

# Lecture 24

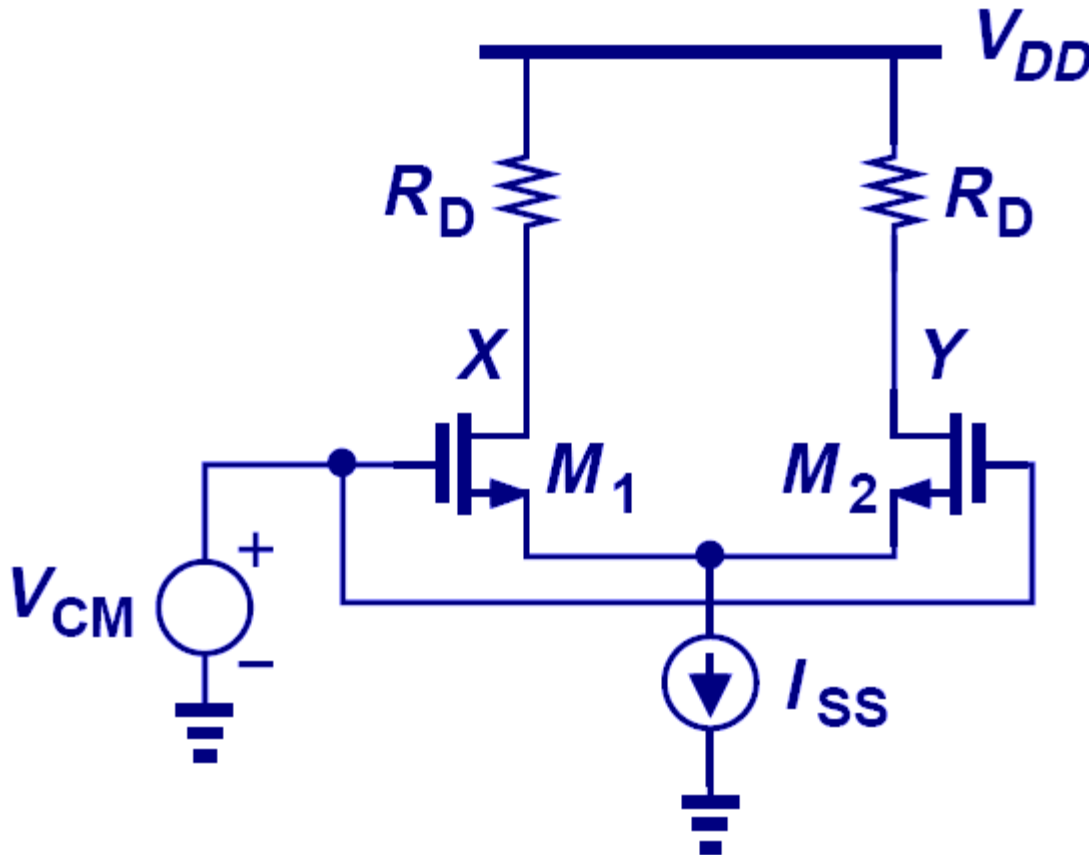
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## OUTLINE

- MOSFET Differential Amplifiers
  
- Reading: Chapter 10.3-10.6

# Common-Mode (CM) Response

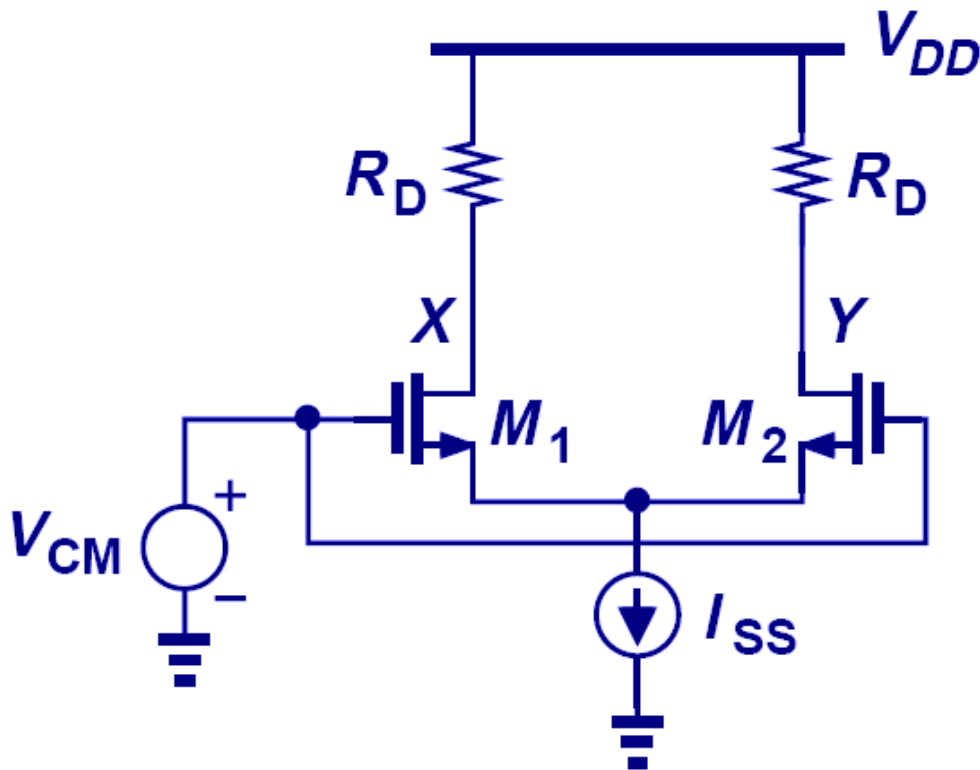
- Similarly to its BJT counterpart, a MOSFET differential pair produces zero differential output



$$V_X = V_Y = V_{DD} - R_D \frac{I_{SS}}{2}$$

# Equilibrium Overdrive Voltage

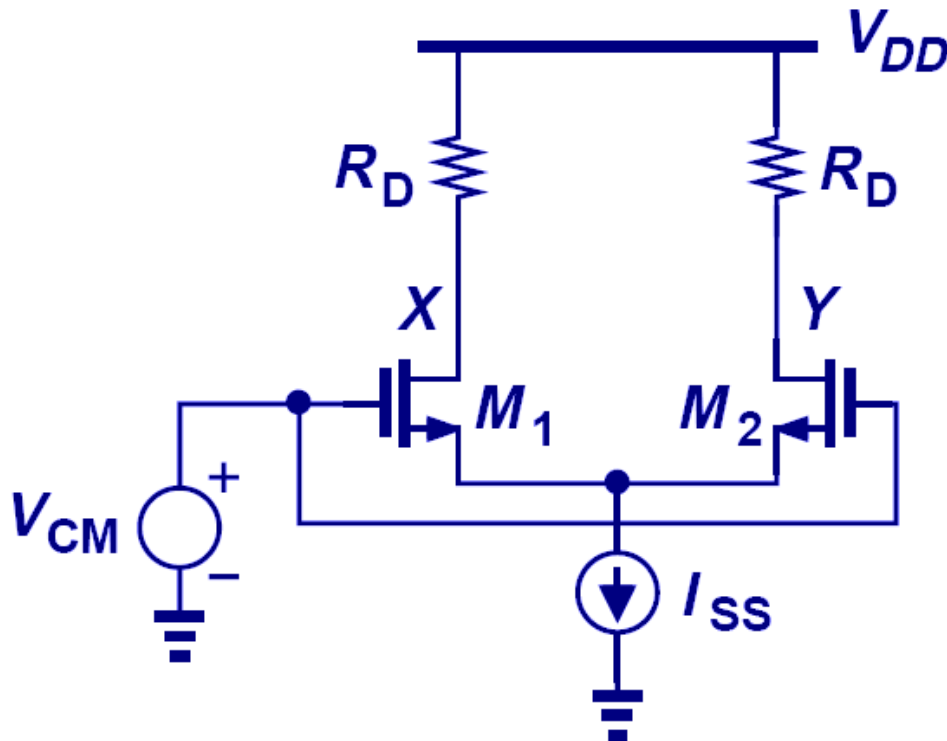
- The *equilibrium overdrive voltage* is defined as  $V_{GS} - V_{TH}$  when  $M_1$  and  $M_2$  each carry a current of  $I_{SS}/2$ .



$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

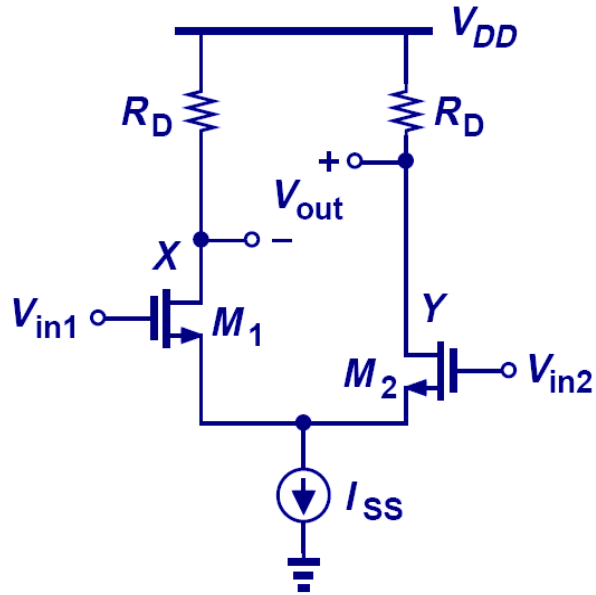
# Minimum CM Output Voltage

- In order to maintain  $M_1$  and  $M_2$  in saturation, the common-mode output voltage cannot fall below  $V_{CM} - V_{TH}$ .
- This value usually limits voltage gain.

