Lecture 2

OUTLINE

- Basic Semiconductor Physics (cont’d)
  - Carrier drift and diffusion
- PN Junction Diodes
  - Electrostatics
  - Capacitance

Reading: Chapter 2.1-2.2

Dopant Compensation

- An N-type semiconductor can be converted into P-type material by counter-doping it with acceptors such that $N_\mathrm{A} > N_\mathrm{D}$.
- A ***compensated semiconductor material*** has both acceptors and donors.

N-type material

($N_\mathrm{D} > N_\mathrm{A}$)

$$n \approx N_D - N_A$$

$$p \approx \frac{n_i^2}{N_D - N_A}$$

P-type material

($N_\mathrm{A} > N_\mathrm{D}$)

$$p \approx N_A - N_D$$

$$n \approx \frac{n_i^2}{N_A - N_D}$$

Types of Charge in a Semiconductor

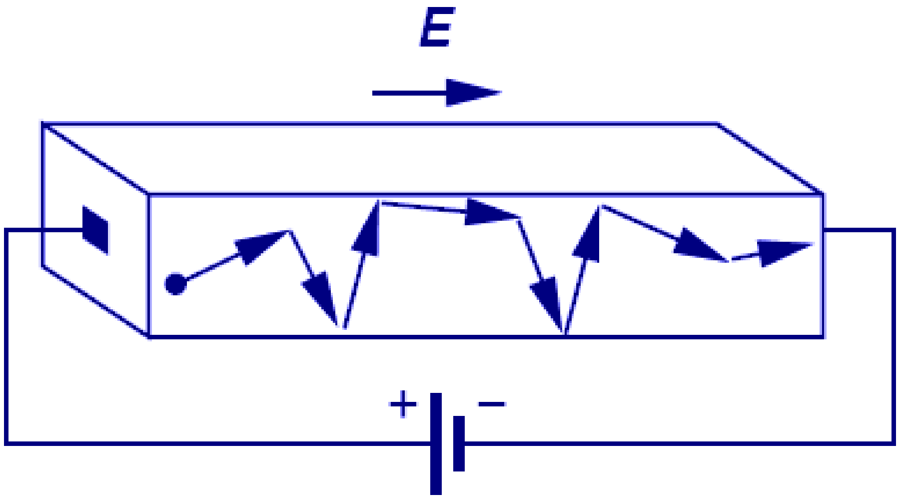
- Negative charges:
  - Conduction electrons (density = n)
  - Ionized acceptor atoms (density = N_A)
- Positive charges:
  - Holes (density = p)
  - Ionized donor atoms (density = N_D)

- The net charge density (C/cm^3) in a semiconductor is

$$\boxed{\rho = q\left(p - n + N_D - N_A\right)}$$

Carrier Drift

- The process in which charged particles move because of an electric field is called ***drift***.
- Charged particles within a semiconductor move with an average velocity proportional to the electric field.
  - The proportionality constant is the carrier ***mobility***.



Hole velocity $\vec{v}_h = \mu_p \vec{E}$

Electron velocity $\vec{v}_e = -\mu_n \vec{E}$

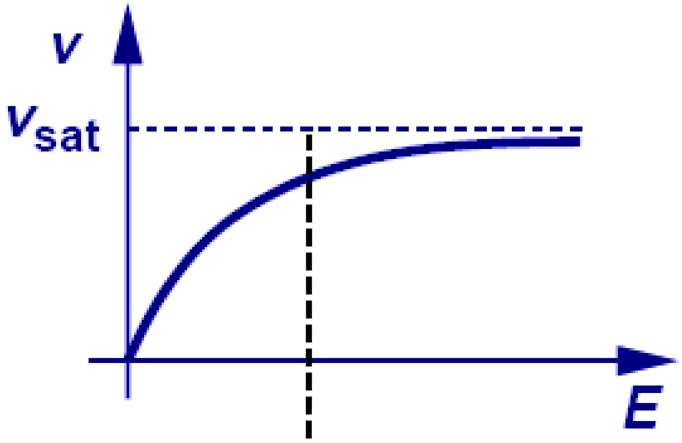
Notation:

$\mu_p \equiv$ hole mobility ($cm^2/V \cdot s$)

$\mu_n \equiv$ electron mobility ($cm^2/V \cdot s$)

Velocity Saturation

- In reality, carrier velocities saturate at an upper limit, called the ***saturation velocity*** (***v***$_{\text{sat}}$).



