Lecture 21

OUTLINE

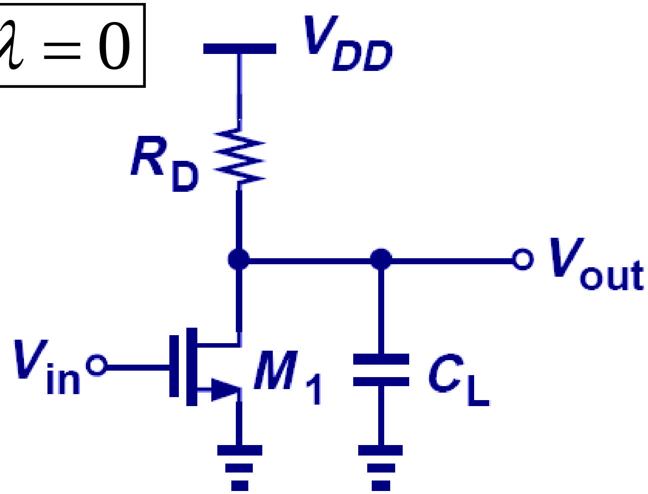
- Frequency Response
  - Review of basic concepts
  - high-frequency MOSFET model
  - CS stage
  - CG stage
  - Source follower
  - Cascode stage

- Reading: Chapter 11

A_v Roll-Off due to C_L

- The impedance of C_L decreases at high frequencies, so that it shunts some of the output current to ground.

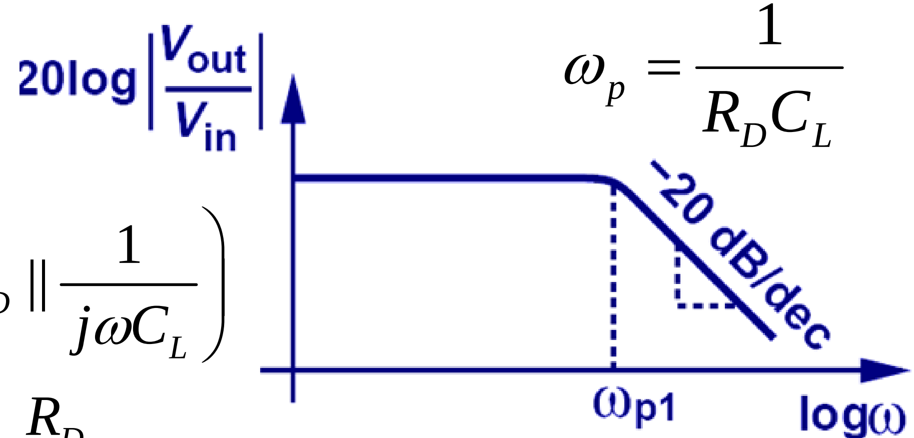
$\lambda = 0$



$$A_v = -g_m\left(R_D \parallel \frac{1}{j\omega C_L}\right)$$

$$= -g_m \frac{R_D}{1 + j\omega R_D C_L}$$

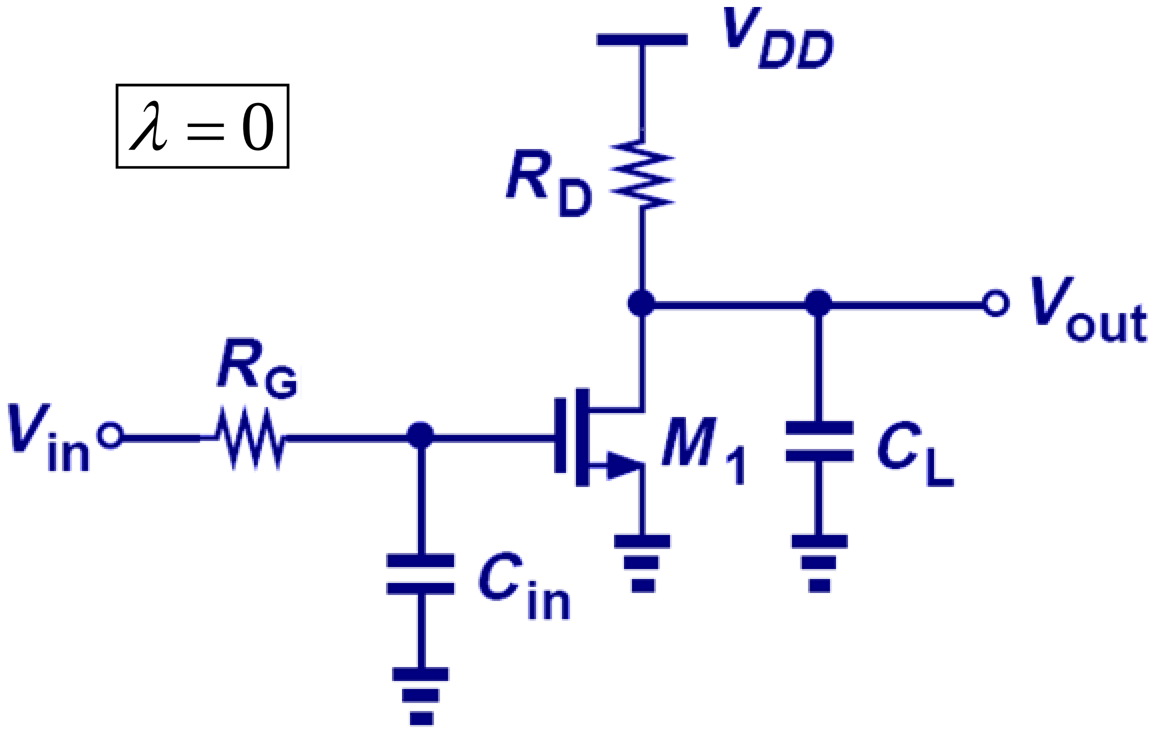
$$\omega_p = \frac{1}{R_D C_L}$$



- In general, if **node *j* in the signal path** has a small-signal resistance of $\boldsymbol{R_j}$ to ground and a capacitance $\boldsymbol{C_j}$ to ground, then it contributes a **pole at frequency $(R_j C_j)^{-1}$**

Pole Identification Example 1

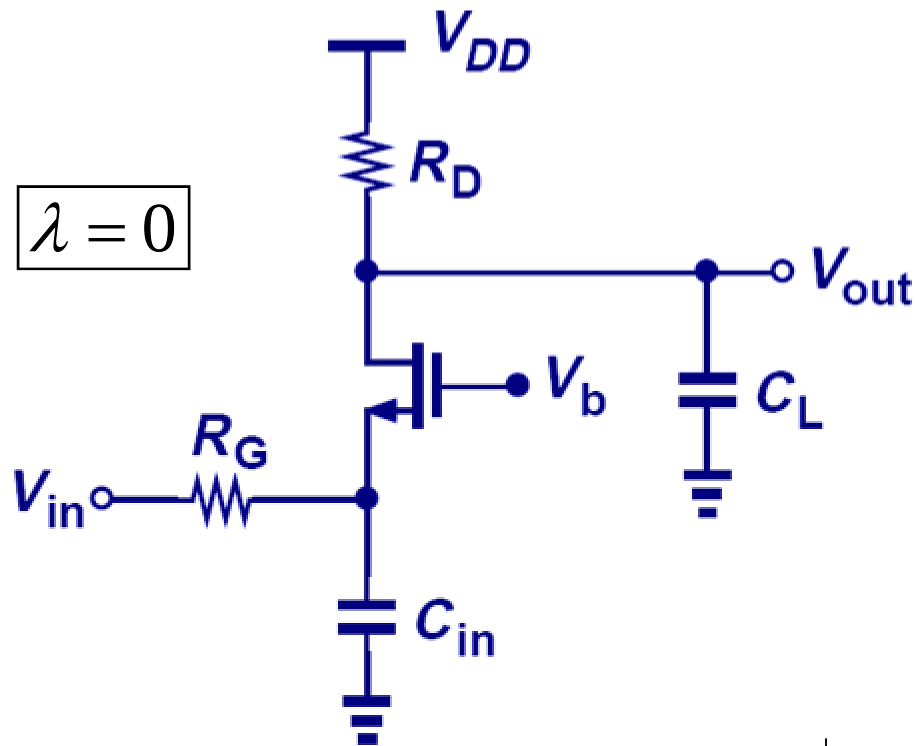
$$\lambda = 0$$



$$|\omega_{p1}| = \frac{1}{R_G C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Pole Identification Example 2



