 1} \| \frac{1}{j\omega(C_{\pi 1} + 2C_{\mu 1})} \qquad Z_{out} = R_L \| \frac{1}{j\omega(C_{\mu 2} + C_{CS2})}$$

Summary of Cascode Stage Benefits

- A cascode stage has high output impedance, which is advantageous for
  - achieving high voltage gain
  - use as a current source
- In a cascode stage, the Miller effect is reduced, for improved performance at high frequencies.