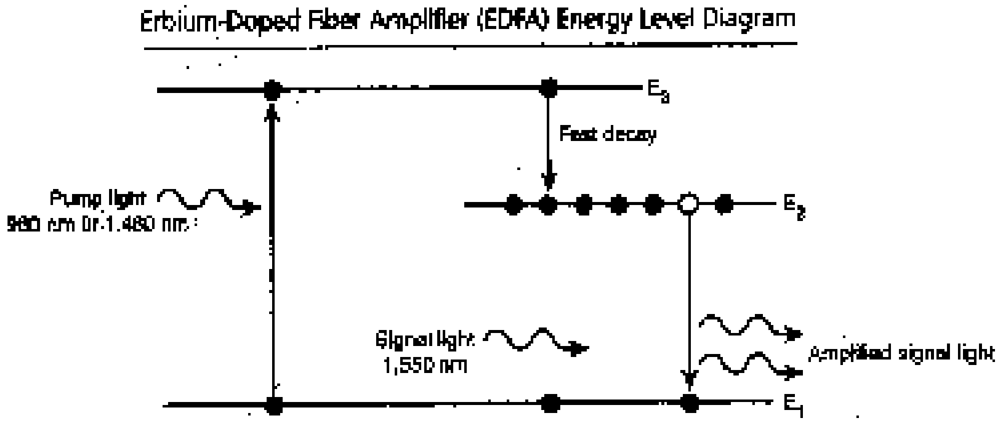
Fiber Optical Amplifiers



In semiconductors, the atoms were excited into a higher state by absorbing energy from an applied current. A different excitation procedure is used in EFDAs. Erbium ions in glass can be excited to move from state 1 to state 3 by a light source (called the pump). In EFDAs designed to amplify 1,550 nm wavelength signals, the pump light usually has a wavelength of either 980 nm or 1,480 nm, and this light is coupled into the Erbium-doped fiber using a simple wave division multiplexer.

Amplification is similar to the optical amplification we saw in a semiconductor laser. The signal light shining on ions in unstable excited states stimulates them to fall back to their lower states and to emit additional light in the process. For very small signal levels, the amount of stimulated emission, and therefore the optical gain of the EDFA, is proportional to the difference in the number of excited state ions in state 2 compared to the number of ions in state 1 and is proportional to the intensity of the 1,550 nm signal light.

However, it is usual to operate EDFAs in a saturated gain mode where the gain is relatively independent of the input signal level.

$2\pi f$

cross-section $= \dfrac{\hbar\omega B(\omega)}{c}$

gives $B \approx 1.45 \times 10^{9}$ $\sec^{-1}\,J^{-1}\,cm^{3}$

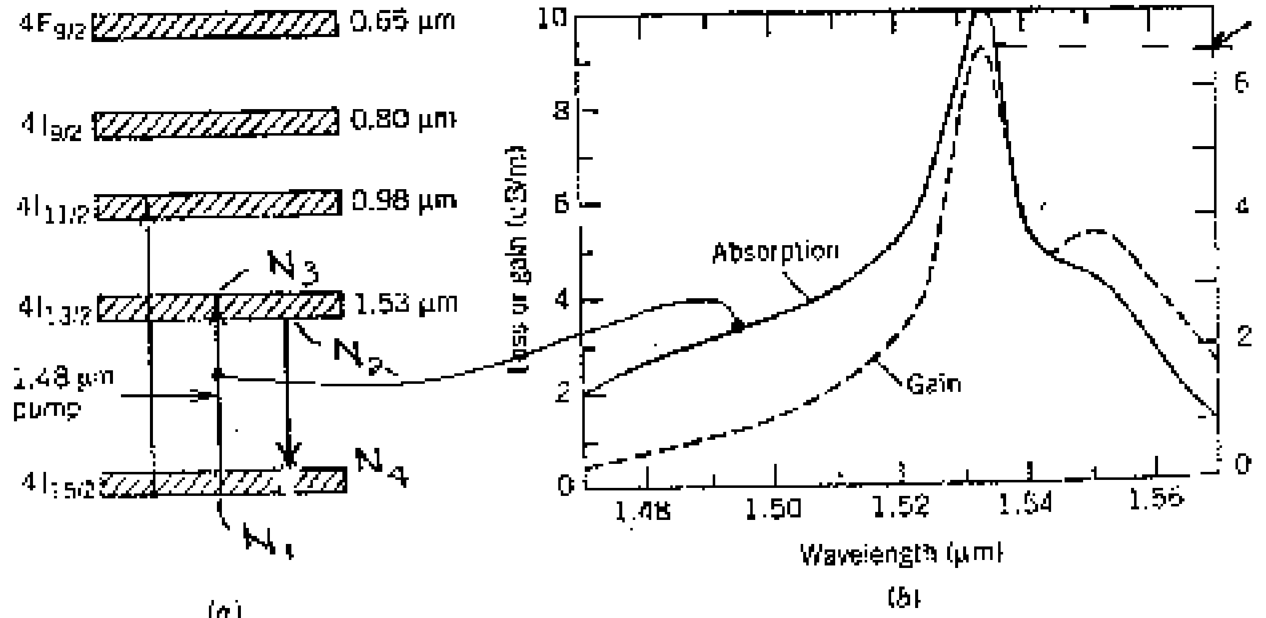
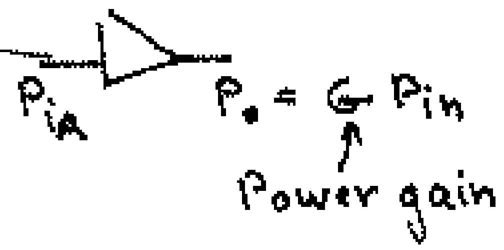