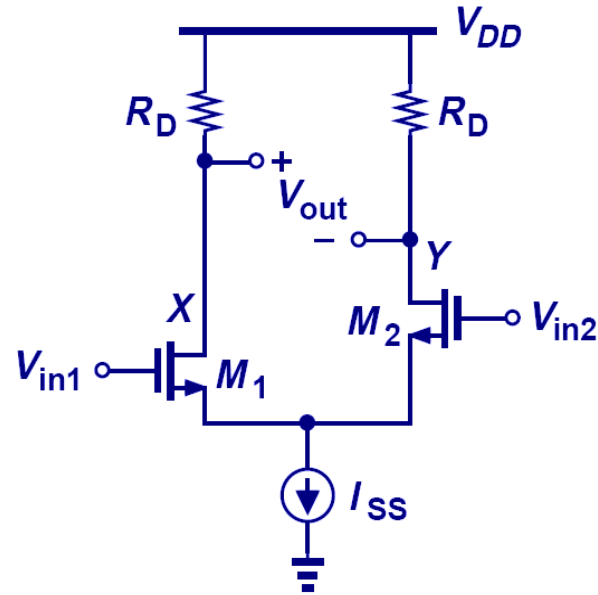


$$V_{DD} - R_D \frac{I_{SS}}{2} > V_{CM} - V_{TH}$$

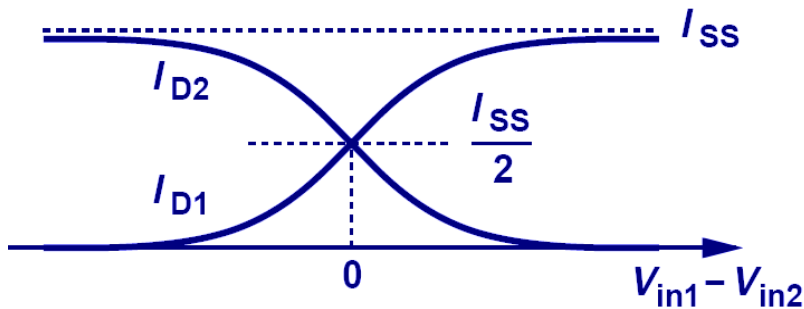
# Differential Response



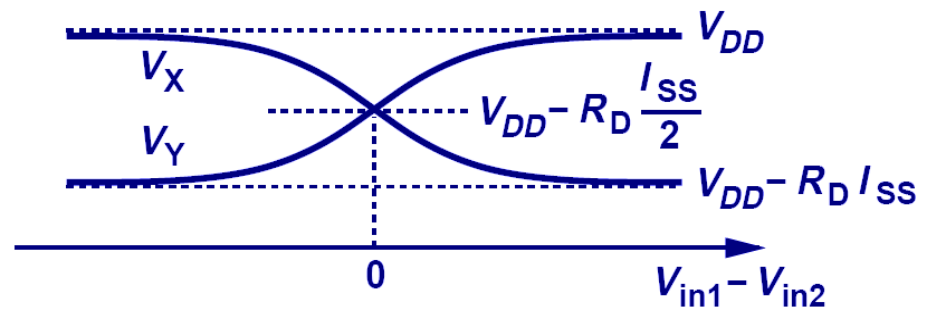
(a)



(b)

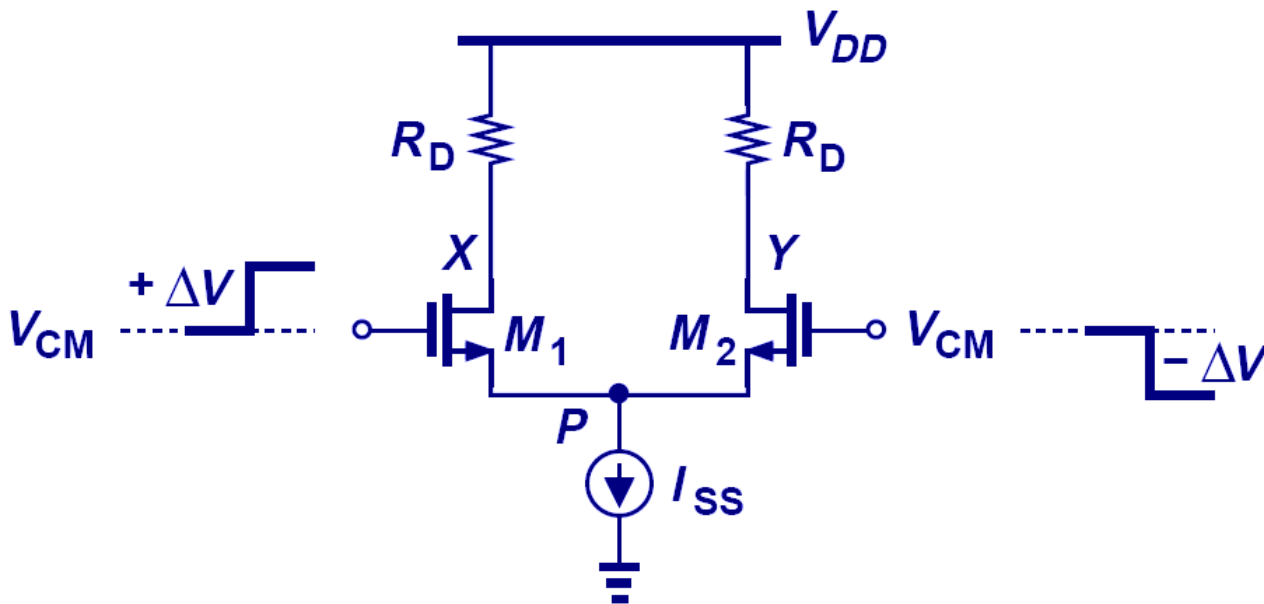


(c)



# Small-Signal Response

- For small input voltages ( $+\Delta V$  and  $-\Delta V$ ), the  $g_m$  values are  $\sim$ equal, so the increase in  $I_{D1}$  and decrease in  $I_{D2}$  are  $\sim$ equal in magnitude. Thus, the voltage at node P is constant and can be considered as AC ground.



$$I_{D1} = \frac{I_{EE}}{2} + \Delta I$$

$$I_{D2} = \frac{I_{EE}}{2} - \Delta I$$

$$\Delta I_{D1} = g_m (\Delta V - \Delta V_P)$$

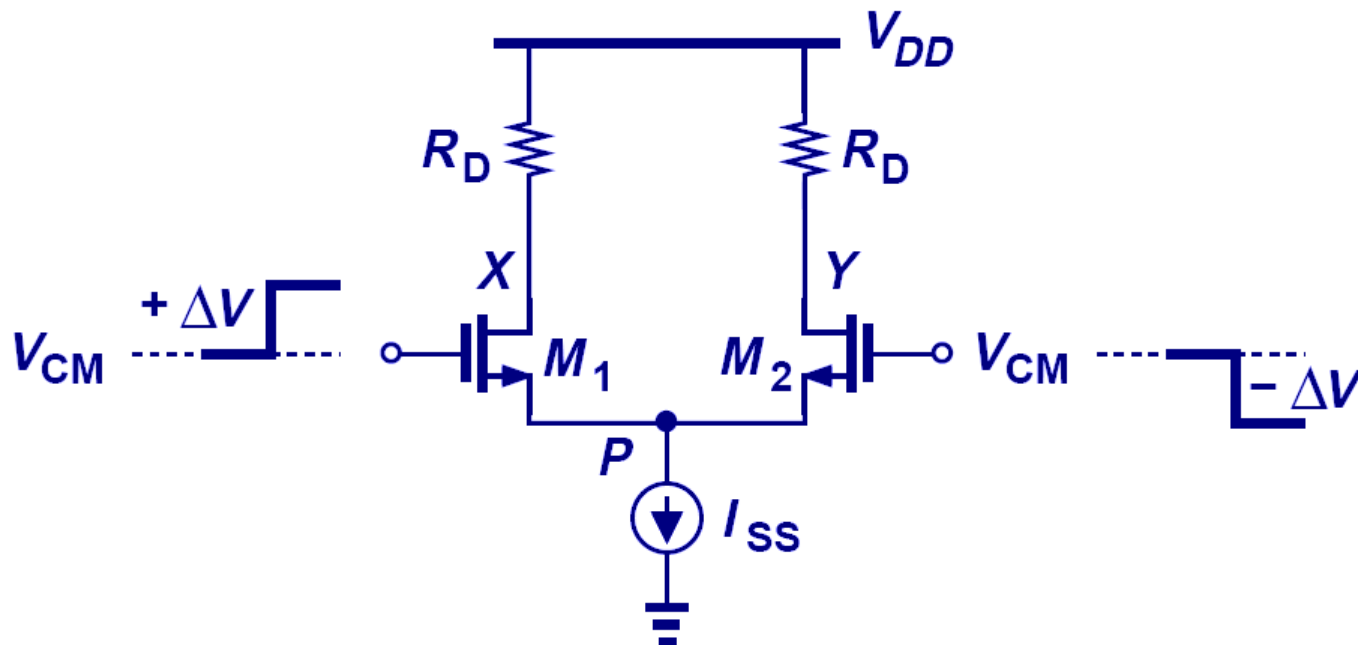
$$\Delta I_{D2} = g_m (-\Delta V - \Delta V_P)$$