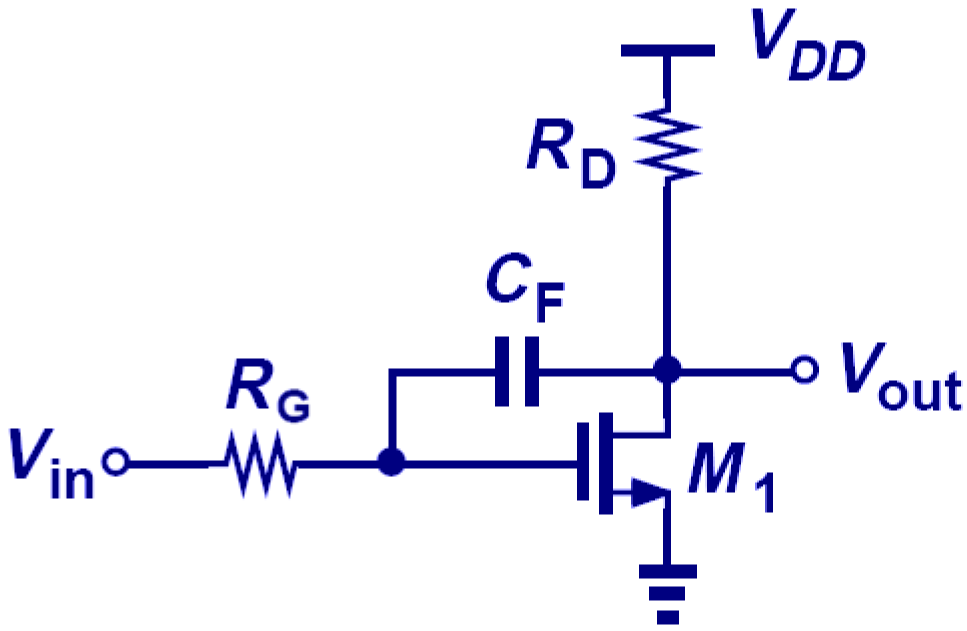
$$\lambda = 0$$

$$\left|\omega_{p1}\right| = \frac{1}{\left(R_G \parallel \frac{1}{g_m}\right)C_{in}}$$

$$\left|\omega_{p2}\right| = \frac{1}{R_D C_L}$$

Dealing with a Floating Capacitance

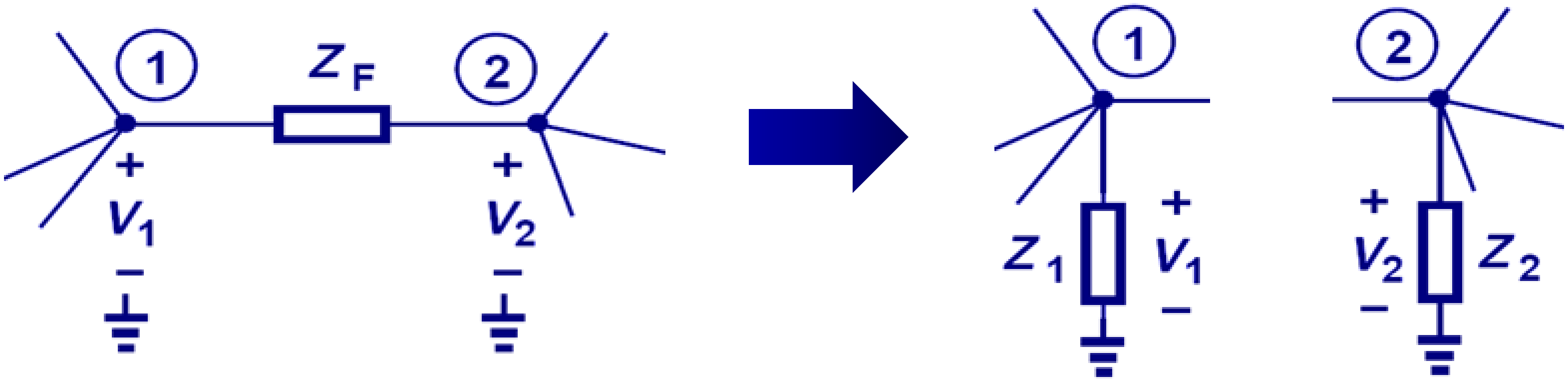
- Recall that a pole is computed by finding the resistance and capacitance between a node and (AC) GROUND.
- It is not straightforward to compute the pole due to C_F in the circuit below, because neither of its terminals is grounded.



Miller's Theorem

- If A_v is the voltage gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2:

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1} \quad \Rightarrow \quad Z_1 = Z_F \frac{V_1}{V_1 - V_2} = Z_F \frac{1}{1 - A_v}$$

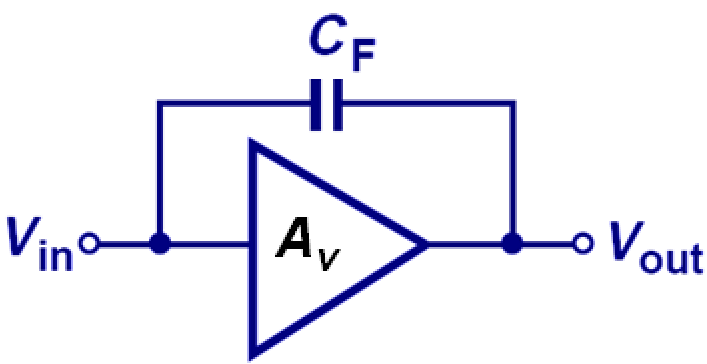


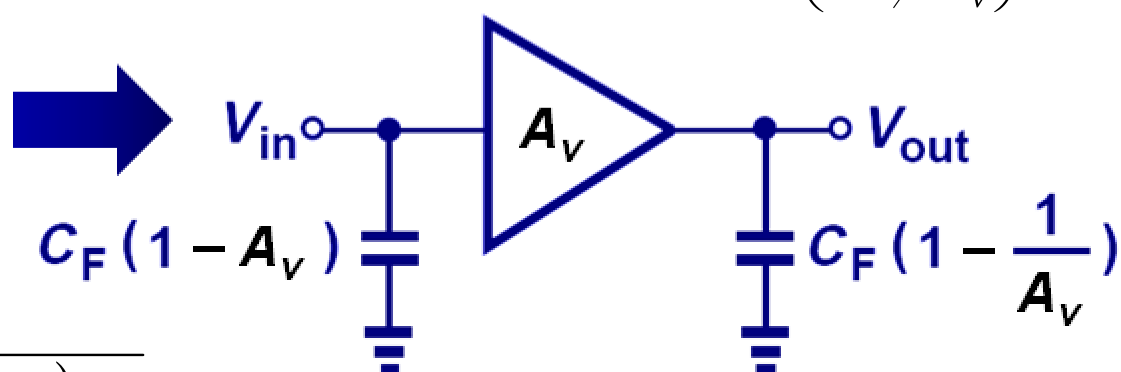
$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2} \quad \Rightarrow \quad Z_2 = -Z_F \frac{V_2}{V_1 - V_2} = Z_F \frac{1}{1 - 1/A_v}$$

Miller Multiplication

- Applying Miller's theorem, we can convert a floating capacitance between the input and output nodes of an amplifier into two grounded capacitances.
- **The capacitance at the input node is larger than the original floating capacitance.**

$$Z_2 = \frac{Z_F}{1-\frac{1}{A_v}} = \frac{\frac{1}{j\omega C_F}}{1-\frac{1}{A_v}} = \frac{1}{j\omega\left(1-\frac{1}{A_v}\right)C_F}$$