Figure 8.17 (a) Energy-level diagram of erbium ions in silica fibers; (b) absorption and gain spectra of an EDFA whose core was codoped with germania to increase the refractive index. (After Ref. [71]. ©1991 IEEE. Reprinted with permission.)

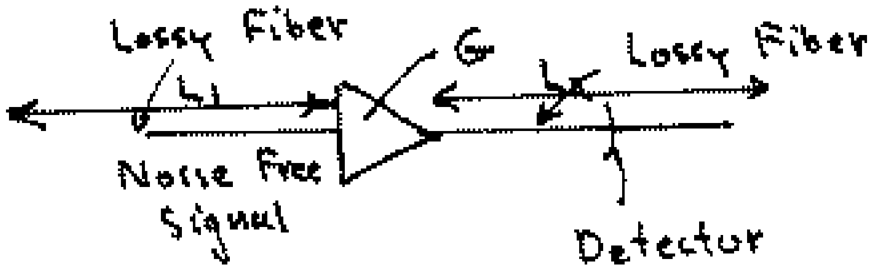
(4)

Problem No. 3 Will Assume That Signal - Spontaneous beat noise dominates. The (noise current)^2 is then

$$\langle \overline{i^2} \rangle_{ASE-sig} = 4 \underbrace{\frac{e^2 \eta^2}{h\nu}}_{\substack{\eta:\ \text{Q.E of Detector} \\ \nu:\ \text{Optical frequency}}} \overset{\text{optical Power}}{P_{iA}} (G-1)\, \overset{\frac{N_2}{N_2-N_1}}{n_{sp}}\, \underset{\text{electrical band-width}}{\Delta f}$$



Assume RIN and Sp-Sp and Spon. Shot Not important.



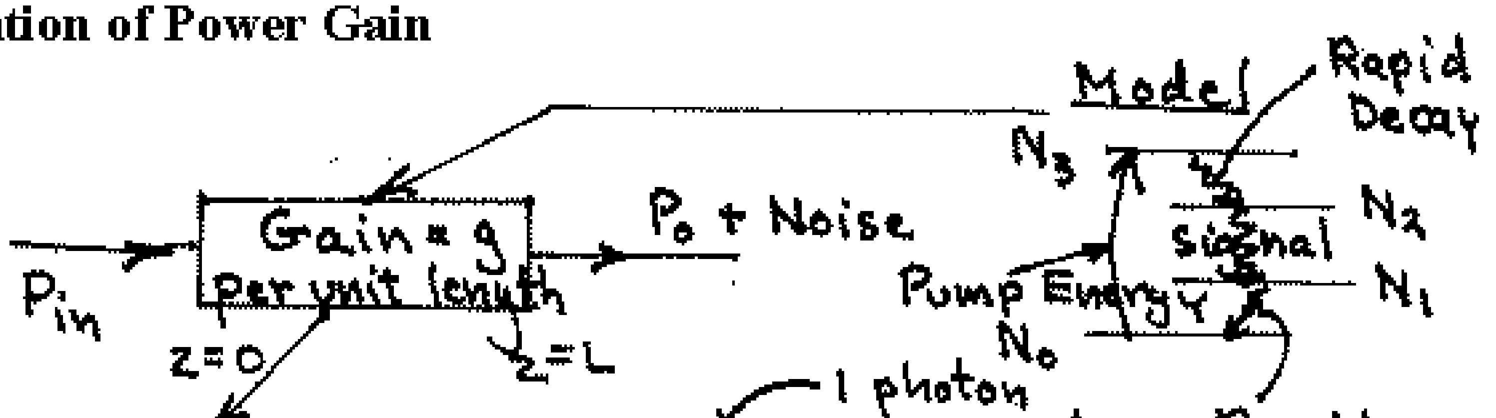
Let $P_{iA} = P_{in}$ be the input power at the amplifier $= P_s e^{-\alpha L_1}$

