Lecture 2

OUTLINE

- Basic Semiconductor Physics (cont'd)
  – Carrier drift and diffusion
- PN Junction Diodes
  – Electrostatics
  – Capacitance

Reading: Chapter 2.1-2.2

Dopant Compensation

- An N-type semiconductor can be converted into P-type material by counter-doping it with acceptors such that $N_A > N_D$.
- A ***compensated semiconductor material*** has both acceptors and donors.

N-type material ($N_D > N_A$)	P-type material ($N_A > N_D$)
$n \approx N_D - N_A$	$p \approx N_A - N_D$
$p \approx \dfrac{n_i^2}{N_D - N_A}$	$n \approx \dfrac{n_i^2}{N_A - N_D}$

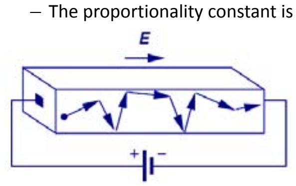
Types of Charge in a Semiconductor

- Negative charges:
  – Conduction electrons (density = n)
  – Ionized acceptor atoms (density = N_A)
- Positive charges:
  – Holes (density = p)
  – Ionized donor atoms (density = N_D)
- The net charge density (C/cm^3) in a semiconductor is

$$\rho = q(p - n + N_D - N_A)$$

Carrier Drift

- The process in which charged particles move because of an electric field is called ***drift***.
- Charged particles within a semiconductor move with an average velocity proportional to the electric field.
  – The proportionality constant is the carrier ***mobility***.



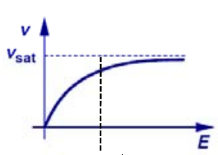
Hole velocity $\vec{v}_h = \mu_p \vec{E}$

Electron velocity $\vec{v}_e = -\mu_n \vec{E}$

Notation:
$\mu_p \equiv$ hole mobility (cm^2/V·s)
$\mu_n \equiv$ electron mobility (cm^2/V·s)

Velocity Saturation

- In reality, carrier velocities saturate at an upper limit, called the ***saturation velocity*** (v_{sat}).



$E_{sat} \approx 10^4$ V/cm

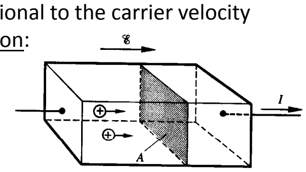
$$\mu = \frac{\mu_0}{1 + bE}$$

$$v_{sat} = \frac{\mu_0}{b}$$

$$v = \frac{\mu_0}{1 + \dfrac{\mu_0 E}{v_{sat}}} E$$

Drift Current

- Drift current is proportional to the carrier velocity and carrier concentration:



$v_h\, t\, A$ = volume from which all holes cross plane in time t

$p\, v_h\, t\, A$ = # of holes crossing plane in time t

$q\, p\, v_h\, t\, A$ = charge crossing plane in time t

$q\, p\, v_h\, A$ = charge crossing plane per unit time = hole current

➔ **Hole current per unit area (*i.e.* current density) $J_{p,drift} = q\, p\, v_h$**

Conductivity and Resistivity

- In a semiconductor, both electrons and holes conduct current:

$$J_{p,drift} = qp\mu_p E \qquad J_{n,drift} = -qn(-\mu_n E)$$

$$J_{tot,drift} = J_{p,drift} + J_{n,drift} = qp\mu_p E + qn\mu_n E$$

$$J_{tot,drift} = q(p\mu_p + n\mu_n)E \equiv \sigma E$$

- Conductivity

$$\boxed{\sigma \equiv qp\mu_p + qn\mu_n \quad \text{[unit: mho/cm = S/cm]}}$$

- Resistivity

$$\rho \equiv \frac{1}{\sigma} \quad [\text{Unit: } \Omega\text{-cm}]$$

- Typical resistivity range for Si: $10^{-3} \sim 10^{3}$ Ω-cm

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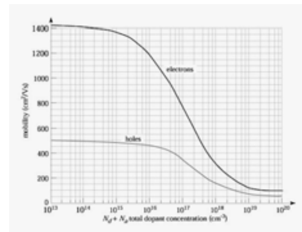
Resistivity Example

- Estimate the resistivity of a Si sample doped with phosphorus to a concentration of 10^{15} cm^{-3} and boron to a concentration of 10^{17} cm^{-3}. The electron mobility and hole mobility are 800 cm^2/Vs and 300 cm^2/Vs, respectively.