$$E_{sat} \approx 10^4 \text{ V/cm}$$

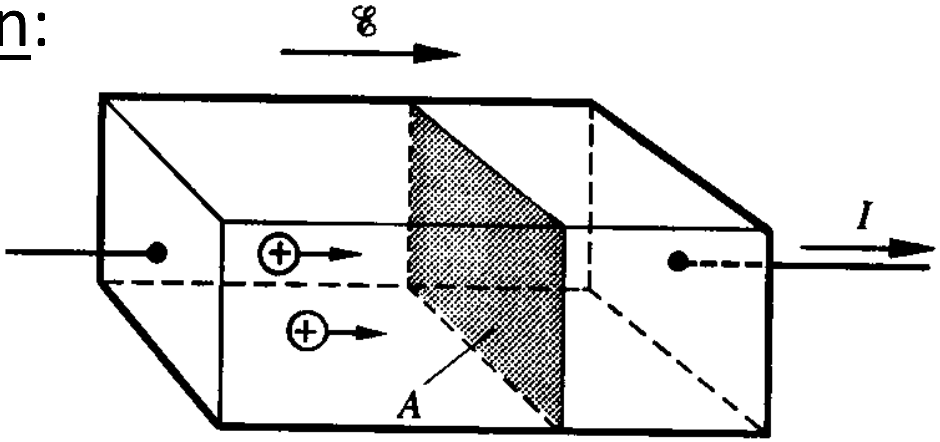
$$\mu = \frac{\mu_0}{1 + bE}$$

$$v_{sat} = \frac{\mu_0}{b}$$

$$v = \frac{\mu_0}{1 + \frac{\mu_0 E}{v_{sat}}} E$$

Drift Current

- Drift current is proportional to the carrier velocity and carrier <u>concentration</u>:



$v_h \, t \, A$ = volume from which all holes cross plane in time t

$p \, v_h \, t \, \mathrm{A}$ = # of holes crossing plane in time t

$q \, p \, v_h \, t \, \mathrm{A}$ = charge crossing plane in time t

$q \, p \, v_h \, A$ = charge crossing plane per unit time = hole current

➔ **Hole current per unit area (*i.e.* current density) $J_{p,drift} = q \, p \, v_h$**

Conductivity and Resistivity

- In a semiconductor, both electrons and holes conduct current:

$$J_{p,drift} = qp\mu_p E \qquad J_{n,drift} = -qn(-\mu_n E)$$

$$J_{tot,drift} = J_{p,drift} + J_{n,drift} = qp\mu_p E + qn\mu_n E$$

$$J_{tot,drift} = q(p\mu_p + n\mu_n)E \equiv \sigma E$$

- Conductivity

$$\boxed{\sigma \equiv qp\mu_p + qn\mu_n \qquad \text{[unit: mho/cm = S/cm]}}$$

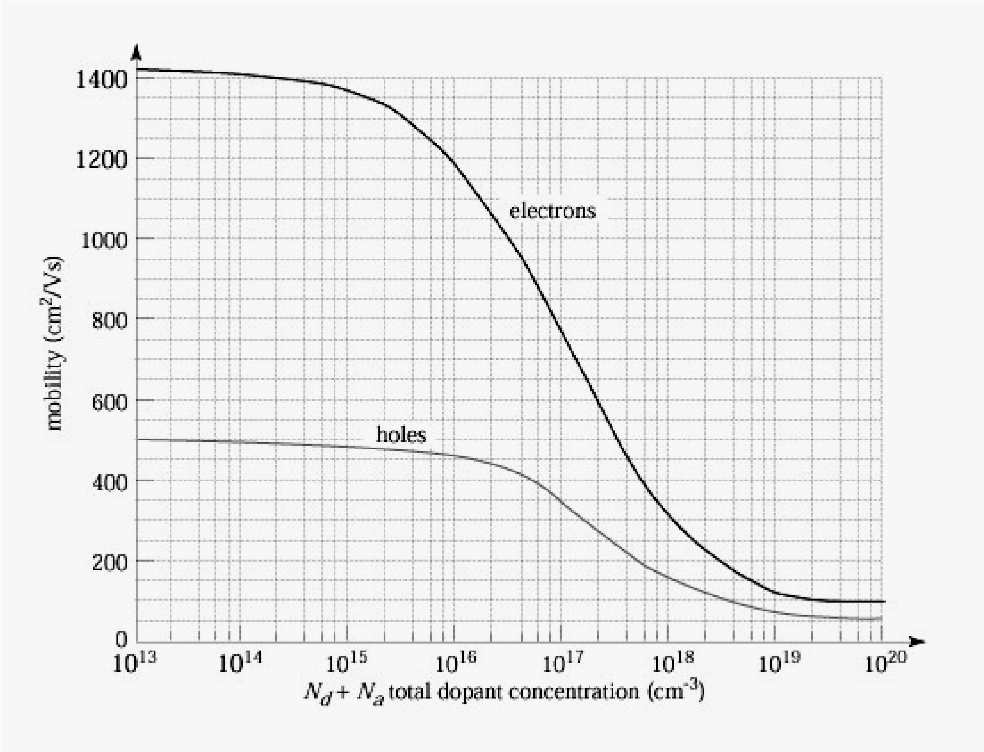
- Resistivity

$$\rho \equiv \frac{1}{\sigma} \qquad \text{[Unit: }\Omega\text{-cm]}$$

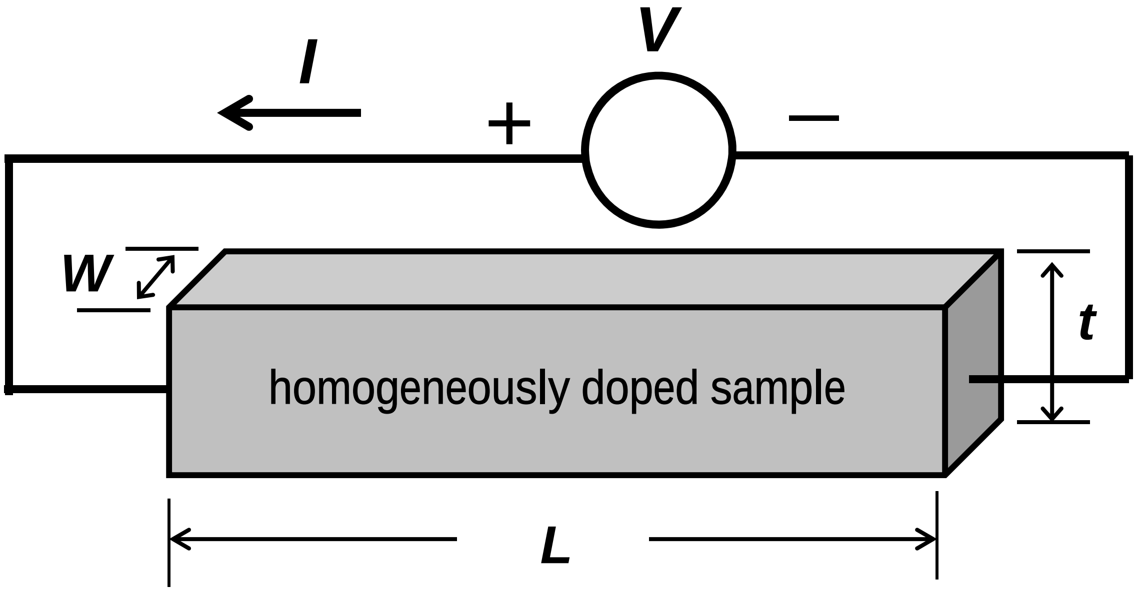
- Typical resistivity range for Si: $10^{-3} \sim 10^{3}$ Ω-cm

Resistivity Example

- Estimate the resistivity of a Si sample doped with phosphorus to a concentration of 10^{15} cm^{-3} and boron to a concentration of 10^{17} cm^{-3}. The electron mobility and hole mobility are 800 cm^2/Vs and 300 cm^2/Vs, respectively.



Electrical Resistance



Resistance

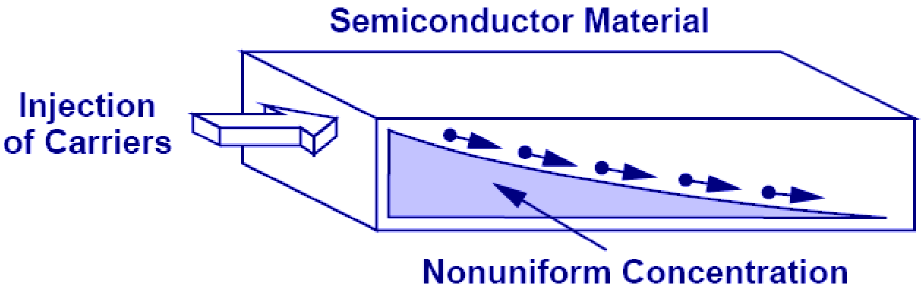
$$\boxed{R \equiv \frac{V}{I} = \rho \frac{L}{Wt}}$$

(Unit: ohms)

where ρ is the resistivity

Carrier Diffusion

- Due to thermally induced random motion, mobile particles tend to move from a region of high concentration to a region of low concentration.
  - Analogy: ink droplet in water
- Current flow due to mobile charge diffusion is proportional to the carrier <u>concentration gradient</u>.
  - The proportionality constant is the ***diffusion constant***.