All except the thermal Noise involve an exponential decay between the amplifier and detector (the shot current noise)^2 by $e^{-\alpha L_2}$ and the (signal current)^2 and (spon-sig current)^2 by $e^{-2\alpha L_2}$ Thus.

$$SNR = \frac{e^2 \eta^2 \frac{P_{in}^2 G^2}{(h\nu)^2} e^{-2\alpha L_2}}{\left[\underbrace{\left(\frac{4kT_e \Delta f}{R}\right)}_{\text{Thermal}} + \underbrace{\left(\frac{2\eta e^2}{h\nu} P_{in} G e^{-\alpha L_2} \Delta f\right)}_{\text{Shot}} + \underbrace{\left(\frac{4e^2\eta^2}{h\nu} P_{in} G (G-1) \eta_{sp} \Delta f\right)}_{\text{Sig-Spontaneous}} \underset{\substack{\text{both Spon + Sig decay} \\ \text{between amp and detector}}}{e^{-2\alpha L_2}} \right]}$$

$$K^2 = SNR = \frac{P_{in} e^{-2\alpha L_2}}{\underbrace{\left(\frac{4kT_e \Delta f (h\nu)^2}{R P_{in} G^2 e^2 \eta^2}\right)}_{\text{Thermal}} + 2\frac{h\nu \Delta f}{\eta}\left[\underbrace{G^{-1} e^{-\alpha L_2}}_{\text{Shot}} + \underbrace{2 n_{sp} \eta \frac{G(G-1)}{G^2} e^{-2\alpha L_2}}_{\text{Sp-Sig}}\right]}$$

Derivation of Power Gain



(1) —— $$\frac{dP}{dz} = gP + g(n_{sp})[hf\,\Delta f]$$

gain factor

1 photon per mode

power

also = $(hf\frac{c}{L})\times(\frac{L}{c}\Delta f)$ Number of modes in Δf

Spontaneous emission noise term

(Note: $g \propto (N_2 - N_1)$ [N_2 must be > N_1 for gain])

(Note: Spontaneous Emission only depends upon N_2 - not N_1; that is why $\left(n_{sp} = \frac{N_2}{N_2 - N_1}\right)$ factor must be included)

(2) —— First term in (1) give signal gain (gP) and second term Amplified Spontaneous Emission (ASE)

To solve for P_o + Noise move gP term to left side and write the equation as (after multiplying by e^{-gz})

$$\frac{d(Pe^{-gz})}{dz} = g n_{sp} hf\Delta f\, e^{-gz}$$

Integrate from z=0 to z=L

$$P_{out}e^{-gL} - P_{in} = \underbrace{g n_{sp} hf\Delta f}_{\text{Constant}}\left(\frac{e^{-g} - 1}{-g}\right)$$

or

$$P_{out} = P_{in}\underbrace{e^{+gL}}_{G} + n_{sp}hf\Delta f\,(G-1)$$

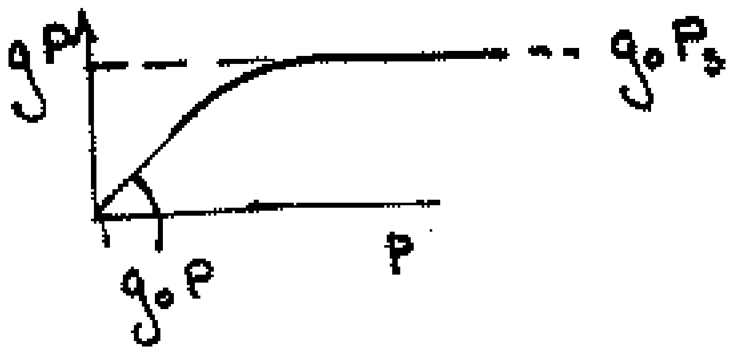
$$P_{out} = \underline{P_o + S_{sp}\Delta f} \quad \text{where } \underline{S_{sp} = (G-1)n_{sp}hf}$$