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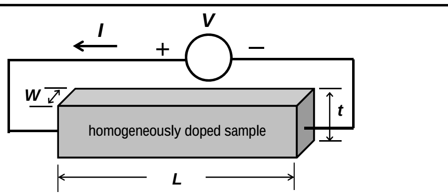
Electrical Resistance



Resistance

$$\boxed{R \equiv \frac{V}{I} = \rho \frac{L}{Wt}}$$

(Unit: ohms)

where ρ is the resistivity

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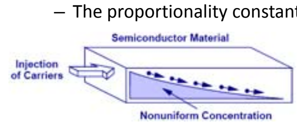
Carrier Diffusion

- Due to thermally induced random motion, mobile particles tend to move from a region of high concentration to a region of low concentration.
  - Analogy: ink droplet in water
- Current flow due to mobile charge diffusion is proportional to the carrier concentration gradient.
  - The proportionality constant is the ***diffusion constant***.



$$J_p = -qD_p \frac{dp}{dx}$$

Notation:
$D_p \equiv$ hole diffusion constant (cm^2/s)
$D_n \equiv$ electron diffusion constant (cm^2/s)

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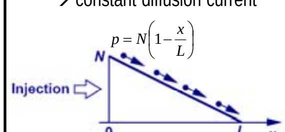
Diffusion Examples

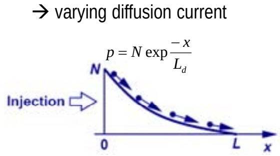
- Linear concentration profile
  → constant diffusion current



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= qD_p \frac{N}{L}$$

- Non-linear concentration profile
  → varying diffusion current



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

$$= \frac{qD_p N}{L_d} \exp\frac{-x}{L_d}$$

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Diffusion Current

- Diffusion current within a semiconductor consists of hole and electron components:

$$J_{p,diff} = -qD_p \frac{dp}{dx} \qquad J_{n,diff} = qD_n \frac{dn}{dx}$$

$$J_{tot,diff} = q(D_n \frac{dn}{dx} - D_p \frac{dp}{dx})$$

- The total current flowing in a semiconductor is the sum of drift current and diffusion current:

$$\boxed{J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff}}$$

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The Einstein Relation

- The characteristic constants for drift and diffusion are related:

$$\frac{D}{\mu} = \frac{kT}{q}$$

- Note that $\frac{kT}{q} \cong 26\text{mV}$ at room temperature (300K)
  – This is often referred to as the "thermal voltage".

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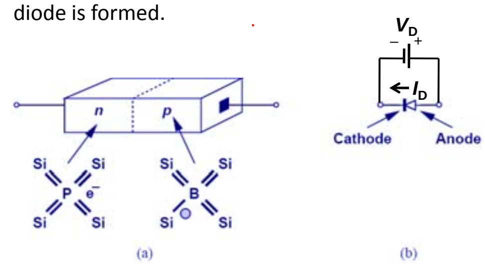
The PN Junction Diode

- When a P-type semiconductor region and an N-type semiconductor region are in contact, a PN junction diode is formed.



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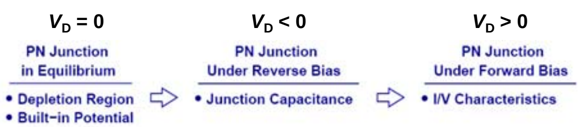
Diode Operating Regions

- In order to understand the operation of a diode, it is necessary to study its behavior in three operation regions: equilibrium, reverse bias, and forward bias.



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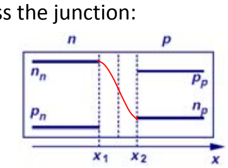
Carrier Diffusion across the Junction

- Because of the difference in hole and electron concentrations on each side of the junction, carriers diffuse across the junction:



Notation:
$n_n \equiv$ electron concentration on N-type side (cm^{-3})
$p_n \equiv$ hole concentration on N-type side (cm^{-3})
$p_p \equiv$ hole concentration on P-type side (cm^{-3})
$n_p \equiv$ electron concentration on P-type side (cm^{-3})

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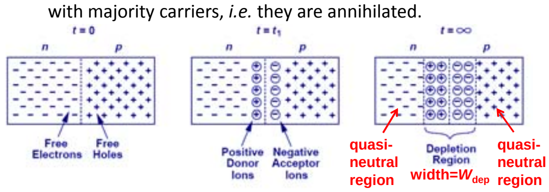
Depletion Region

- As conduction electrons and holes diffuse across the junction, they leave behind ionized dopants. Thus, a region that is depleted of mobile carriers is formed.
  – The charge density in the depletion region is not zero.
  – The carriers which diffuse across the junction recombine with majority carriers, *i.e.* they are annihilated.