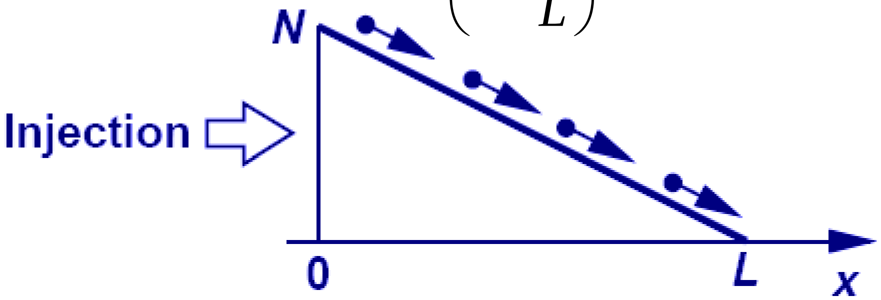
$$J_p = -qD_p \frac{dp}{dx}$$

<u>Notation</u>:
$D_p \equiv$ hole diffusion constant (cm^2/s)
$D_n \equiv$ electron diffusion constant (cm^2/s)

Diffusion Examples

- Linear concentration profile
  → constant diffusion current

$$p = N\left(1 - \frac{x}{L}\right)$$

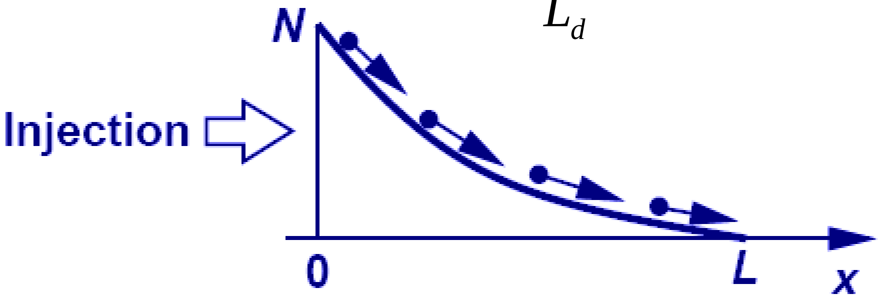


$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= qD_p \frac{N}{L}$$

- Non-linear concentration profile
  → varying diffusion current

$$p = N \exp \frac{-x}{L_d}$$



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= \frac{qD_p N}{L_d} \exp \frac{-x}{L_d}$$

Diffusion Current

- Diffusion current within a semiconductor consists of hole and electron components:

$$J_{p,diff} = -qD_p \frac{dp}{dx} \qquad J_{n,diff} = qD_n \frac{dn}{dx}$$

$$J_{tot,diff} = q(D_n \frac{dn}{dx} - D_p \frac{dp}{dx})$$

- The total current flowing in a semiconductor is the sum of drift current and diffusion current:

$$\boxed{J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff}}$$

The Einstein Relation

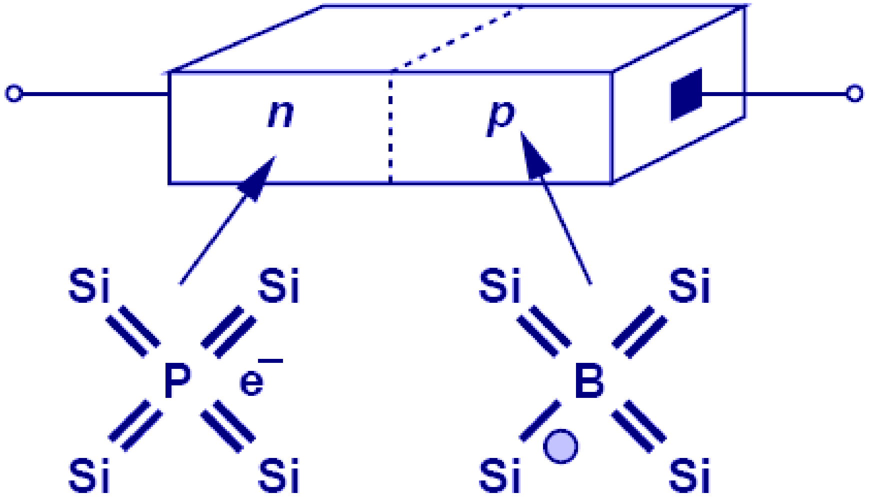
- The characteristic constants for drift and diffusion are related:

$$\boxed{\frac{D}{\mu} = \frac{kT}{q}}$$

- Note that $\frac{kT}{q} \cong 26\text{mV}$ at room temperature (300K)
  - This is often referred to as the “thermal voltage”.

The PN Junction Diode

- When a P-type semiconductor region and an N-type semiconductor region are in contact, a PN junction diode is formed.



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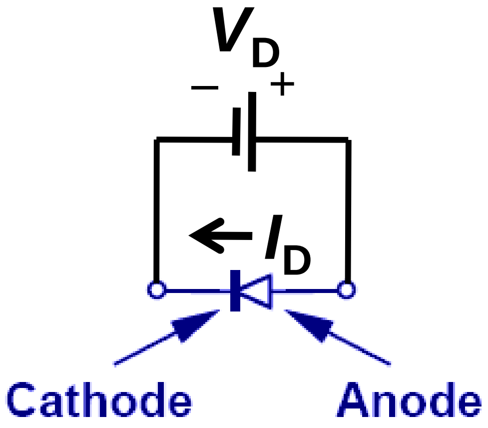

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Diode Operating Regions

- In order to understand the operation of a diode, it is necessary to study its behavior in three operation regions: equilibrium, reverse bias, and forward bias.