$$\Delta I_{D1} = -\Delta I_{D2}$$

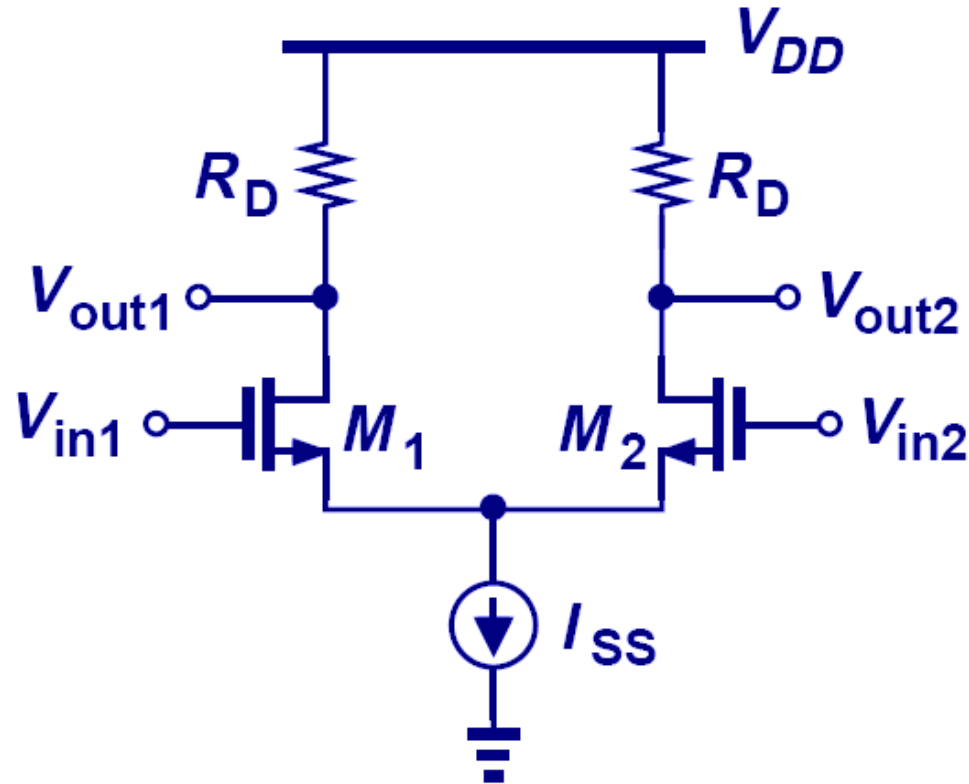
$$\Rightarrow \boxed{\Delta V_P = 0}$$

# Small-Signal Differential Gain

- Since the output signal changes by  $-2g_m\Delta VR_D$  when the input signal changes by  $2\Delta V$ , the small-signal voltage gain is  $-g_m R_D$ .
- Note that the voltage gain is the same as for a CS stage, but that the power dissipation is doubled.



# Large-Signal Analysis

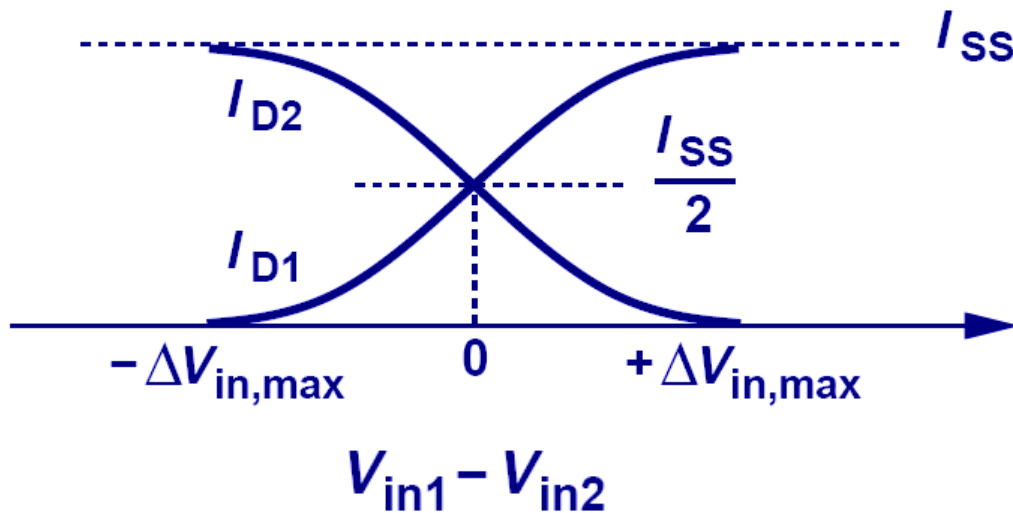


$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$



# Maximum Differential Input Voltage

- There exists a finite differential input voltage that completely steers the tail current from one transistor to the other. This value is known as the *maximum differential input voltage*.



If all current flows through  $M_2$  :

$$V_{GS2} = V_{TH} + \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$I_{D1} = 0 \Rightarrow V_{GS1} = V_{TH}$$

$$|V_{in1} - V_{in2}|_{max} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

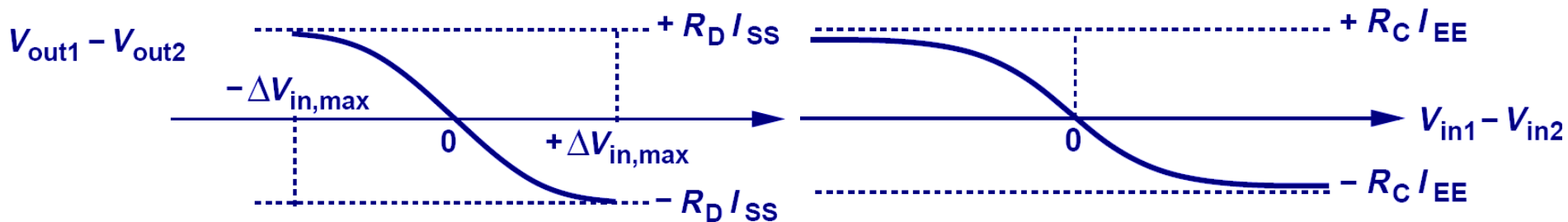
$$= \sqrt{2} (V_{GS} - V_{TH})_{equil}$$

# MOSFET vs. BJT Differential Pairs

- In a MOSFET differential pair, there exists a finite differential input voltage to completely switch the current from one transistor to the other, whereas in a BJT differential pair that voltage is infinite.