Amplified Signal

Amplified Spontaneous Emission

Saturation of the Amplifier

In Eq. (1) $gP = \dfrac{g_0}{1 + P/P_s} P$ where P_s = saturation Power



Thus (1) becomes

$$\frac{dP}{dz} = \frac{g_0}{1+P/P_s} P + \frac{g_0}{1+P/P_s} n_{sp} [hf\Delta f] = \frac{g_0}{1+P/P_s} (P + n_{sp} hf \Delta f)$$

Consider the signal term on the Right side of this equation:

$$\frac{dP}{dz} = \frac{g_0 P}{1 + P/P_s} \quad \text{or} \quad \frac{dP}{P}\left(1 + \frac{P}{P_s}\right) = g_0 \, dz$$

Integrating gives: $\ln \dfrac{P_o}{P_{in}} + \dfrac{P_o - P_{in}}{P_s} = g_0 L$

or $$\frac{P_o}{P_{in}} = e^{-\left(\frac{P_o - P_{in}}{P_s}\right)} G_0$$

where $G_0 = e^{g_0 L}$

Letting $P_o = G P_{in}$ we have
$$\frac{P_o - P_{in}}{P_s} = \frac{P_o - P_o/G}{P_s} = \frac{P_o}{P_s}\left(1 - \frac{1}{G}\right)$$

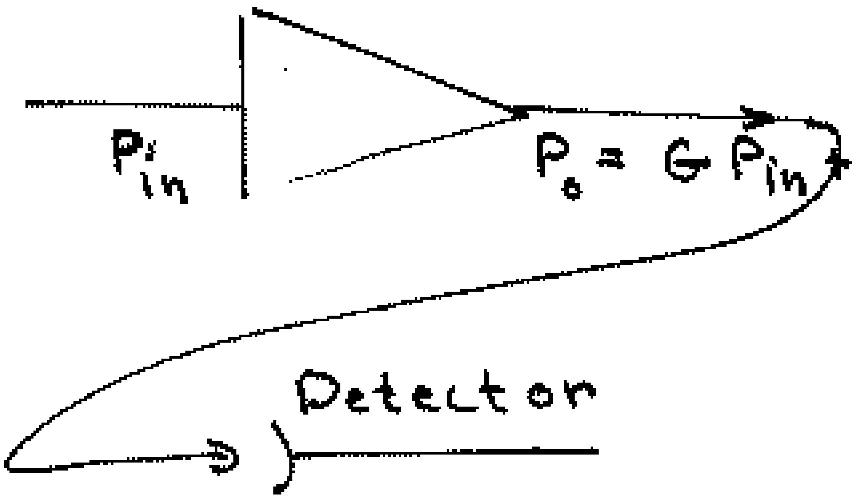
Thus
$$\frac{P_o}{P_{in}} = G = \underbrace{e^{-\left(1 - \frac{1}{G}\right)\frac{P_o}{P_s}} G_0}_{\text{Implicit Relation For the Saturated Gain}}$$

when the noise term is included this becomes

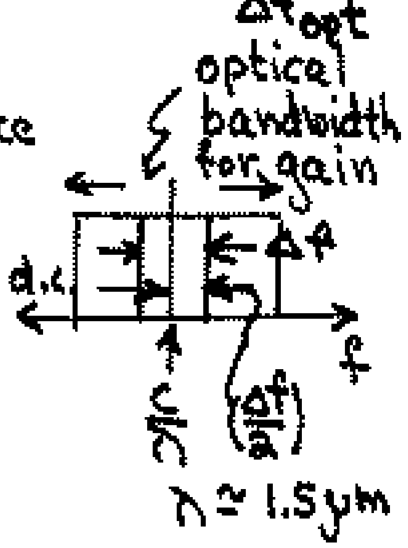
$$P_o = P_{in} G + (G-1) n_{sp} hf \Delta f$$

where $$G = e^{-\left[\left(1 - \frac{1}{G}\right)\frac{P_o}{P_s - hf n_{sp}\Delta f}\right]} G_0 \quad \text{with } G_0 = e^{\left(\frac{g_0 L}{P_s - hf n_{sp} \Delta f}\right)}$$