$V_D = 0$

PN Junction in Equilibrium

- Depletion Region
- Built-in Potential

⇨

$V_D < 0$

PN Junction Under Reverse Bias

- Junction Capacitance

⇨

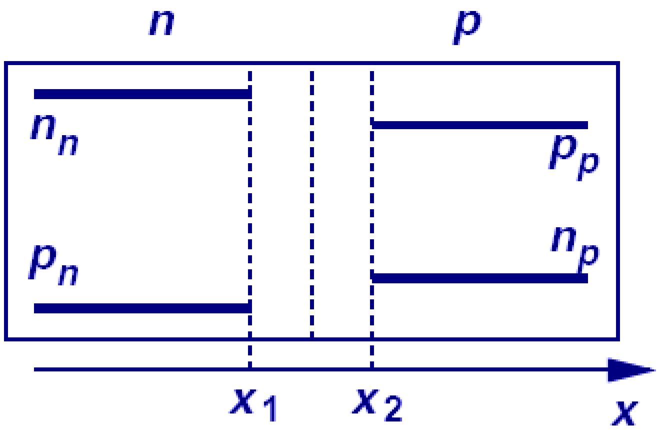
$V_D > 0$

PN Junction Under Forward Bias

- I/V Characteristics

Carrier Diffusion across the Junction

- Because of the difference in hole and electron concentrations on each side of the junction, carriers diffuse across the junction:

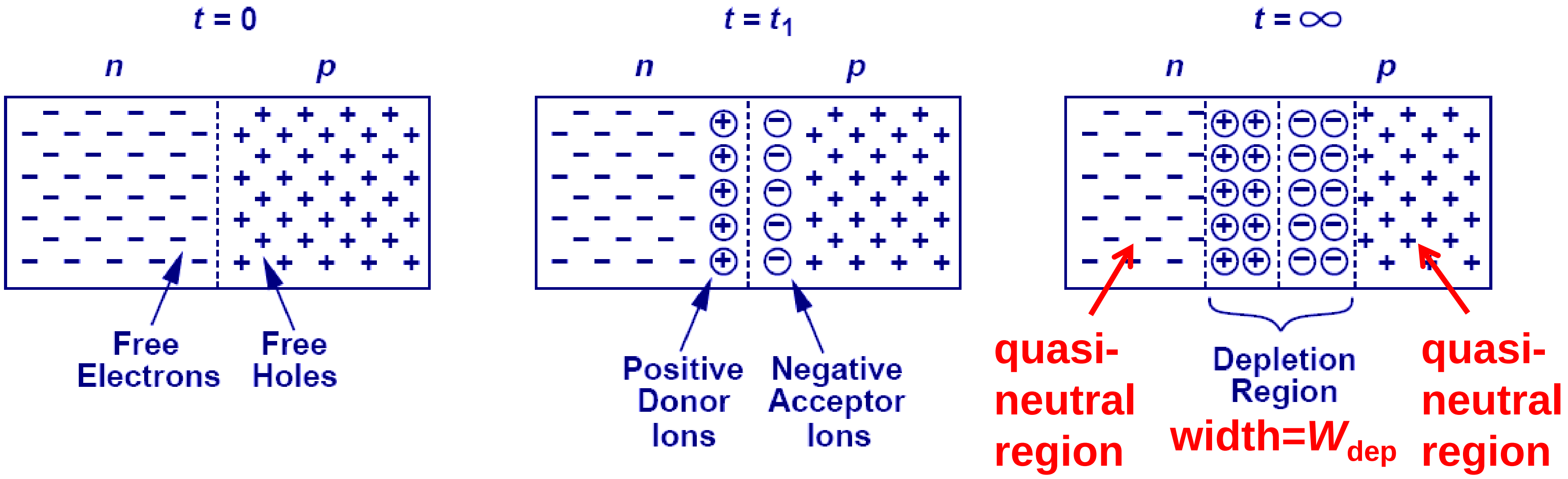


Notation:

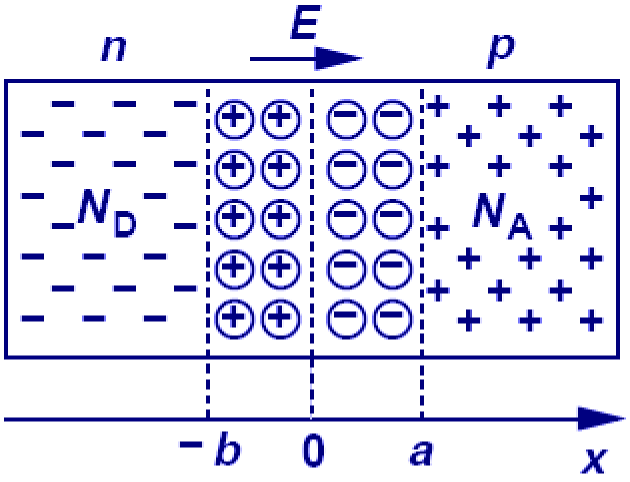
$n_n \equiv$ electron concentration on N-type side (cm^{-3})
$p_n \equiv$ hole concentration on N-type side (cm^{-3})
$p_p \equiv$ hole concentration on P-type side (cm^{-3})
$n_p \equiv$ electron concentration on P-type side (cm^{-3})