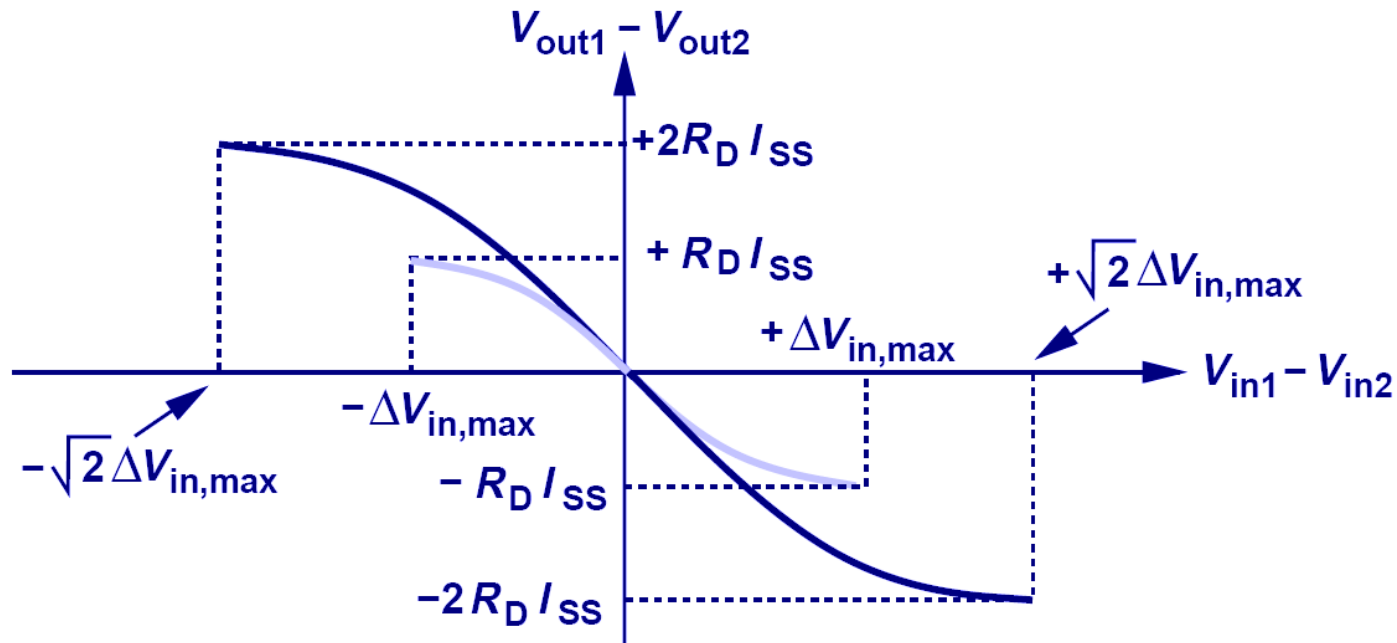
MOSFET Differential Pair

BJT Differential Pair



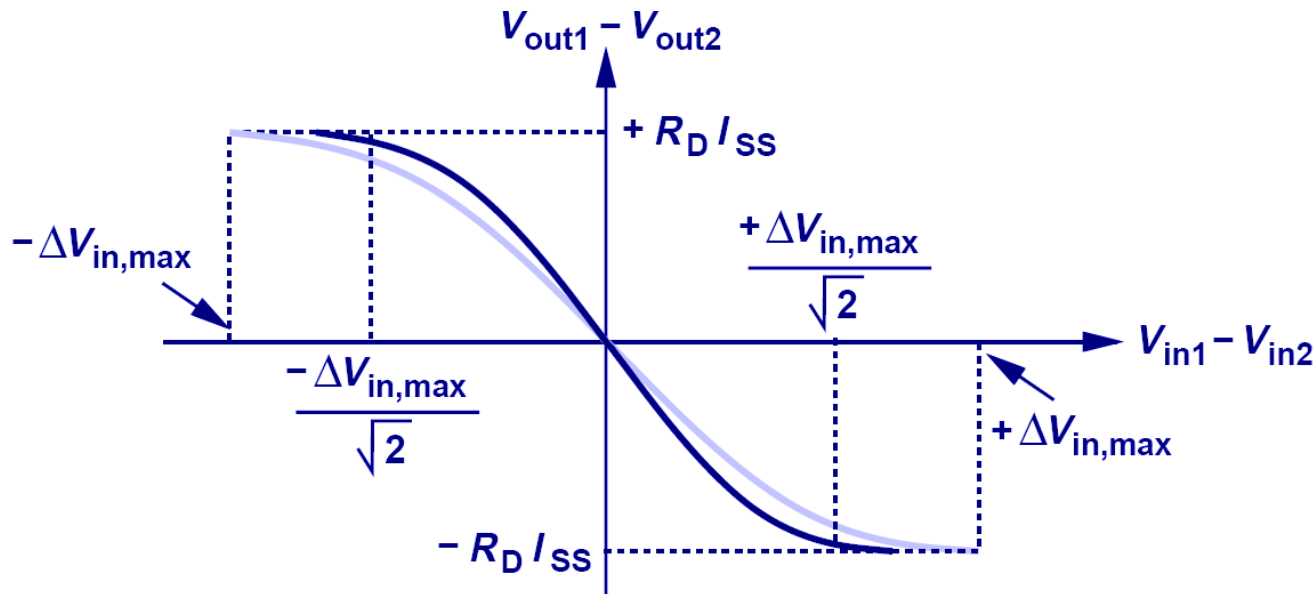
# Effect of Doubling the Tail Current

- If  $I_{SS}$  is doubled, the equilibrium overdrive voltage for each transistor increases by  $\sqrt{2}$ , thus  $\Delta V_{in,max}$  increases by  $\sqrt{2}$  as well. Moreover, the differential output swing will double.



# Effect of Doubling $W/L$

- If  $W/L$  is doubled, the equilibrium overdrive voltage is lowered by  $\sqrt{2}$ , thus  $\Delta V_{in,max}$  will be lowered by  $\sqrt{2}$  as well. The differential output swing will be unchanged.



# Small-Signal Analysis

- When the input differential signal is small compared to  $4I_{SS}/\mu_n C_{ox}(W/L)$ , the output differential current is  $\sim$  linearly proportional to it:

$$I_{D1} - I_{D2} \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} (V_{in1} - V_{in2})$$

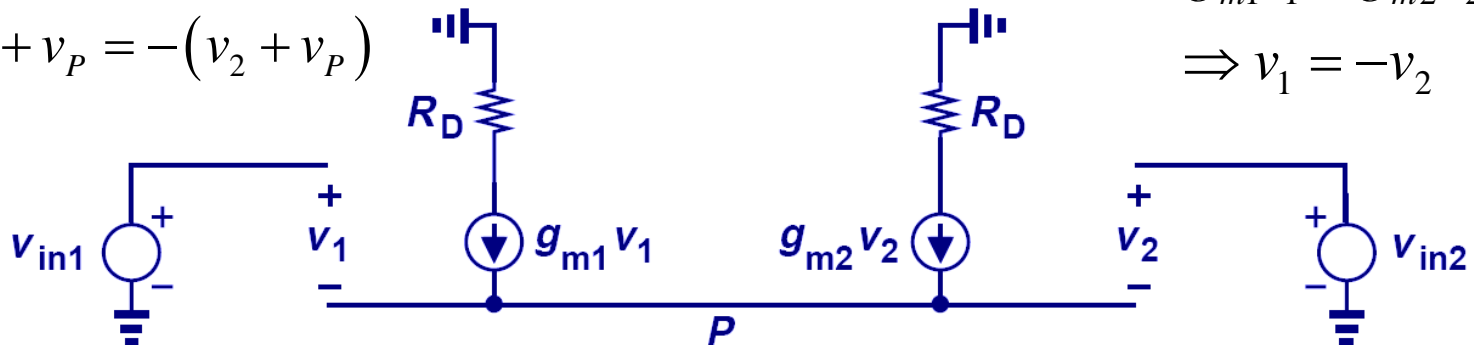
- We can use the small-signal model to prove that the change in tail node voltage ( $v_p$ ) is zero:

$$v_{in1} = -v_{in2}$$

$$\Rightarrow v_1 + v_p = -(v_2 + v_p)$$

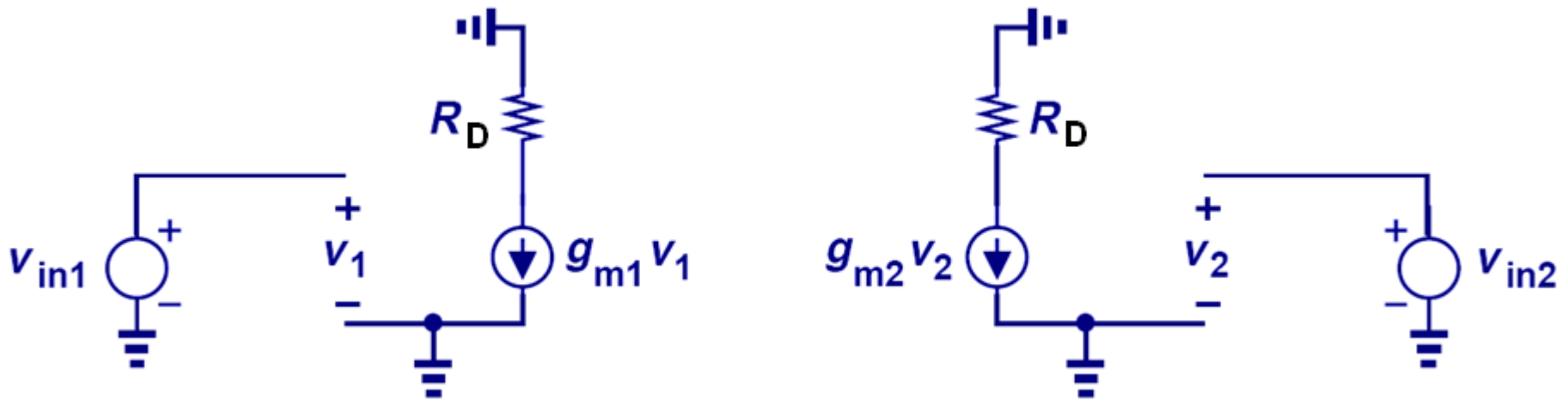
$$g_{m1}v_1 + g_{m2}v_2 = 0$$

$$\Rightarrow v_1 = -v_2$$



# Virtual Ground and Half Circuit

- Since the voltage at node  $P$  does not change for small input signals, the half circuit can be used to calculate the voltage gain.