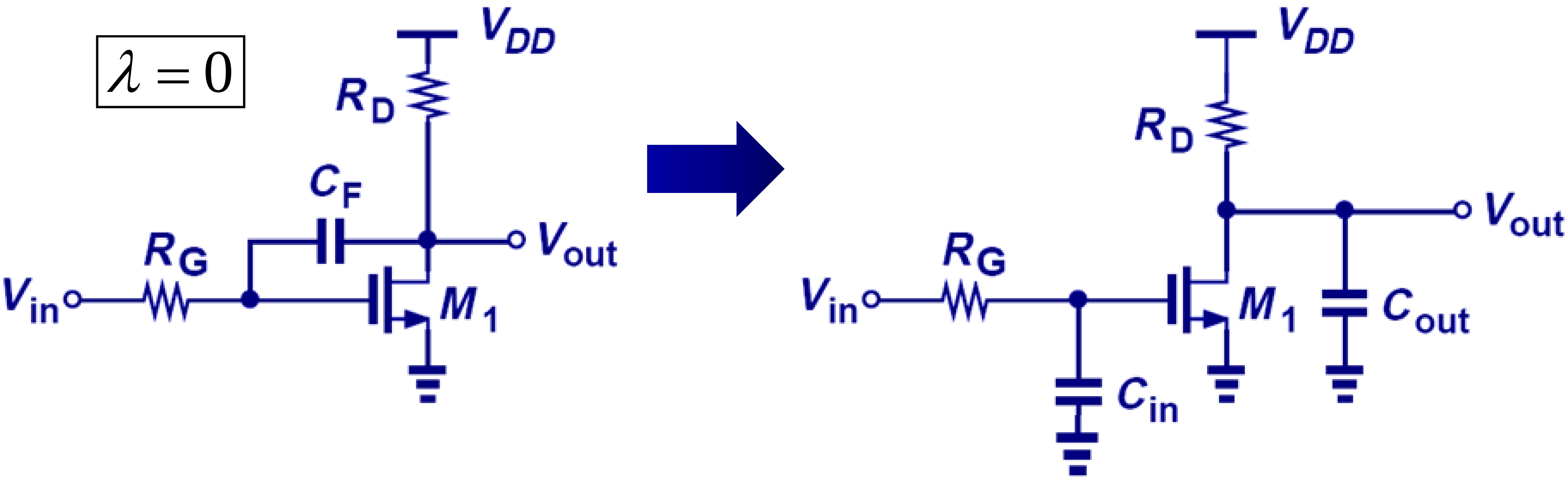




$$Z_1 = \frac{Z_F}{1-A_v} = \frac{\frac{1}{j\omega C_F}}{1-A_v} = \frac{1}{j\omega(1-A_v)C_F}$$

Application of Miller's Theorem

$$\lambda = 0$$



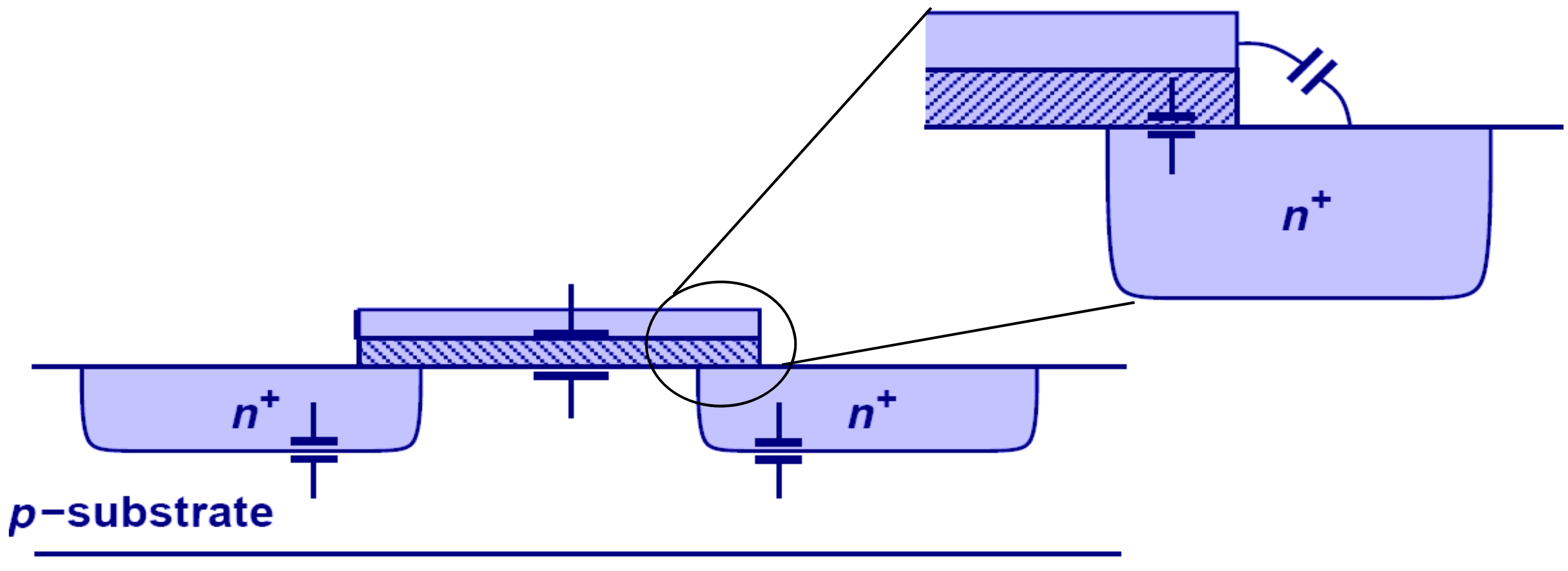
$$\omega_{in} = \frac{1}{R_G(1 + g_m R_D)C_F}$$

$$\omega_{out} = \frac{1}{R_D\left(1 + \frac{1}{g_m R_D}\right)C_F}$$

MOSFET Intrinsic Capacitances

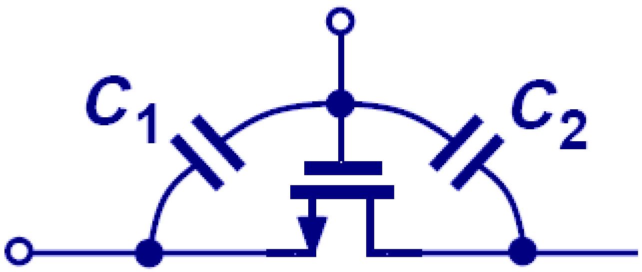
The MOSFET has intrinsic capacitances which affect its performance at high frequencies:

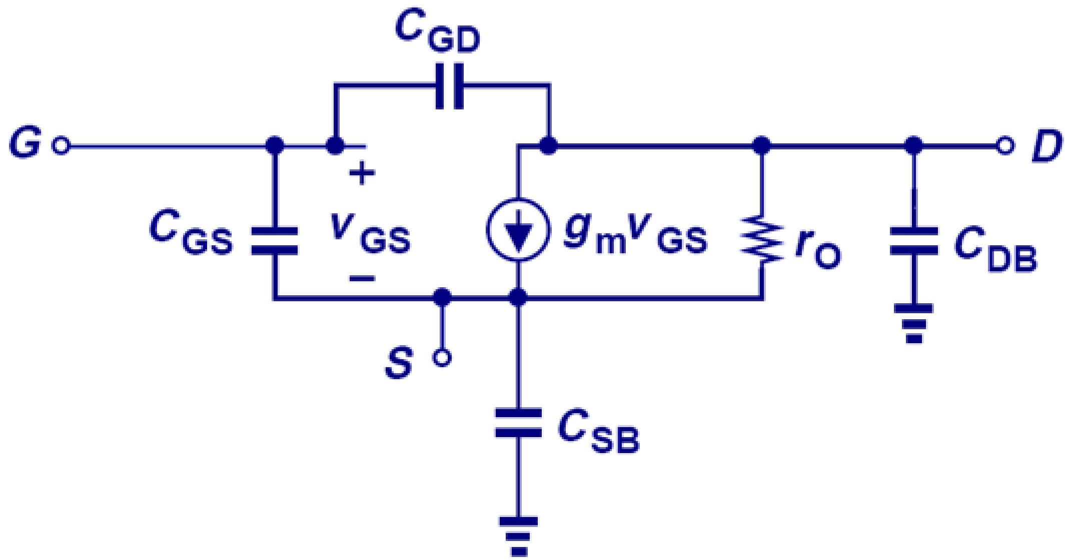
1. gate oxide capacitance between the gate and channel,
2. overlap and fringing capacitances between the gate and the source/drain regions, and
3. source-bulk & drain-bulk junction capacitances (C_{SB} & C_{DB}).