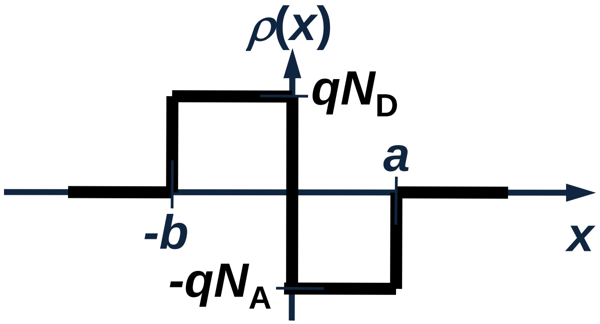
Depletion Region

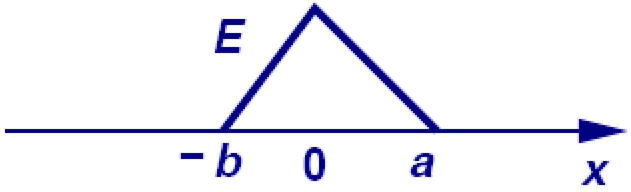
- As conduction electrons and holes diffuse across the junction, they leave behind ionized dopants. Thus, a region that is depleted of mobile carriers is formed.
  - The charge density in the depletion region is not zero.
  - The carriers which diffuse across the junction recombine with majority carriers, *i.e.* they are annihilated.



The Depletion Approximation







In the depletion region on the **N side**:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}} = \frac{qN_D}{\varepsilon_{si}}$$

Gauss's Law

$$E = \frac{qN_D}{\varepsilon_{si}}(x+b)$$

$$\varepsilon_{si} = 10^{12} \text{ F/cm}$$

In the depletion region on the **P side**:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}} = \frac{-qN_A}{\varepsilon_{si}}$$

$$E = \frac{qN_A}{\varepsilon_{si}}(a-x)$$

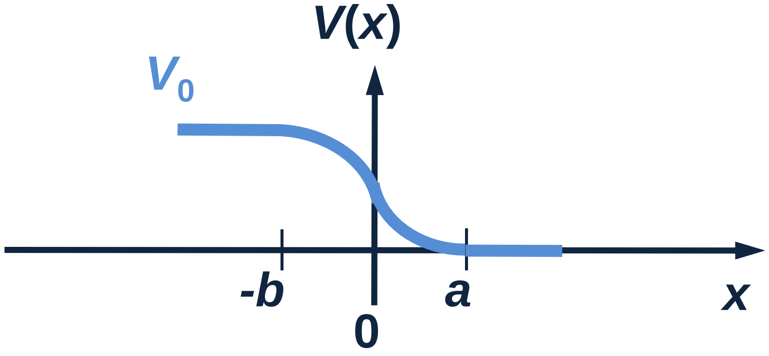
$$\boxed{aN_A = bN_D}$$

Potential Distribution

- In the depletion region, the electric potential is quadratic since the electric field is linear
- The potential difference between the N and the P side is called built-in potential, V_0

$$E = -\frac{dV}{dx}$$

$$V = -\int E \cdot dx$$



PN Junction in Equilibrium

- In equilibrium, the drift and diffusion components of current are balanced; therefore the net current flowing across the junction is zero.

$$J_{p,drift} = -J_{p,diff}$$

$$J_{n,drift} = -J_{n,diff}$$

$$J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff} = 0$$

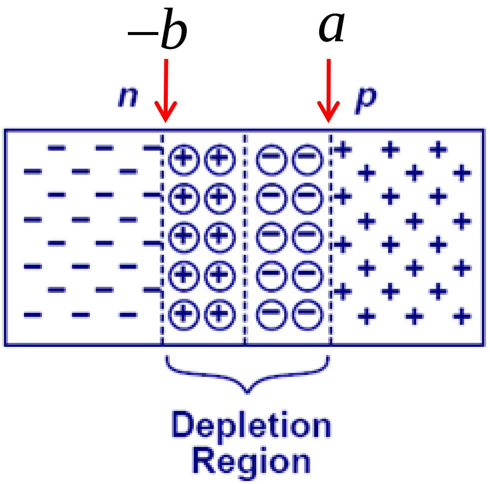
Built-in Potential, V_0

- Because of the electric field in the depletion region, there exists a potential drop across the junction:

$$qp\mu_p E = qD_p \frac{dp}{dx} \quad \Rightarrow \quad p\mu_p\left(-\frac{dV}{dx}\right) = D_p \frac{dp}{dx}$$

$$\Rightarrow \quad -\mu_p \int_{-b}^{a} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}$$

$$\Rightarrow \quad V(-b) - V(a) = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \frac{N_A}{\left(n_i^2 / N_D\right)}$$