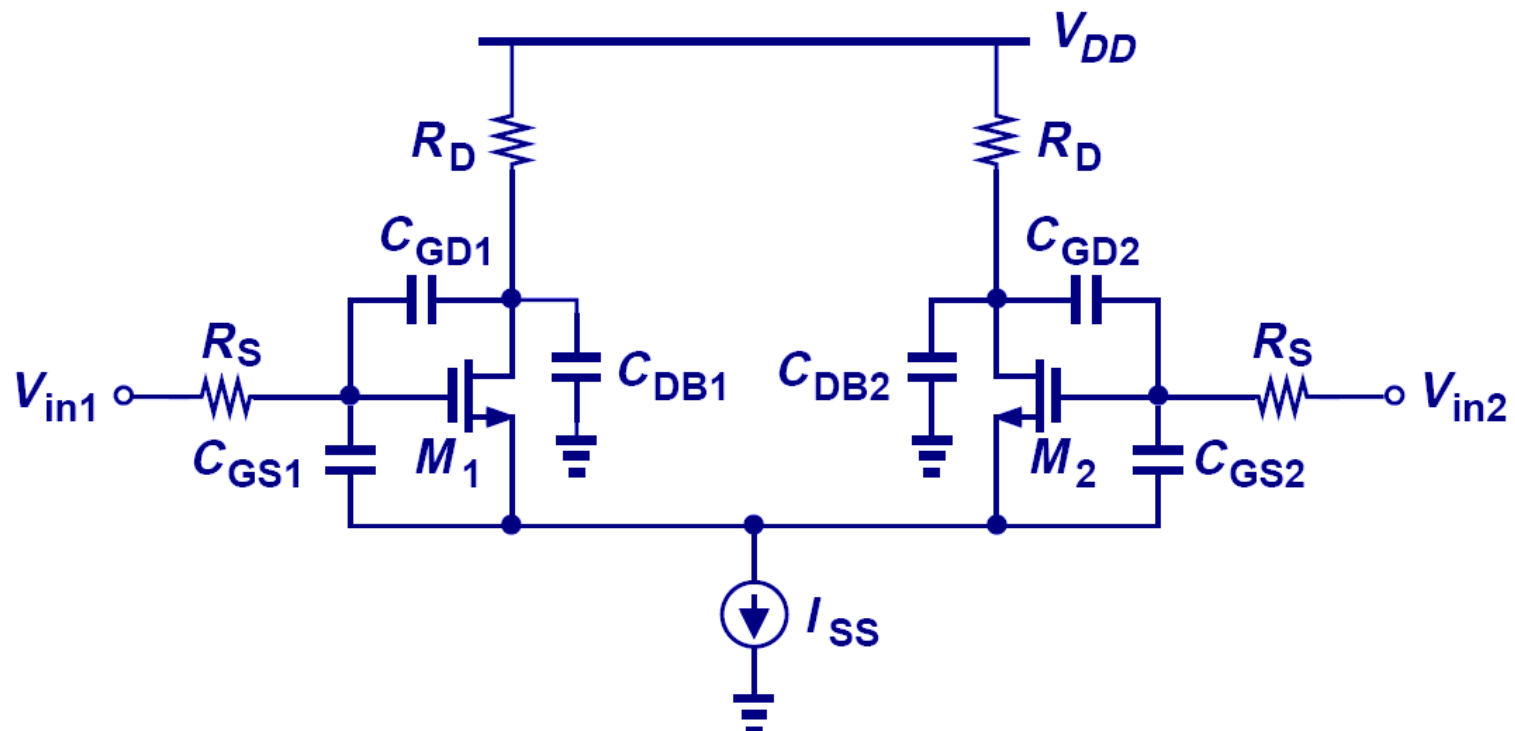


$$v_P = 0$$

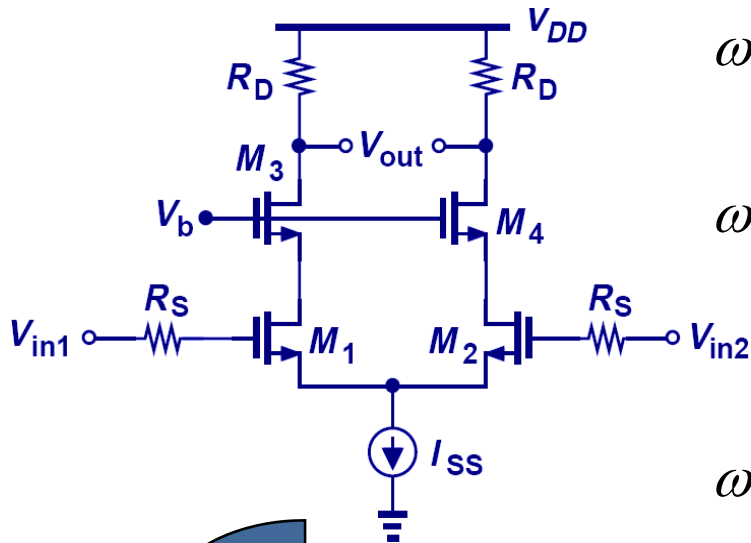
$$A_v = -g_m R_D$$

# MOSFET Diff. Pair Frequency Response

- Since the MOSFET differential pair can be analyzed using its half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.



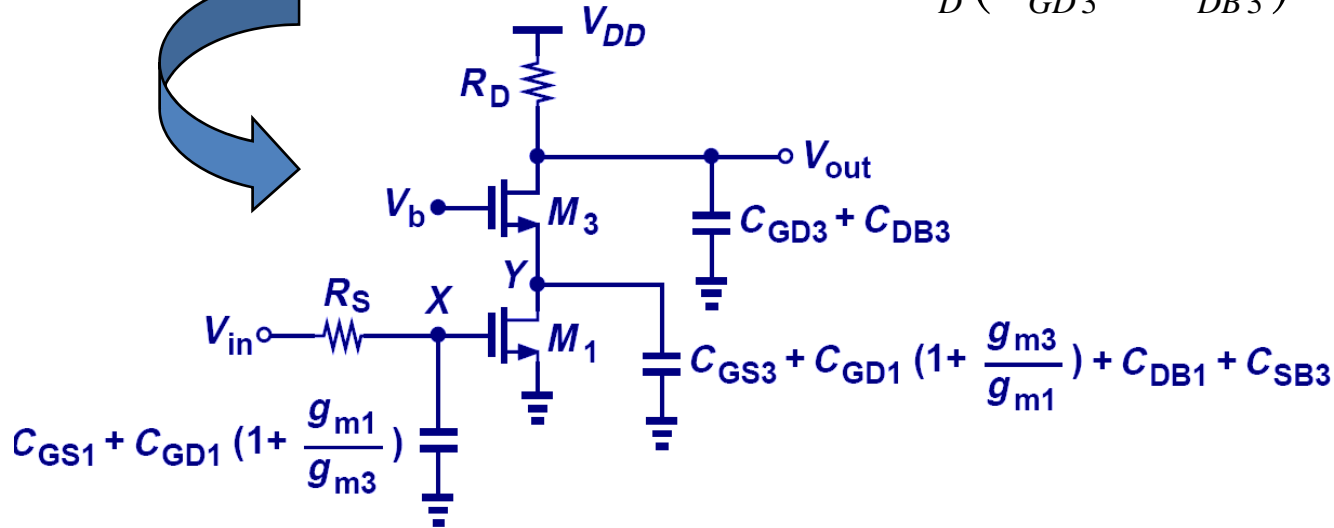
# Example



$$\omega_{p,X} = \frac{1}{R_S [C_{GS1} + (1 + g_{m1} / g_{m3}) C_{GD1}]}$$

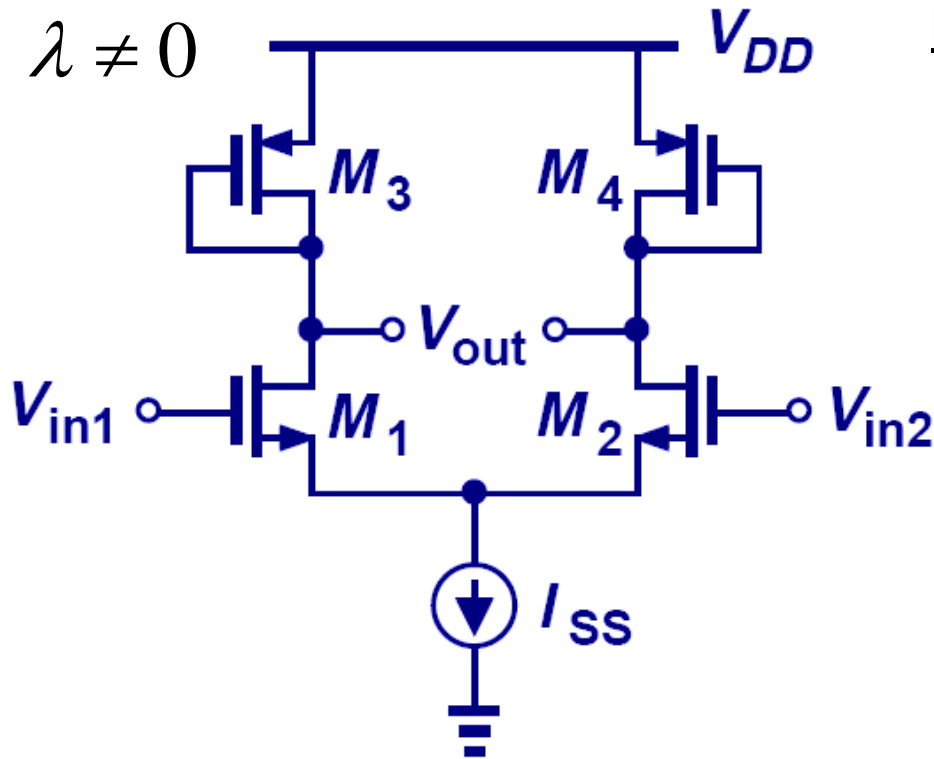
$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m3}} \left[ C_{GS3} + C_{GD1} \left( 1 + \frac{g_{m3}}{g_{m1}} \right) + C_{DB1} + C_{SB3} \right]}$$