High-Frequency MOSFET Model

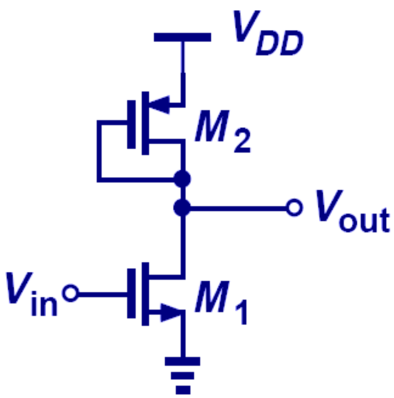
- The gate oxide capacitance can be decomposed into a capacitance between the gate and the source (C_1) and a capacitance between the gate and the drain (C_2).
  - In saturation, $C_1 \cong (2/3)\times C_{gate}$, and $C_2 \cong 0$. ($C_{gate}=C_{ox}WL$)
  - C_1 in parallel with the source overlap/fringing capacitance → C_{GS}
  - C_2 in parallel with the drain overlap/fringing capacitance → C_{GD}



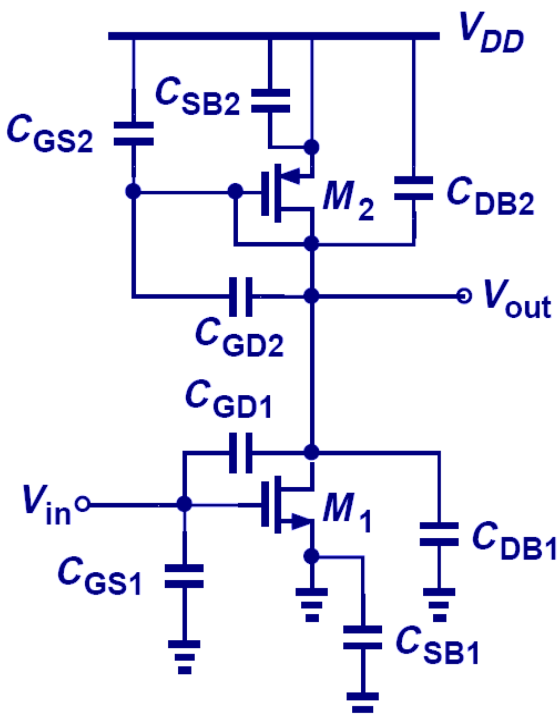


Example

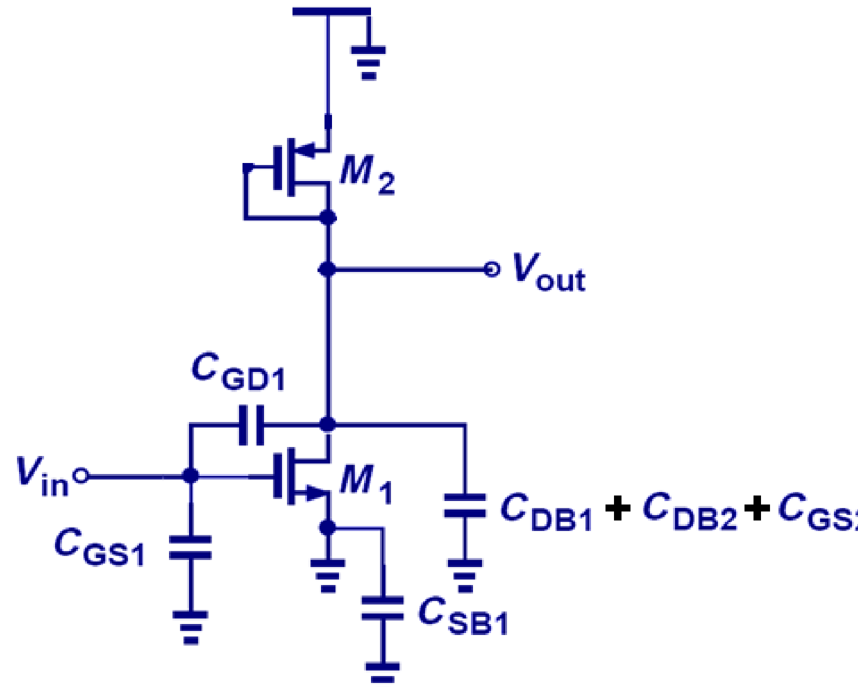
CS stage



…with MOSFET capacitances explicitly shown



Simplified circuit for high-frequency analysis



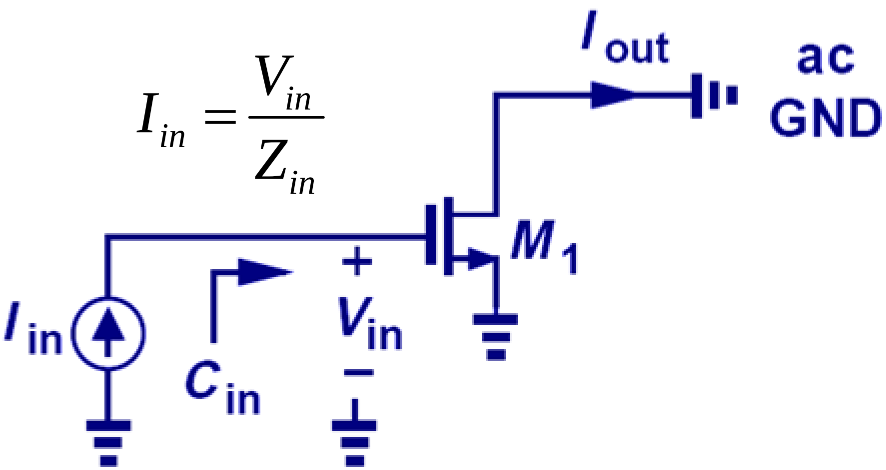
Transit Frequency

- The "transit" or "cut-off" frequency, f_T, is a measure of the intrinsic speed of a transistor, and is defined as the frequency where the current gain falls to 1.

Conceptual set-up to measure f_T

$$I_{out} = g_m V_{in}$$

$$I_{in} = \frac{V_{in}}{Z_{in}}$$



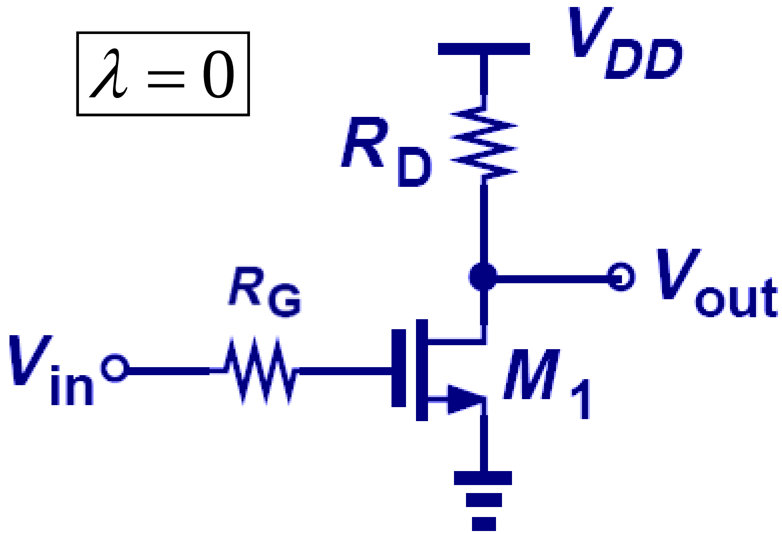
$$\left|\frac{I_{out}}{I_{in}}\right| = \left|g_m Z_{in}\right| = \left|g_m\left(\frac{1}{j\omega_T C_{in}}\right)\right| = 1$$

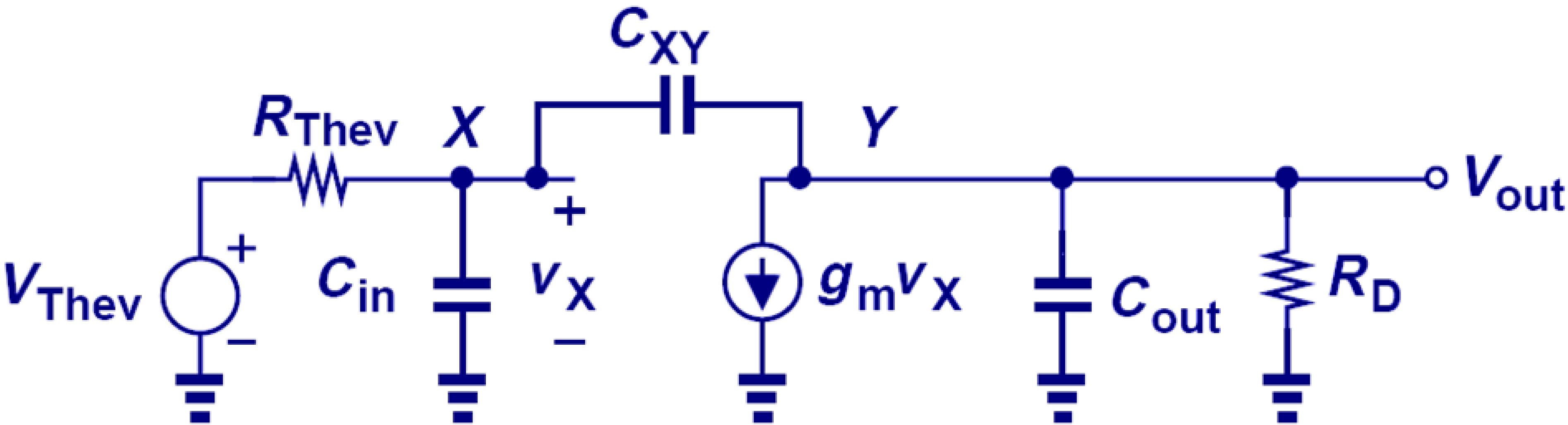
$$\Rightarrow \omega_T = \frac{g_m}{C_{in}}$$