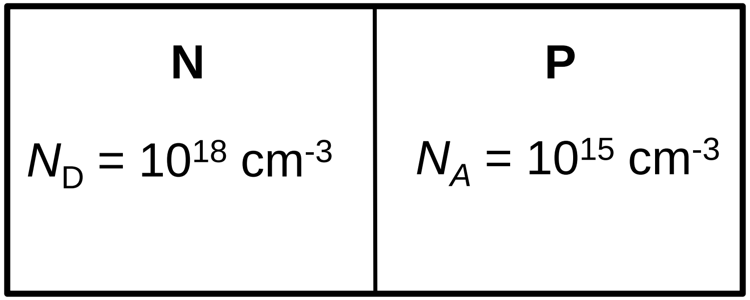


$$\boxed{V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}}$$

(Unit: Volts)

Built-In Potential Example

- Estimate the built-in potential for PN junction below.



N

$N_D = 10^{18}\ \text{cm}^{-3}$

P

$N_A = 10^{15}\ \text{cm}^{-3}$

$$V_0 = \frac{kT}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right) = (26\text{mV})\ln\left(\frac{10^{18}10^{15}}{10^{20}}\right) = (26\text{mV})\ln\left(10^{13}\right)$$

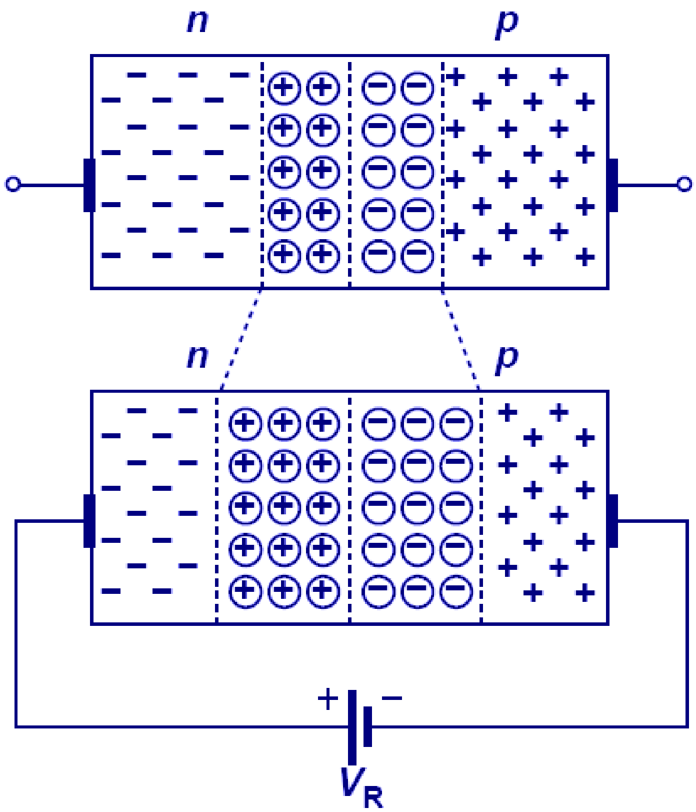
$$\text{Note: } \frac{kT}{q}\ln(10) \cong 26\text{mV} \times 2.3 \cong 60\text{mV}$$

$$V_0 = 60\text{mV} \times 13 = 780\text{mV}$$

PN Junction under Reverse Bias

- A reverse bias increases the potential drop across the junction. As a result, the magnitude of the electric field increases and the width of the depletion region widens.

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}}$$

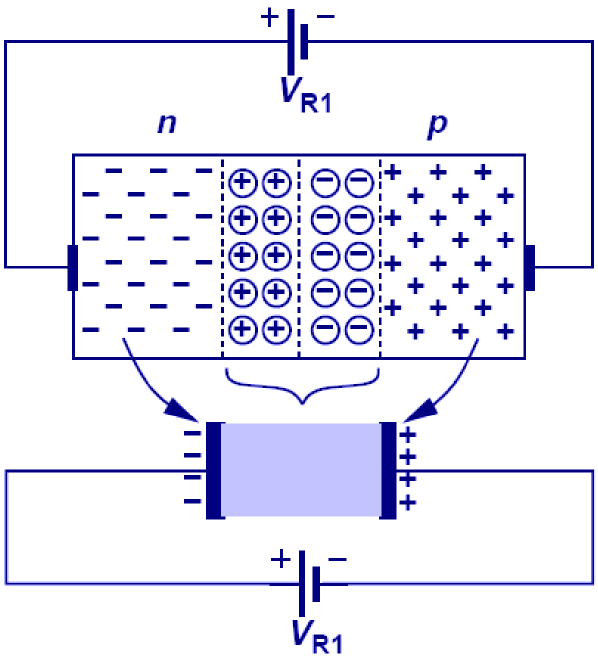


Diode Current under Reverse Bias

- In equilibrium, the built-in potential effectively prevents carriers from diffusing across the junction.
- Under reverse bias, the potential drop across the junction increases; therefore, negligible diffusion current flows. **A very small drift current flows**, limited by the rate at which minority carriers diffuse from the quasi-neutral regions into the depletion region.