$$\omega_{p,out} = \frac{1}{R_D (C_{GD3} + C_{DB3})}$$

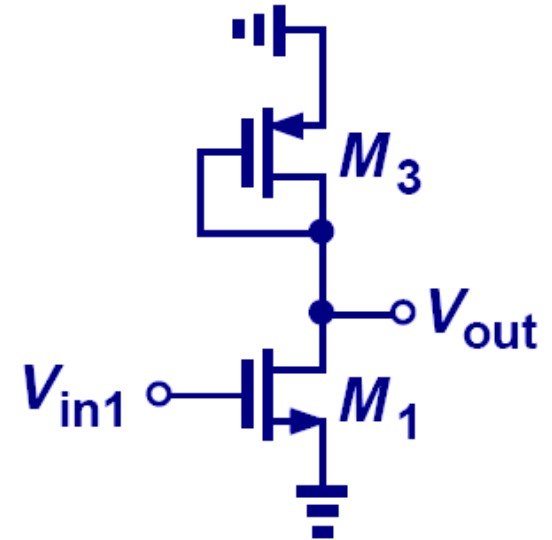




# Half Circuit Example 1

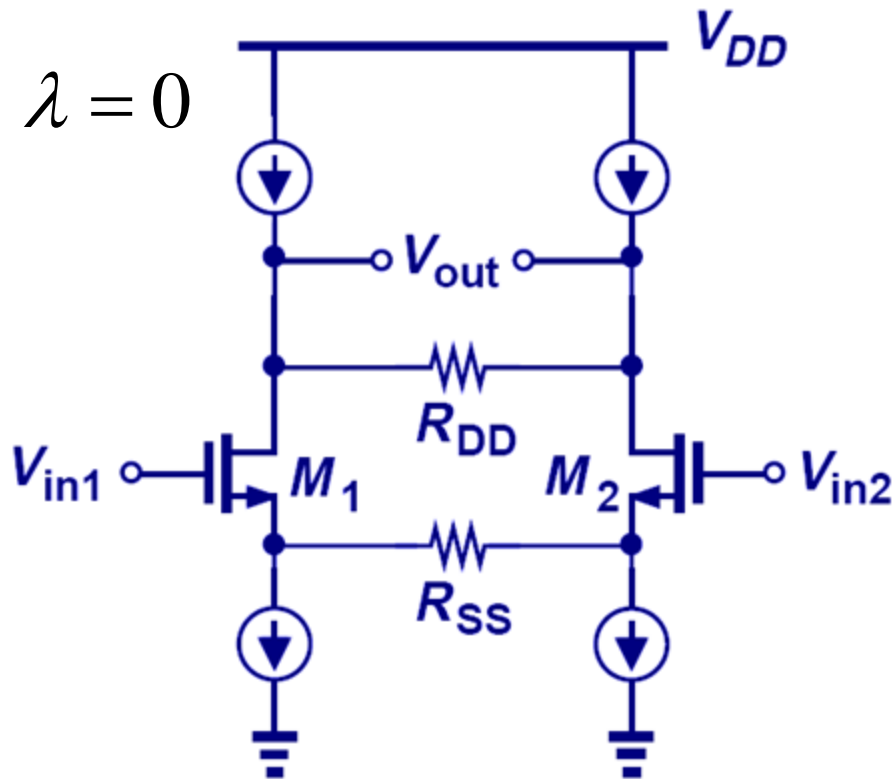


Half circuit for small-signal analysis

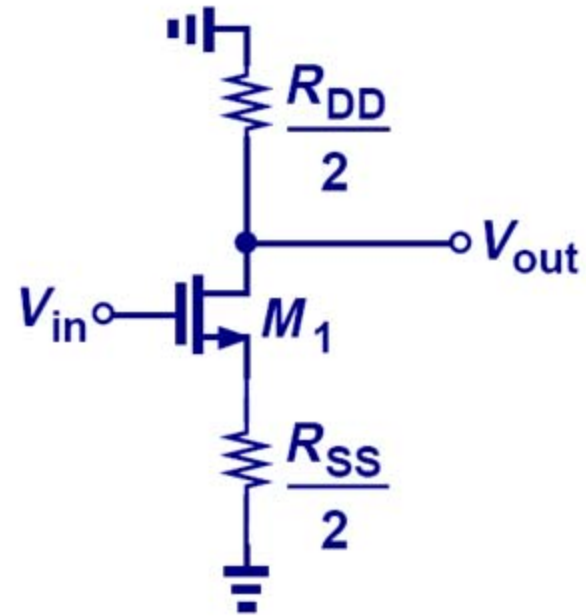


$$A_v = -g_{m1} \left( \frac{1}{g_{m3}} \parallel r_{O3} \parallel r_{O1} \right)$$