$$\boxed{2\pi f_T = \frac{g_m}{C_{GS}}}$$

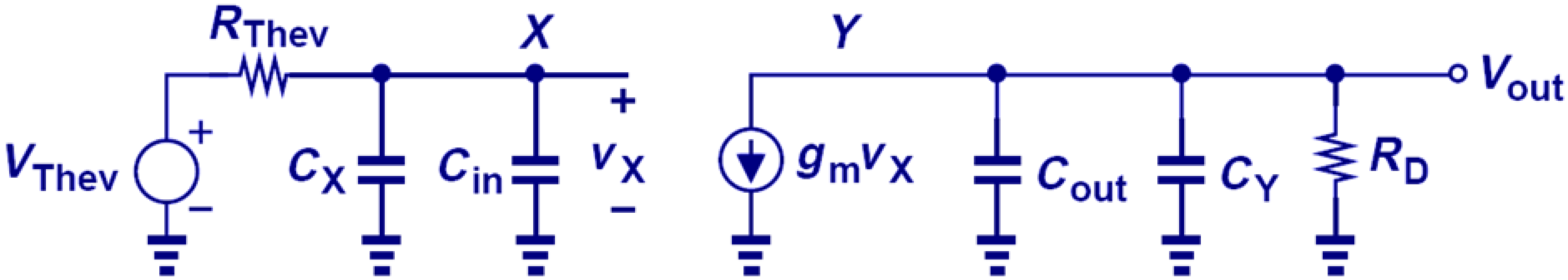
Small-Signal Model for CS Stage

$\lambda = 0$





… Applying Miller's Theorem



$V_{Thev} = V_{in}$

$R_{Thev} = R_G$

$C_X = C_{GD}\,(1 + g_m R_D)$

$C_Y = C_{GD}\,(1 + \frac{1}{g_m R_D})$

$$\omega_{p,in} = \frac{1}{R_{Thev}\left(C_{in} + (1 + g_m R_D)C_{GD}\right)}$$

$$\omega_{p,out} = \frac{1}{R_D\left(C_{out} + \left(1 + \frac{1}{g_m R_D}\right)C_{GD}\right)}$$

Note that $\omega_{p,out} > \omega_{p,in}$

Direct Analysis of CS Stage

- Direct analysis yields slightly different pole locations and an extra zero:

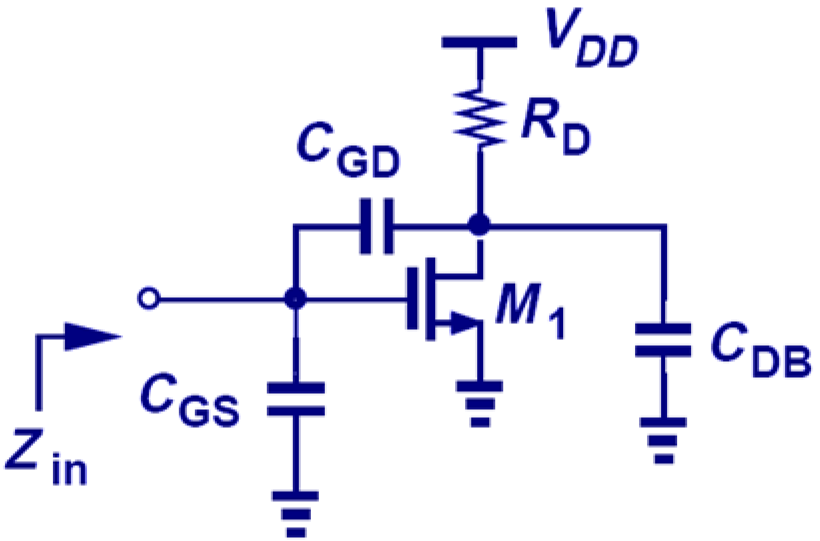
$$\omega_z = \frac{g_m}{C_{XY}}$$

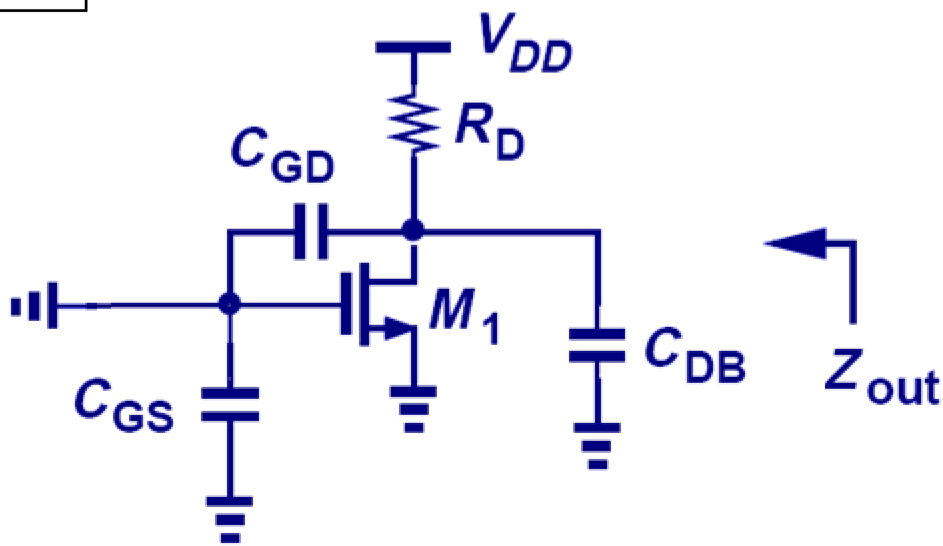
$$\omega_{p1} = \frac{1}{(1+g_m R_D)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_D(C_{XY}+C_{out})}$$

$$\omega_{p2} = \frac{(1+g_m R_D)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_D(C_{XY}+C_{out})}{R_{Thev}R_D(C_{in}C_{XY}+C_{out}C_{XY}+C_{in}C_{out})}$$

I/O Impedances of CS Stage

$\lambda = 0$





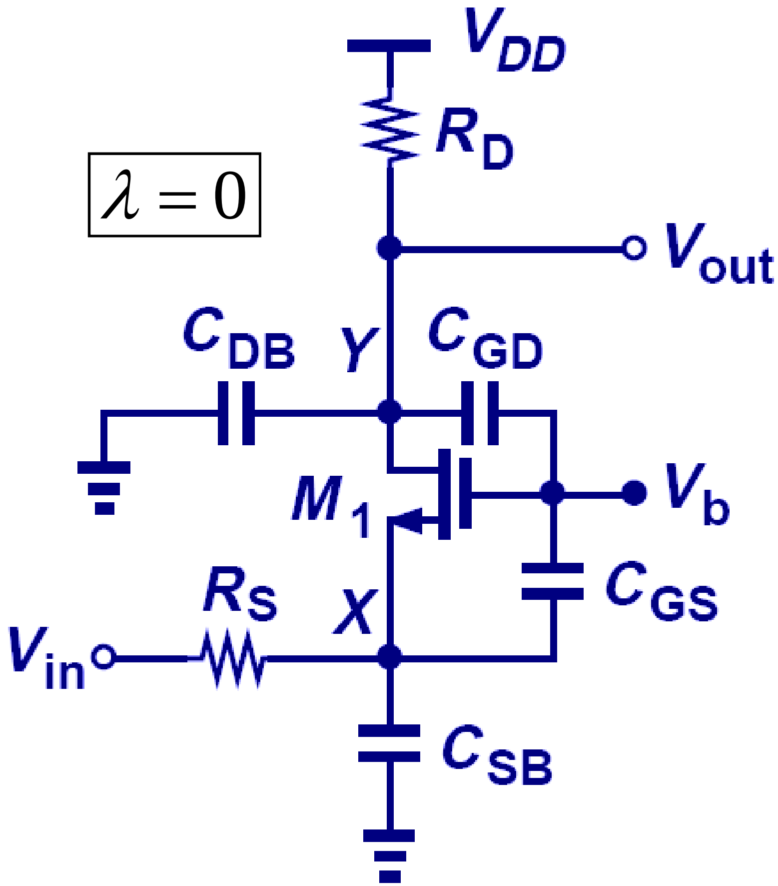
$$Z_{in} \approx \frac{1}{j\omega[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