PN Junction Capacitance

- A reverse-biased PN junction can be viewed as a capacitor. The depletion width (W_{dep}) and hence the junction capacitance (C_j) varies with V_R.



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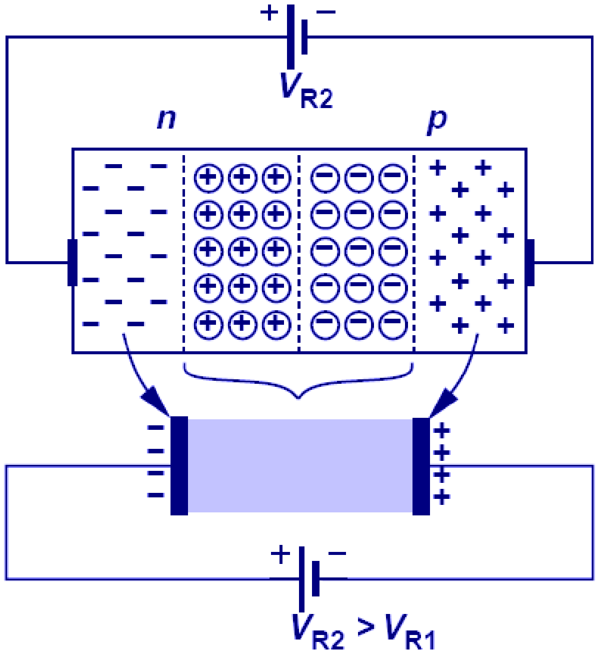

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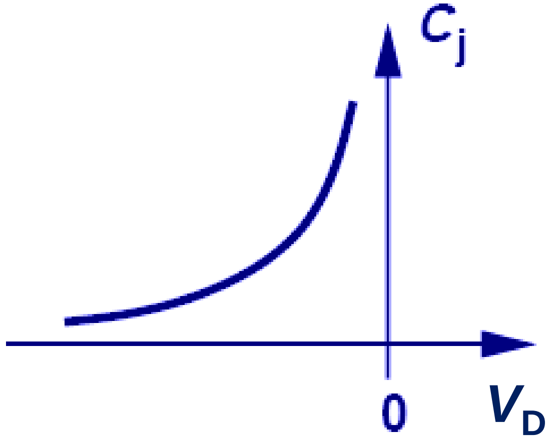
$$\boxed{C_j = \frac{\varepsilon_{si}}{W_{dep}} \quad [\mathrm{F/cm}^2]}$$

Voltage-Dependent Capacitance



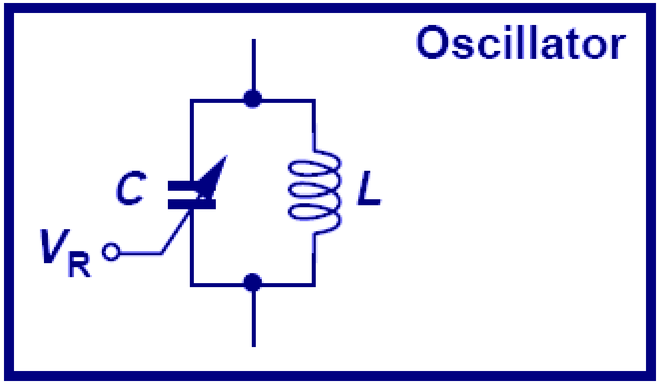
$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

$$C_{j0} = \sqrt{\frac{\varepsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0}}$$

$\varepsilon_{si} \cong 10^{-12}$ F/cm is the permittivity of silicon

Reverse-Biased Diode Application

- A very important application of a reverse-biased PN junction is in a voltage controlled oscillator (VCO), which uses an LC tank. By changing V_R, we can change C, which changes the oscillation frequency.



$$f_{res} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Summary

- Current flowing in a semiconductor is comprised of drift and diffusion components: $J_{tot} = qp\mu_p E + qn\mu_n E + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$
- A region depleted of mobile charge exists at the junction between P-type and N-type materials.
  - A built-in potential drop (V_0) across this region is established by the charge density profile; it opposes diffusion of carriers across the junction. A reverse bias voltage serves to enhance the potential drop across the depletion region, resulting in very little (drift) current flowing across the junction.
  - The width of the depletion region (W_{dep}) is a function of the bias voltage (V_D).

$$W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 - V_D)} \qquad V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^{\,2}}$$