# Half Circuit Example 2

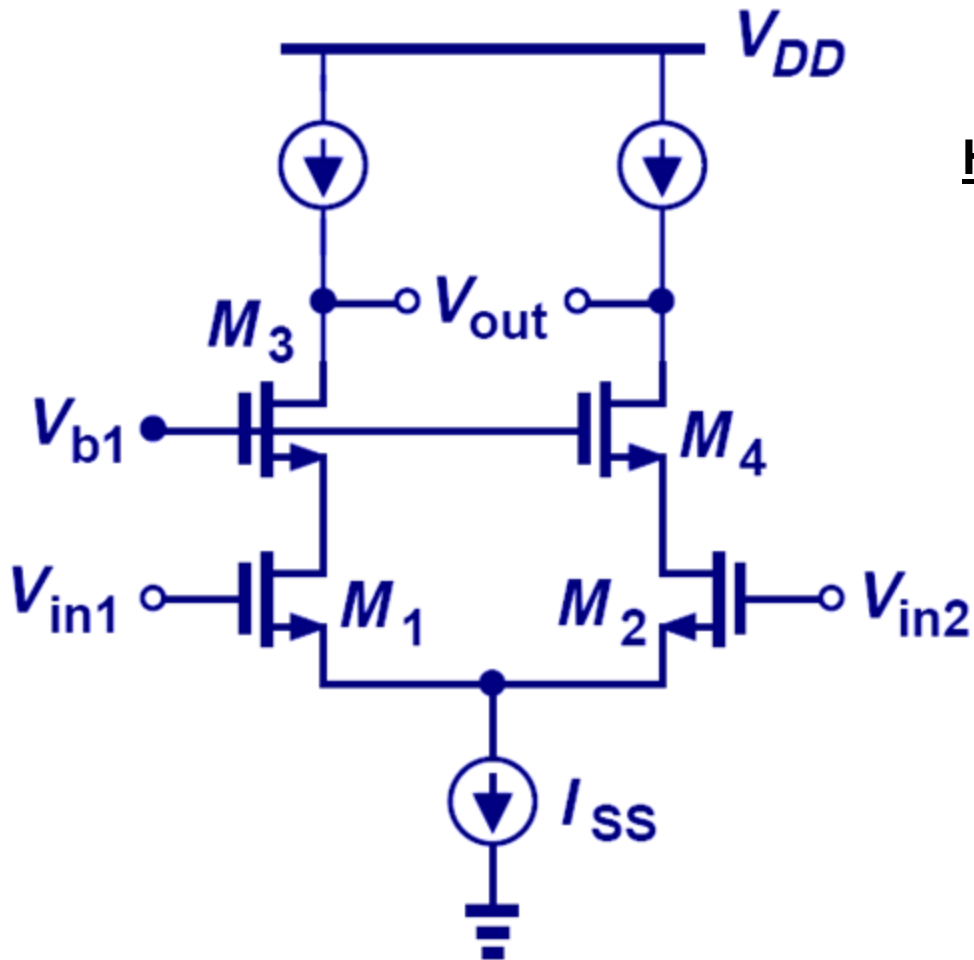


Half circuit for small-signal analysis

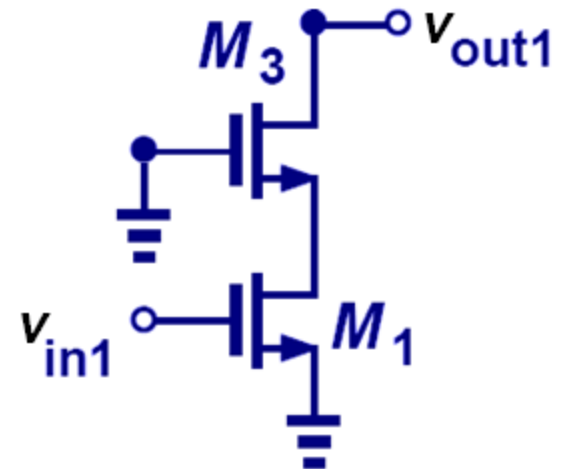


$$A_v = -\frac{R_{DD}/2}{(1/g_m) + (R_{SS}/2)}$$

# MOSFET Cascode Differential Pair

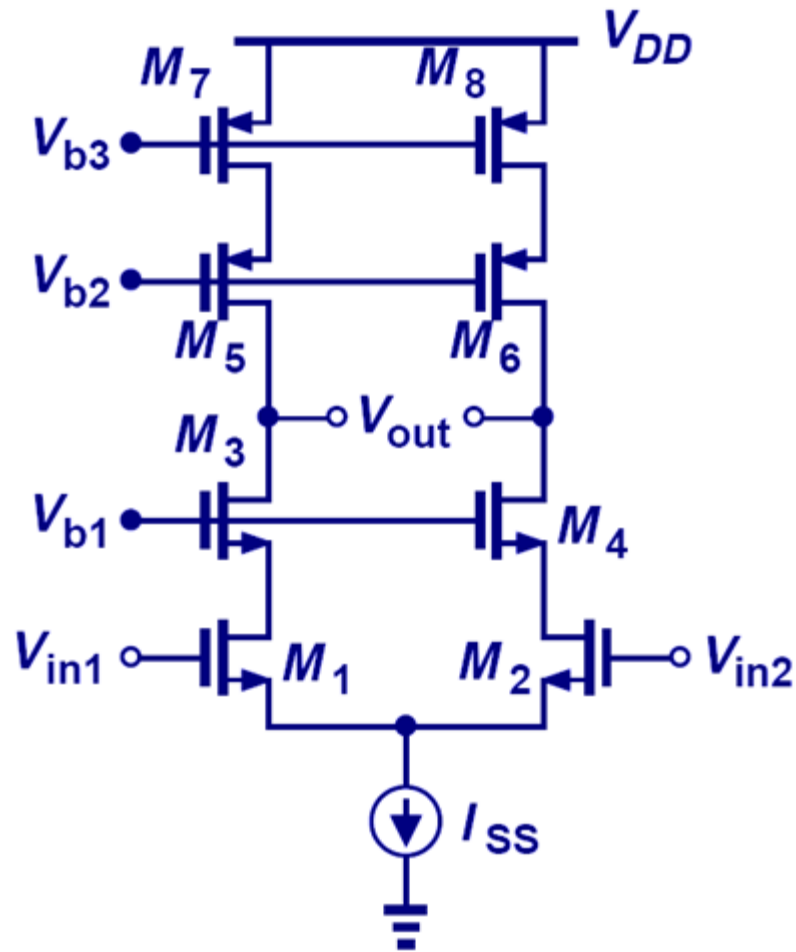


Half circuit for small-signal analysis

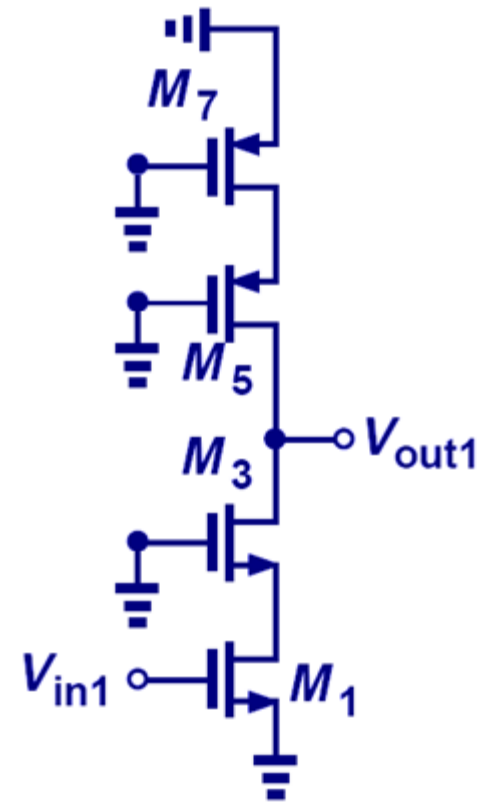


$$A_v \approx -g_{m1} r_{O3} g_{m3} r_{O1}$$

# MOSFET Telescopic Cascode Amplifier



Half circuit for small-signal analysis



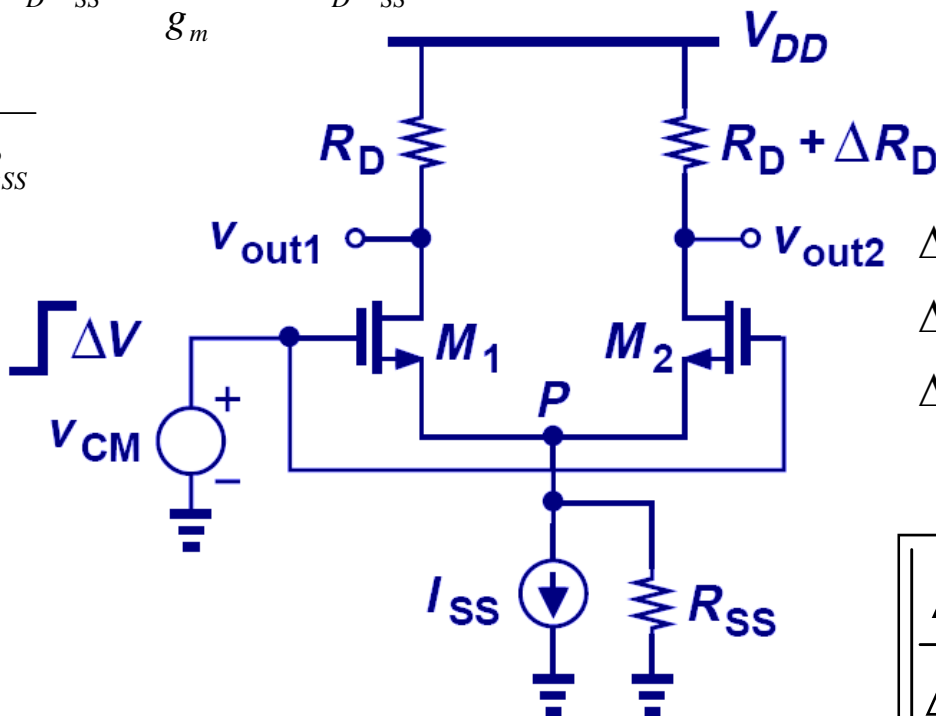
$$A_v \approx -g_{m1} \left[ (g_{m3} r_{O3} r_{O1}) \parallel (g_{m5} r_{O5} r_{O7}) \right]$$

# CM to DM Conversion Gain, $A_{CM-DM}$

- If finite tail impedance and asymmetry are both present, then the differential output signal will contain a portion of the input common-mode signal.

$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D R_{SS} = \frac{\Delta I_D}{g_m} + 2\Delta I_D R_{SS}$$

$$\Rightarrow \Delta I_D = \frac{\Delta V_{CM}}{\frac{1}{g_m} + 2R_{SS}}$$



$$\Delta V_{out1} = -\Delta I_D R_D$$

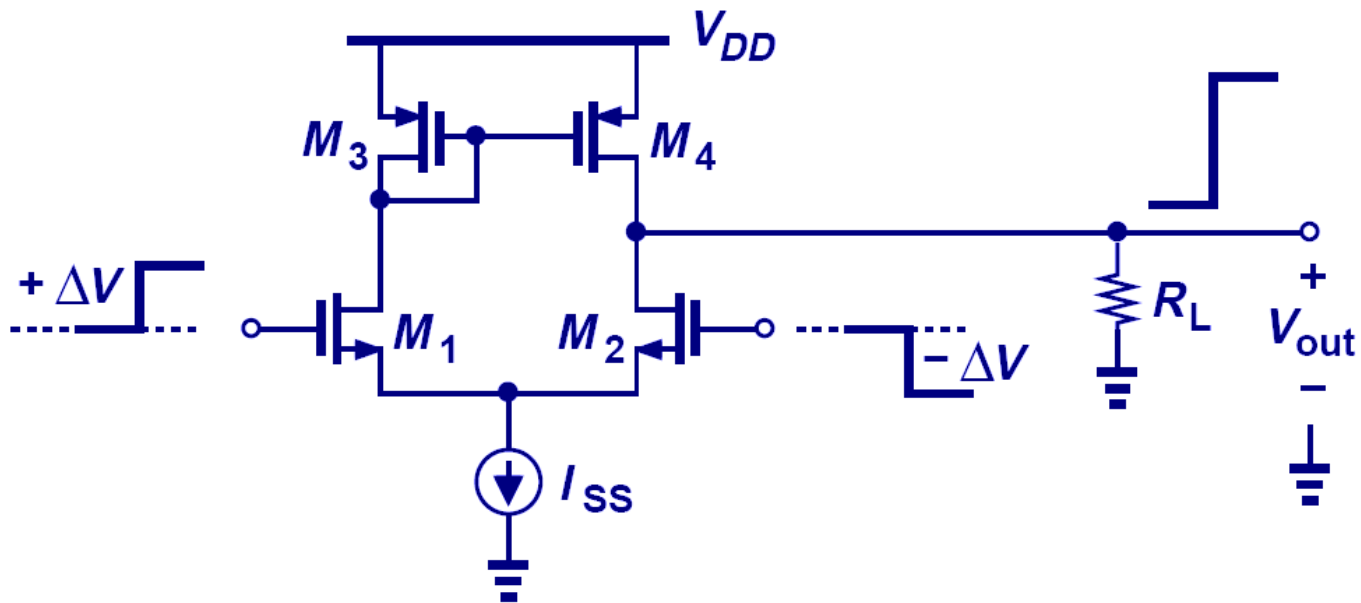
$$\Delta V_{out2} = -\Delta I_D (R_D + \Delta R_D)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} = -\Delta I_D \Delta R_D$$

$$\left| \frac{\Delta V_{out}}{\Delta V_{CM}} \right| = \frac{\Delta R_D}{\left( \frac{1}{g_m} \right) + 2R_{SS}}$$