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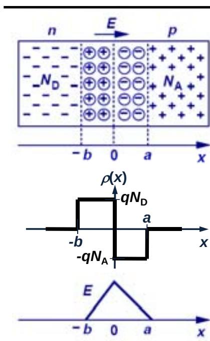
The Depletion Approximation



In the depletion region on the **N side**:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}} = \frac{qN_D}{\varepsilon_{si}} \qquad \text{Gauss's Law}$$

$$E = \frac{qN_D}{\varepsilon_{si}}(x+b) \qquad \varepsilon_{si} = 10^{12}\ \text{F/cm}$$

In the depletion region on the **P side**:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_{si}} = \frac{-qN_A}{\varepsilon_{si}}$$

$$E = \frac{qN_A}{\varepsilon_{si}}(a-x)$$

$$aN_A = bN_D$$

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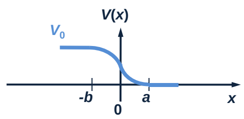
Potential Distribution

- In the depletion region, the electric potential is quadratic since the electric field is linear
- The potential difference between the N and the P side is called built-in potential, V_0

$$E = -\frac{dV}{dx}$$
$$V = -\int E \cdot dx$$



PN Junction in Equilibrium

- In equilibrium, the drift and diffusion components of current are balanced; therefore the net current flowing across the junction is zero.

$$J_{p,drift} = -J_{p,diff}$$
$$J_{n,drift} = -J_{n,diff}$$

$$J_{tot} = J_{p,drift} + J_{n,drift} + J_{p,diff} + J_{n,diff} = 0$$

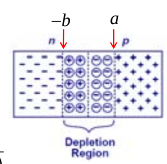
Built-in Potential, V_0

- Because of the electric field in the depletion region, there exists a potential drop across the junction:

$$qp\mu_p E = qD_p \frac{dp}{dx} \quad \Rightarrow \quad p\mu_p\left(-\frac{dV}{dx}\right) = D_p \frac{dp}{dx}$$

$$\Rightarrow \quad -\mu_p \int_{-b}^{a} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}$$

$$\Rightarrow \quad V(-b) - V(a) = \frac{D_p}{\mu_p} \ln \frac{p_p}{p_n} = \frac{kT}{q} \ln \frac{N_A}{\left(n_i^2 / N_D\right)}$$



$$\boxed{V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}}$$

(Unit: Volts)

Built-In Potential Example

- Estimate the built-in potential for PN junction below.

N	P
$N_D = 10^{18}$ cm^{-3}	$N_A = 10^{15}$ cm^{-3}

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = (26\text{mV}) \ln\left(\frac{10^{18} 10^{15}}{10^{20}}\right) = (26\text{mV}) \ln\left(10^{13}\right)$$

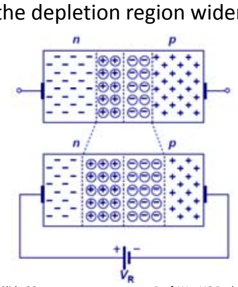
$$\text{Note: } \frac{kT}{q} \ln(10) \cong 26\text{mV} \times 2.3 \cong 60\text{mV}$$

$$V_0 = 60\text{mV} \times 13 = 780\text{mV}$$

PN Junction under Reverse Bias

- A reverse bias increases the potential drop across the junction. As a result, the magnitude of the electric field increases and the width of the depletion region widens.

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}}$$

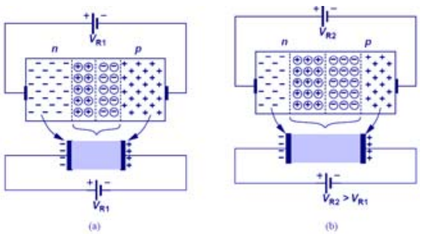


Diode Current under Reverse Bias

- In equilibrium, the built-in potential effectively prevents carriers from diffusing across the junction.
- Under reverse bias, the potential drop across the junction increases; therefore, negligible diffusion current flows. **A very small drift current flows**, limited by the rate at which minority carriers diffuse from the quasi-neutral regions into the depletion region.

PN Junction Capacitance

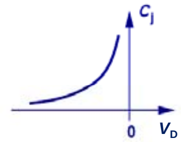
- A reverse-biased PN junction can be viewed as a capacitor. The depletion width (W_{dep}) and hence the junction capacitance (C_j) varies with V_R.



$$C_j = \frac{\varepsilon_{si}}{W_{dep}}$$

[F/cm^2]

Voltage-Dependent Capacitance



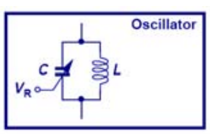
$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

$$C_{j0} = \sqrt{\frac{\varepsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_0}}$$

$\varepsilon_{si} \cong 10^{-12}$ F/cm is the permittivity of silicon

Reverse-Biased Diode Application

- A very important application of a reverse-biased PN junction is in a voltage controlled oscillator (VCO), which uses an LC tank. By changing V_R, we can change *C*, which changes the oscillation frequency.



$$f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

Summary

- Current flowing in a semiconductor is comprised of drift and diffusion components: $J_{tot} = qp\mu_p E + qn\mu_n E + qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx}$
- A region depleted of mobile charge exists at the junction between P-type and N-type materials.
  - A built-in potential drop (V_0) across this region is established by the charge density profile; it opposes diffusion of carriers across the junction. A reverse bias voltage serves to enhance the potential drop across the depletion region, resulting in very little (drift) current flowing across the junction.
  - The width of the depletion region (W_{dep}) is a function of the bias voltage (V_D).

$$W_{dep} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 - V_D)} \qquad V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$