$$Z_{out} = \frac{1}{j\omega[C_{GD} + C_{DB}]} \parallel R_D$$

CG Stage: Pole Frequencies

CG stage with MOSFET capacitances shown

$$\lambda = 0$$



$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right) C_X}$$

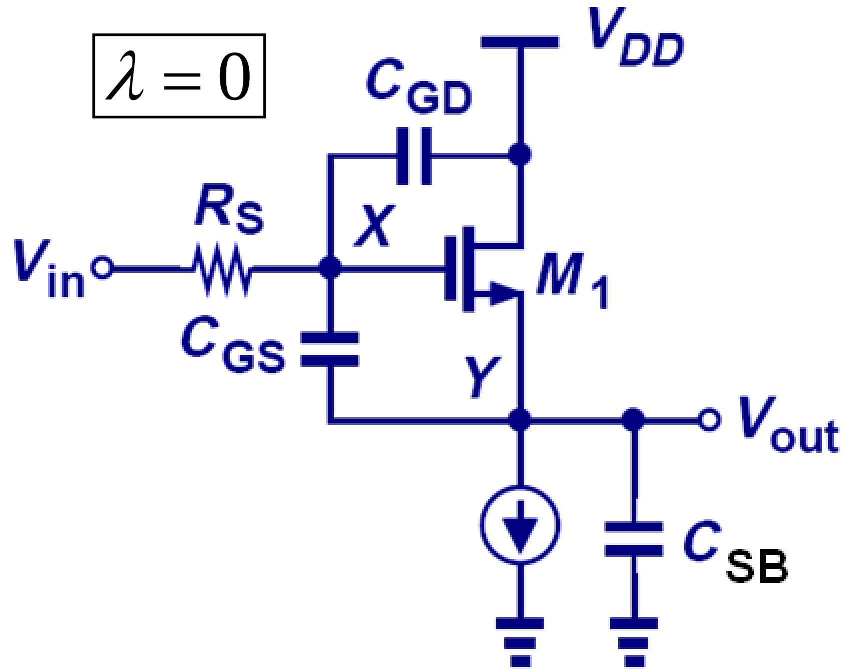
$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_D C_Y}$$

$$C_Y = C_{GD} + C_{DB}$$

AC Analysis of Source Follower

$\lambda = 0$



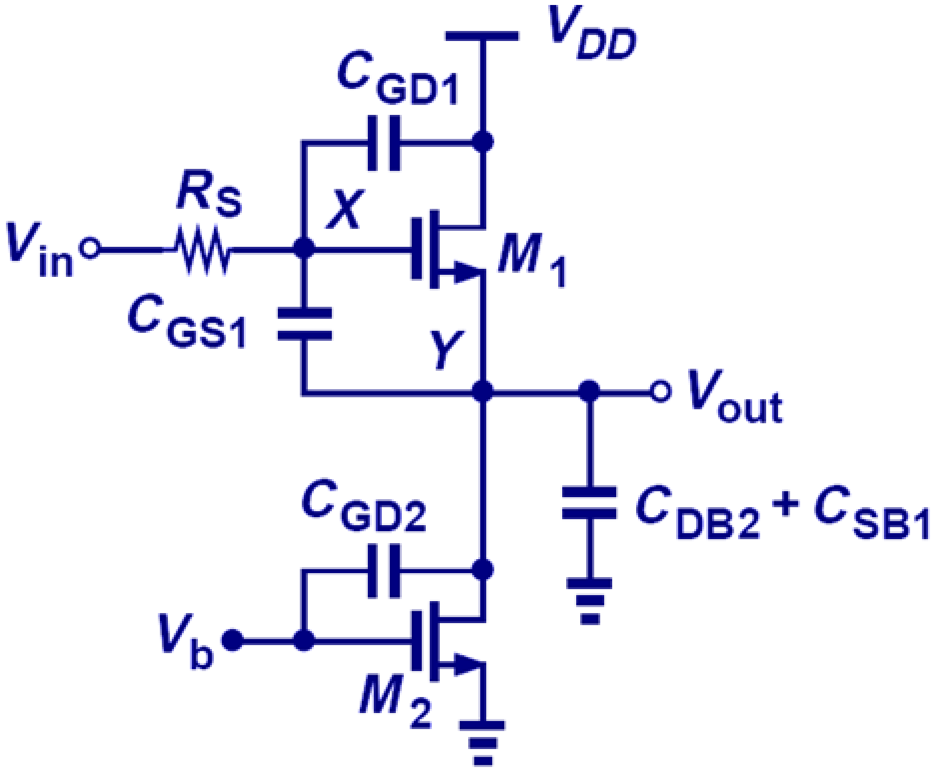
- The transfer function of a source follower can be obtained by direct AC analysis, similarly as for the emitter follower

$$\frac{v_{out}}{v_{in}} = \frac{1 + (j\omega)\dfrac{C_{GS}}{g_m}}{a(j\omega)^2 + b(j\omega) + 1}$$

$$a = \frac{R_S}{g_m}\left(C_{GD}C_{GS} + C_{GD}C_{SB} + C_{GS}C_{SB}\right)$$

$$b = R_S C_{GD} + \frac{C_{GD} + C_{SB}}{g_m}$$

Example



$$\frac{v_{out}}{v_{in}} = \frac{1 + (j\omega)\dfrac{C_{GS}}{g_m}}{a(j\omega)^2 + b(j\omega) + 1}$$

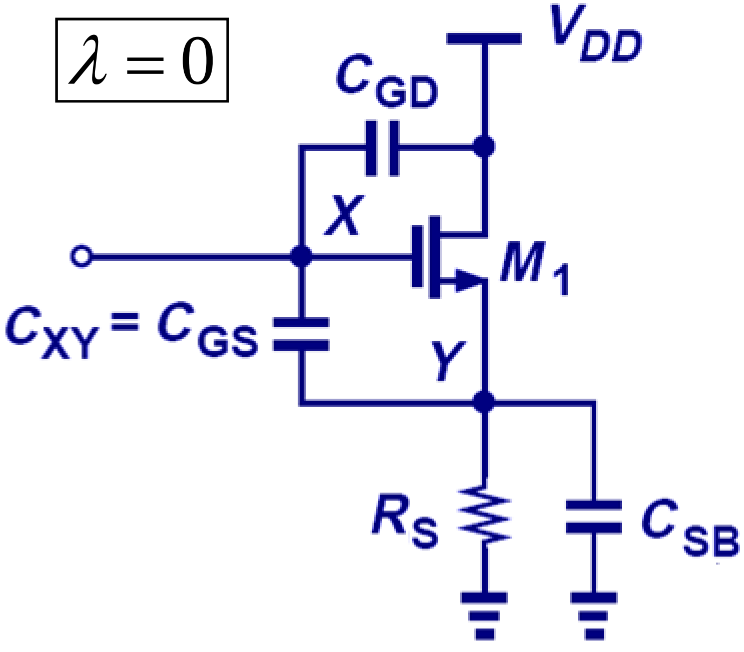
$$a = \frac{R_S}{g_{m1}}\left[C_{GD1}C_{GS1} + (C_{GD1} + C_{GS1})(C_{SB1} + C_{GD2} + C_{DB2})\right]$$

$$b = R_S C_{GD1} + \frac{C_{GD1} + C_{SB1} + C_{GD2} + C_{DB2}}{g_{m1}}$$

Source Follower: Input Capacitance

- Recall that the voltage gain of a source follower is $A_v = \dfrac{R_S}{\dfrac{1}{g_m} + R_S}$