Spontaneous - Signal Beat Noise - Simplified



$P_o = G P_{in} + \underbrace{(G-1) n_{sp} hf \, \Delta f}_{\text{Noise Power}}$ Note



Detector current $I(t) = \dfrac{\eta P_o}{hf} e = \mathcal{R} P_o$ (1) $\quad \mathcal{R} = \dfrac{\eta e}{hf}$

For two fields Electric Field $= \underset{\text{Signal}}{E_s} + \underset{\text{Noise}}{E_N}$

$E_s = E_1 \cos(\omega_1 t + \varphi_1) = \dfrac{E_1 e^{i(\omega_1 t + \varphi_1)}}{2} + \text{c.c.}$

$E_N = E_2 \cos(\omega_2 t + \varphi_2) = \dfrac{E_2 e^{i(\omega_2 t + \varphi_2)}}{2} + \text{c.c.}$

Random Phase Associated With Noise

$\therefore P_o = \dfrac{(E_s + E_N)^2}{\sqrt{\mu/\epsilon}} \Bigg|_{\text{Low Frequency Terms}}$

$= \dfrac{(E_s^2 + E_N^2 + 2E_s E_N)}{\sqrt{\mu/\epsilon}} \Bigg|_{\text{Low Frequency Terms}}$

$= \left(\dfrac{1}{\sqrt{\mu/\epsilon}}\right) \left(\tfrac{1}{2} E_1 E_1^* + \tfrac{1}{2} E_2^2 + E_1 E_2 \cos((\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2))\right)$

But $P_1 \Big|_{\text{Low Frequency}} = \tfrac{1}{2} E_1^2 / \sqrt{\mu/\epsilon}$; $P_2 \Big|_{\text{Low Freq.}} = \tfrac{1}{2} E_2^2 / \sqrt{\mu/\epsilon}$

Thus $P_o = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos((\omega_1 - \omega_2)t + (\varphi_1 - \varphi_2))$ - (2)

Now $P_1 = G P_{in}$ and $P_2 = n_{sp}(G-1) hf \, \Delta f$

The $\cos((\omega_1 - \omega_2)t + \varphi_1 - \varphi_2)$ term when squared and averaged over a random φ_2 gives $\frac{1}{2}$. Thus

$\overline{I^2_{N\,spon\text{-}sig}} = \left(\dfrac{\eta e}{hf}\right)^2 4 G P_{in} (G-1) hf \, n_{sp} \left(\dfrac{\Delta f}{2}\right)$ → electrical band width used

Dominant Noise Terms When An Optical Amplifier is Used With a Detector

Shot Noise (Signal) $\overline{i_s^2} = q\,2\frac{q\eta}{hf}\,G\,P_{in}\,\Delta f$

Spontaneous-Spontaneous Beat Noise $\overline{i^2_{sp-sp}} = hf\left(2\frac{q\eta}{hf}(G-1)n_{sp}\right)^2 \Delta f_{opt}(\Delta f\ hf)$

Signal-Spontaneous Beat Noise

$$\boxed{\overline{i^2_{sig-sp}} = 4\left(\frac{q\eta}{hf}G\,P_{in}\right)\left(\frac{q\eta}{hf}(G-1)n_{sp}\right)hf\,\Delta f}$$

electrical bandwidth

Usually Dominant
(Approximately $2G\times$ shot noise)

Noise Figure of An Optical Amplifier

Let $\mathcal{R} = \frac{\eta q}{hf}$ = Responsivity

when $G = 1$ the dominant noise (minimum) is the shot noise

$$(SNR)_{in} = \frac{(\mathcal{R}P_{in})^2}{2q\,\mathcal{R}P_{in}\,\Delta f} = \frac{P_{in}\,\eta}{2\,hf\,\Delta f}$$

with the amplifier Sp-Sig Noise dominant

$$\frac{\overline{i^2_{sig-sp}}}{\overline{i_s^2}} = 2\eta(G-1)n_{sp}$$

$$(SNR)_{out} = \frac{(\mathcal{R}G\,P_{in})^2}{4\,\mathcal{R}^2 G\,P_{in}(G-1)\,n_{sp}\,hf\,\Delta f}$$

$$= \frac{G}{4(G-1)}\,\frac{P_{in}}{n_{sp}\,hf\,\Delta f}$$

Thus $F_n = \frac{(SNR)_{in}}{(SNR)_{out}} = 2n_{sp}\frac{(G-1)}{G} \rightarrow 2$ at best