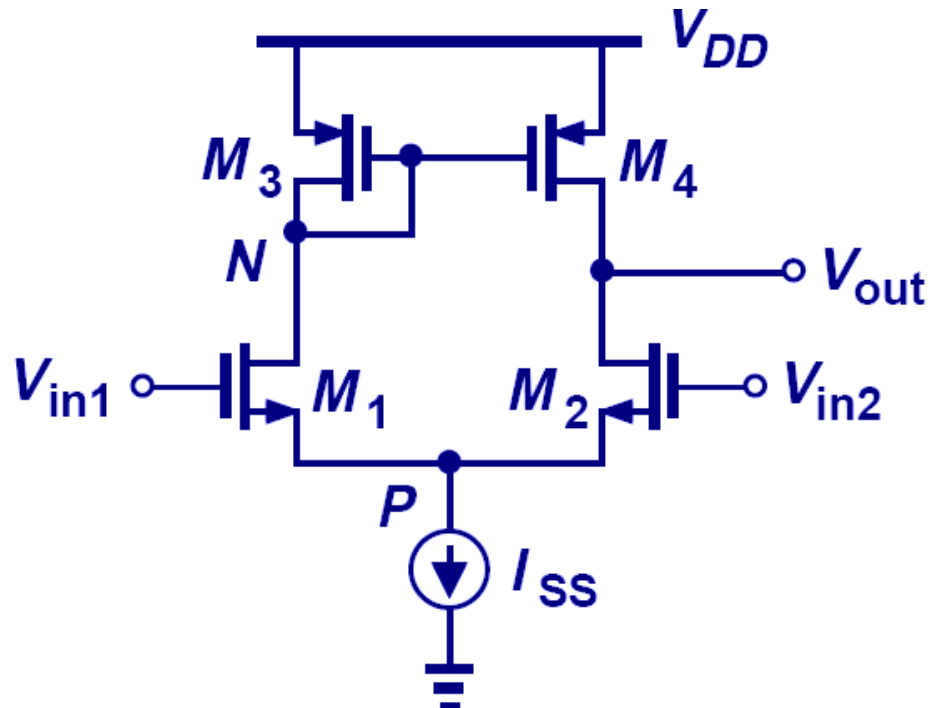
# MOS Diff. Pair with Active Load

- Similarly to its BJT counterpart, a MOSFET differential pair can use an active load to enhance its single-ended output.

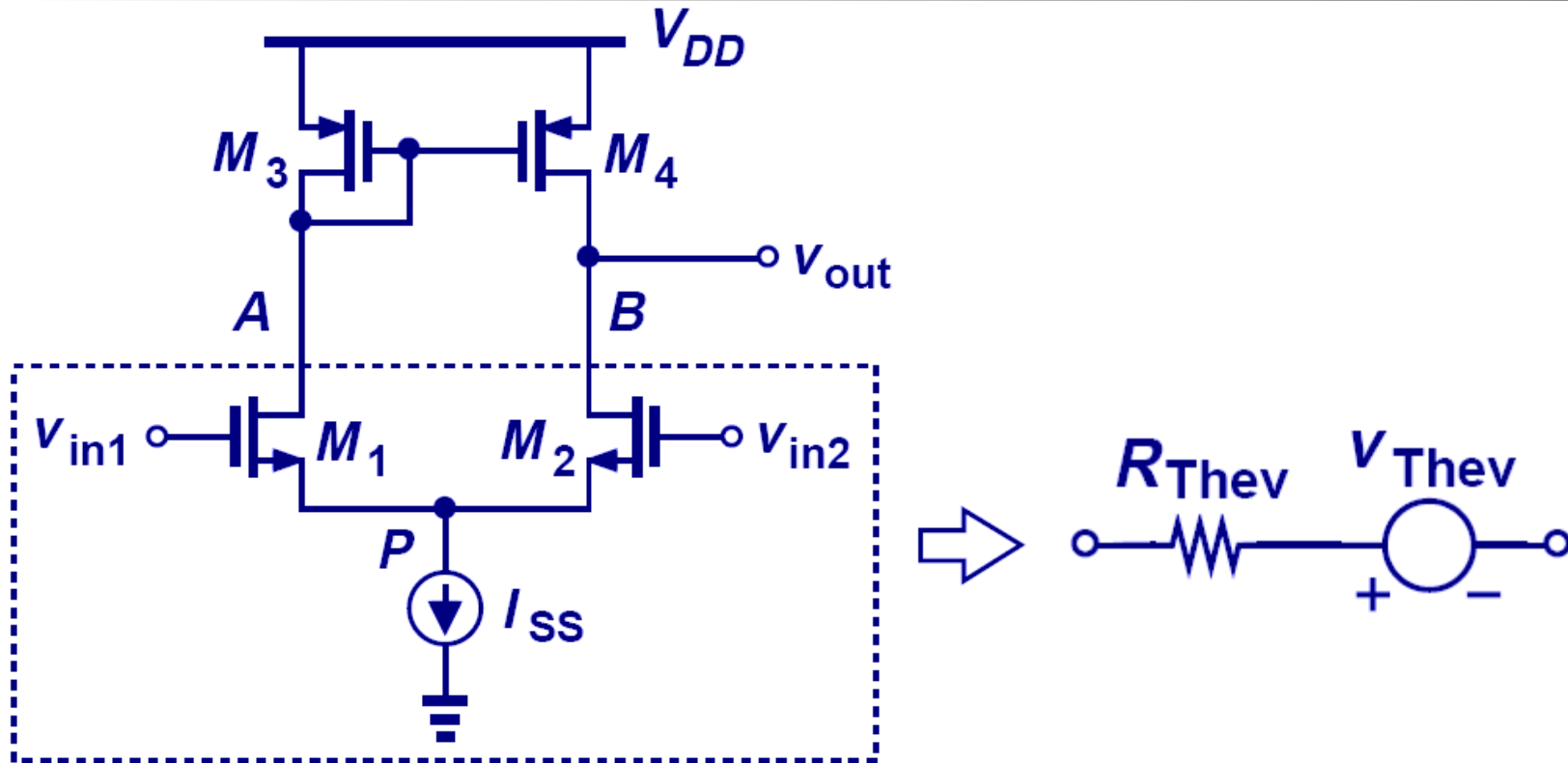


# Asymmetric Differential Pair

- Because of the vast difference in magnitude of the resistances seen at the drains of  $M_1$  and  $M_2$ , the voltage swings at these two nodes are different and therefore node  $P$  cannot be viewed as a virtual ground...



# Thevenin Equivalent of the Input Pair



$$v_{Thev} = -g_{mN} r_{oN} (v_{in1} - v_{in2})$$

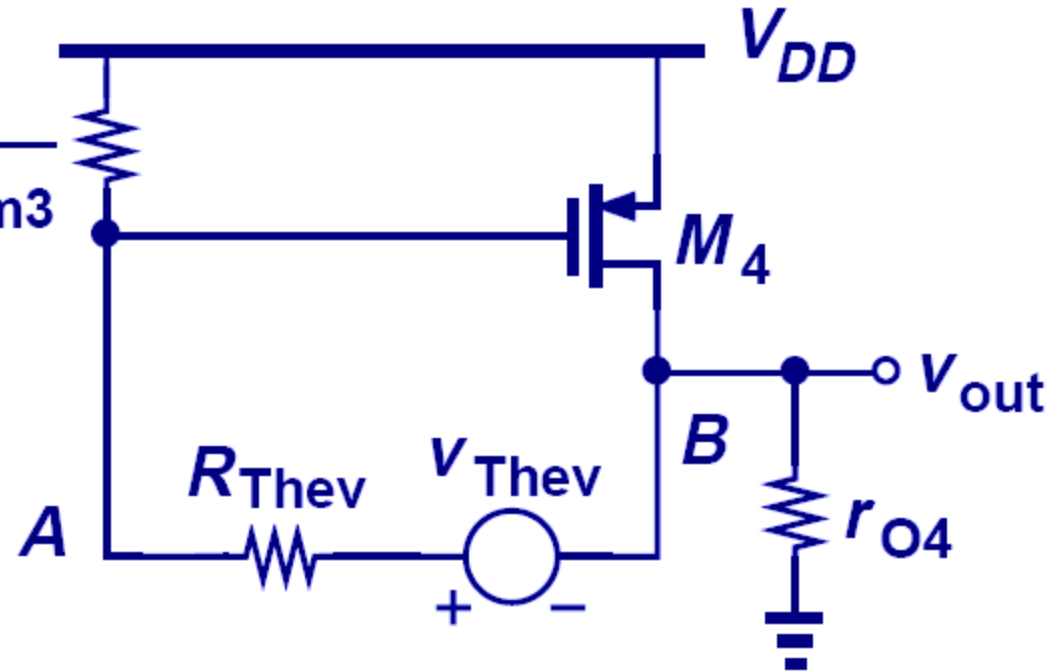
$$R_{Thev} = 2r_{oN}$$



# Simplified Diff. Pair w/ Active Load

$$v_A = (v_{out} + v_{Thev}) \frac{\frac{1}{g_{m3}}}{\frac{1}{g_{m3}} + R_{Thev}}$$

$$\approx \frac{v_{out} + v_{Thev}}{g_{m3} \cdot 2r_{oN}}$$



$$\text{KCL at } v_{out} : g_{m4} v_A + \frac{v_{out}}{r_{o4}} + \frac{v_{out} + v_{Thev}}{\frac{1}{g_{m3}} + R_{Thev}} = 0$$

$$\frac{v_{out}}{v_{in1} - v_{in2}} = g_{mN} (r_{oN} \parallel r_{oP})$$