Follower stage with MOSFET capacitances shown

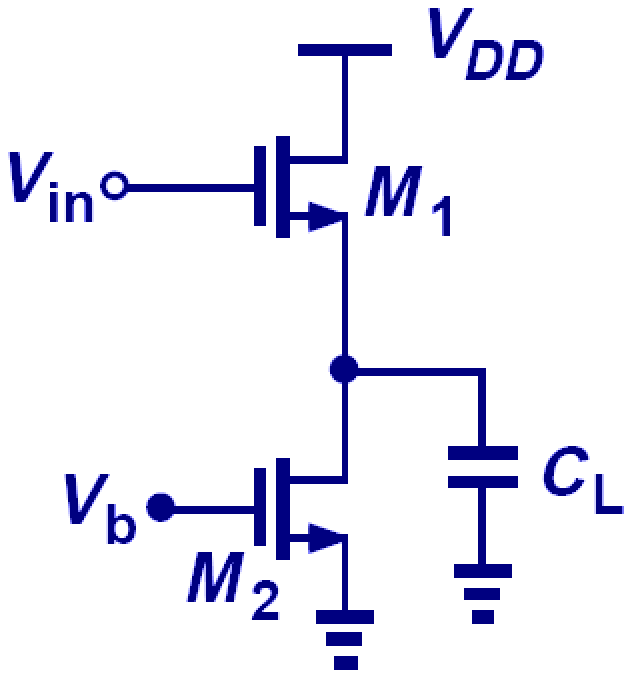


- C_{XY} can be decomposed into C_X and C_Y at the input and output nodes, respectively:

$$C_X = (1 - A_v)C_{GS} = \frac{C_{GS}}{1 + g_m R_S}$$

$$C_{in} = C_{GD} + \frac{C_{GS}}{1 + g_m R_S}$$

Example

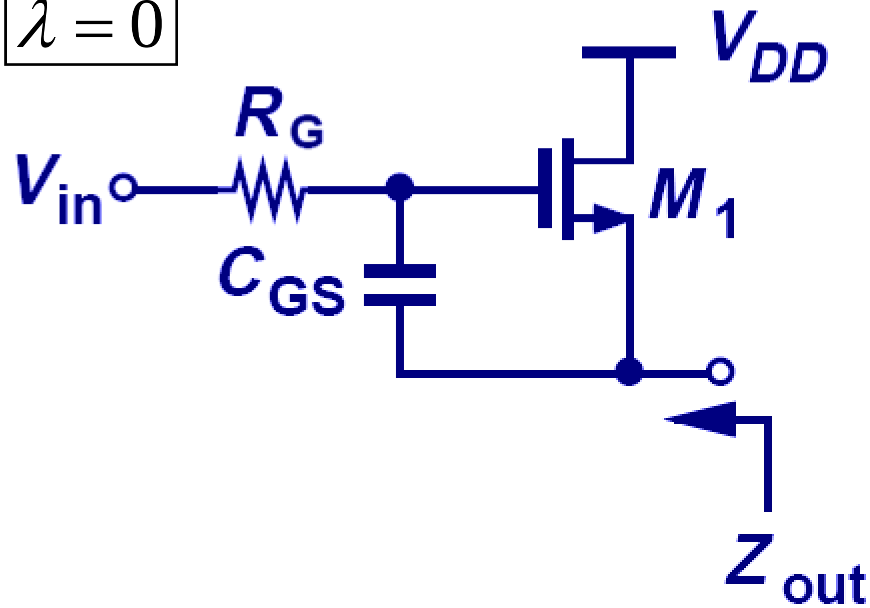


$\lambda \neq 0$

$$C_{in} = C_{GD1} + \frac{1}{1 + g_{m1}\left(r_{O1} \parallel r_{O2}\right)} C_{GS1}$$

Source Follower: Output Impedance

$\lambda = 0$



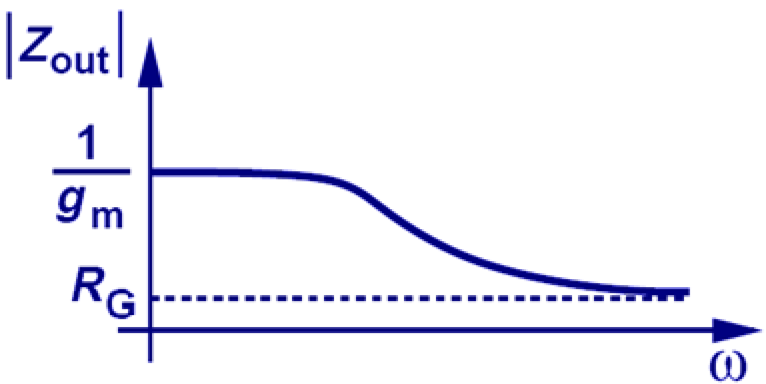
- The output impedance of a source follower can be obtained by direct AC analysis of small-signal model, similarly as for the emitter follower

$$\frac{v_X}{i_X} = \frac{j\omega R_G C_{GS} + 1}{j\omega C_{GS} + g_m}$$

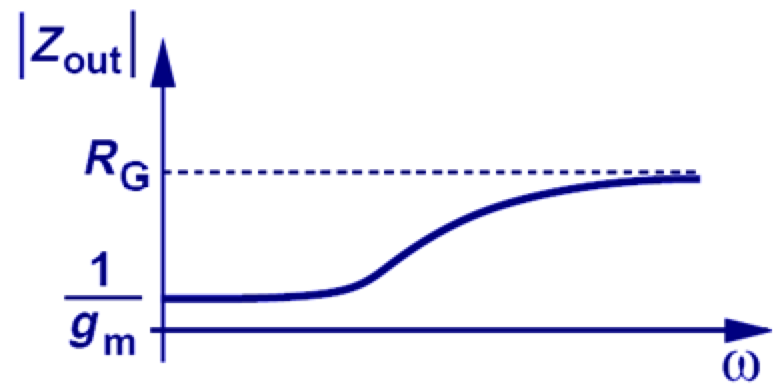
Source Follower as Active Inductor

$$Z_{out} = \frac{j\omega R_G C_{GS} + 1}{j\omega C_{GS} + g_m}$$

<u>CASE 1</u>: $R_G < 1/g_m$

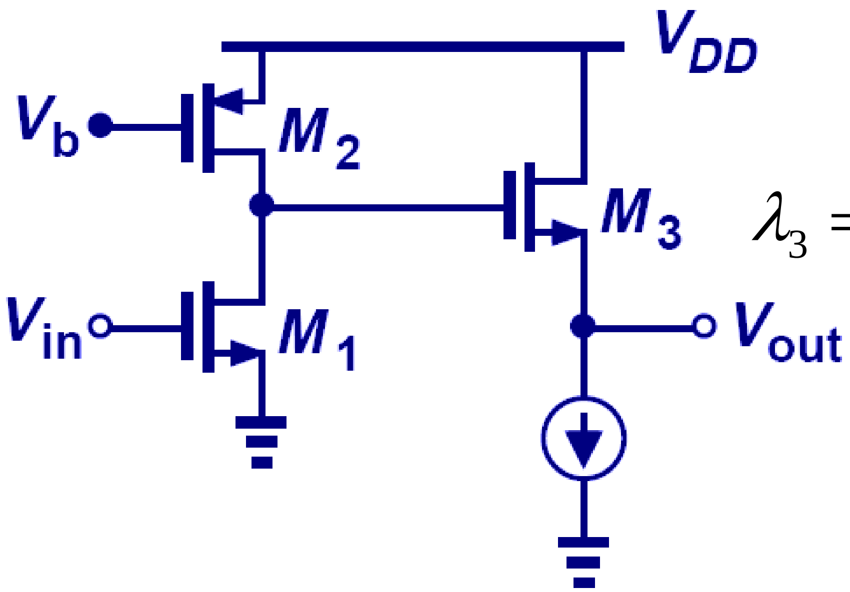


<u>CASE 2</u>: $R_G > 1/g_m$

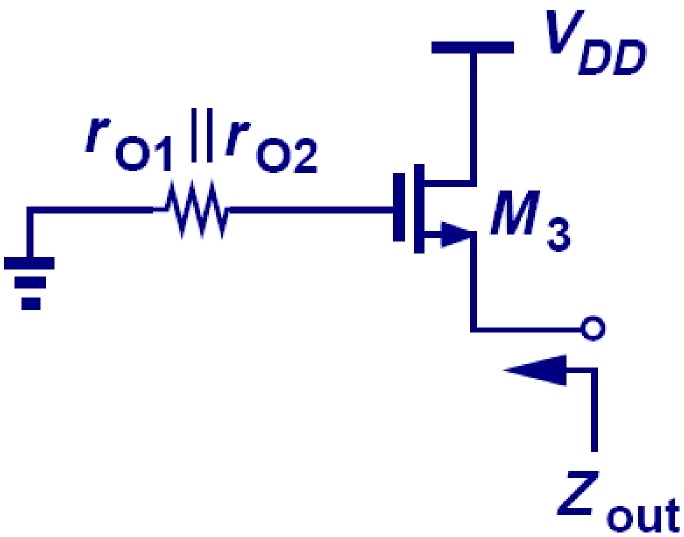


- A follower is typically used to lower the driving impedance, *i.e.* R_G is large compared to $1/g_m$, so that the "active inductor" characteristic on the right is usually observed.

Example



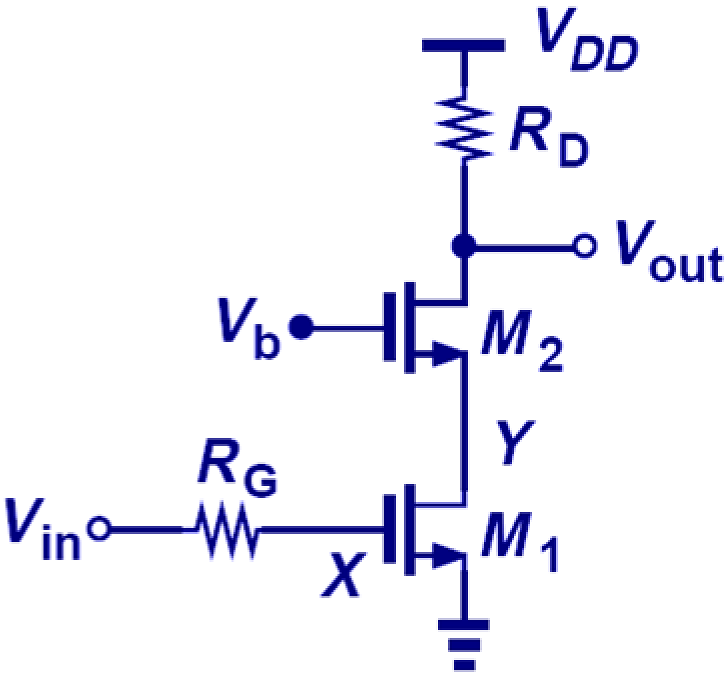
$\lambda_3 = 0$



$$Z_{out} = \frac{j\omega(r_{O1} \,||\, r_{O2})C_{GS3} + 1}{j\omega C_{GS3} + g_{m3}}$$

MOS Cascode Stage

- For a cascode stage, Miller multiplication is smaller than in the CS stage.



$$A_{v,XY} \equiv \frac{v_X}{v_Y} = -g_{m1}\left(\frac{1}{g_{m2}}\right) \approx -1$$

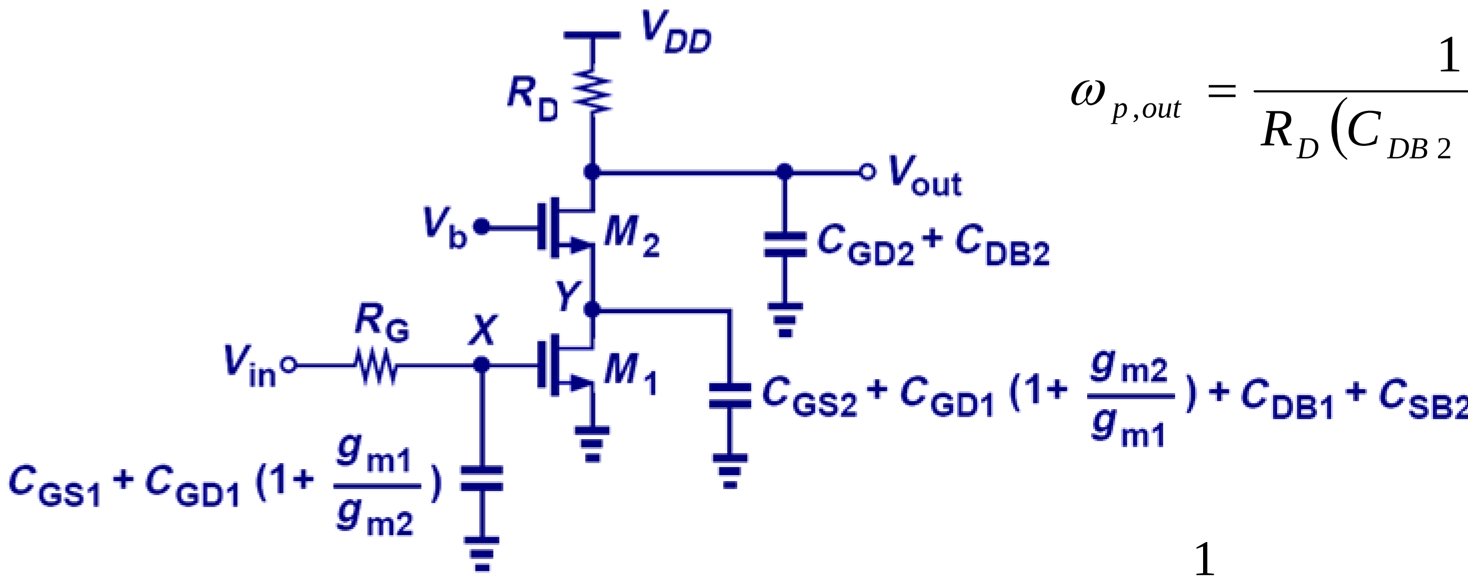
$$\Rightarrow \quad C_X \approx 2C_{XY}$$

Cascode Stage: Pole Frequencies

$\lambda = 0$

Cascode stage with MOSFET capacitances shown

(Miller approximation applied)



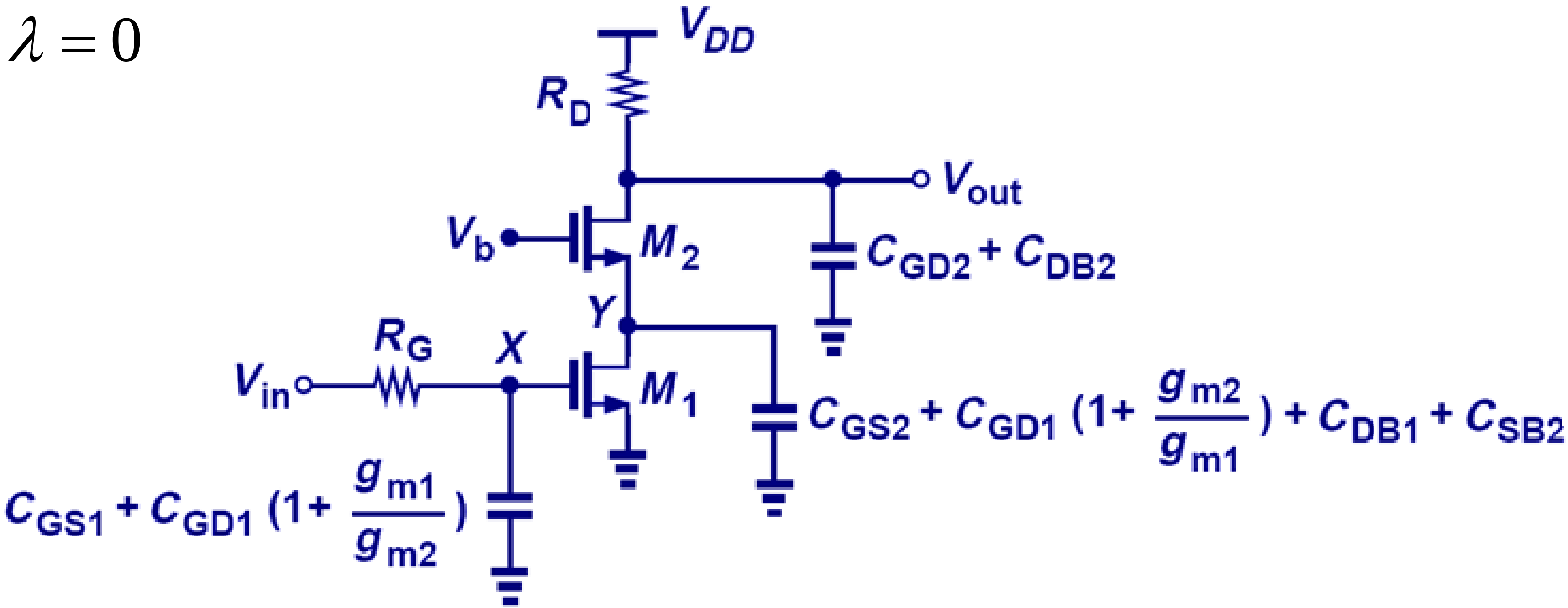
$$\omega_{p,out} = \frac{1}{R_D\left(C_{DB2} + C_{GD2}\right)}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}\left[C_{DB1} + C_{GS2} + \left(1+\frac{g_{m2}}{g_{m1}}\right)C_{GD1} + C_{SB2}\right]}$$

$$\omega_{p,X} = \frac{1}{R_G\left[C_{GS1} + \left(1+\frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

Cascode Stage: I/O Impedances

$\lambda = 0$



$$Z_{in} = \frac{1}{j\omega\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{GD1}\right]}$$

$$Z_{out} = R_D \,\|\, \frac{1}{j\omega(C_{GD2} + C_{DB2})}$$