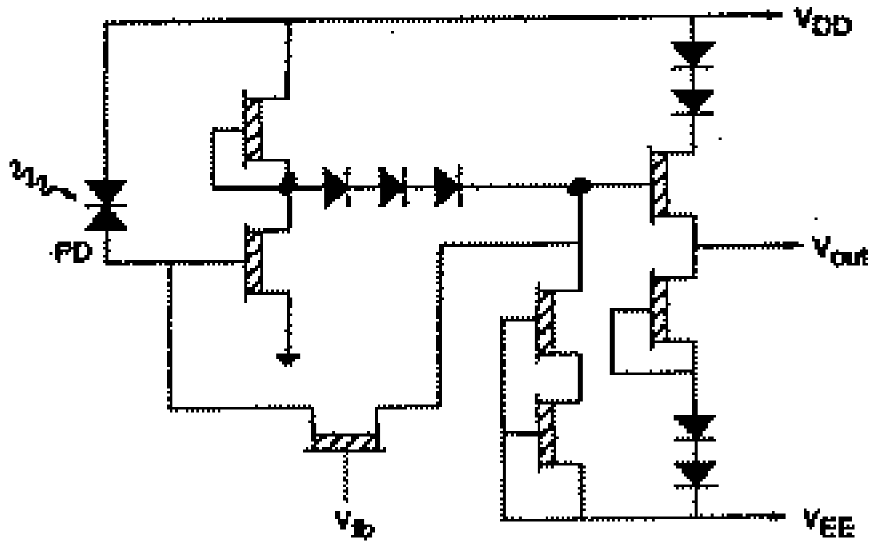
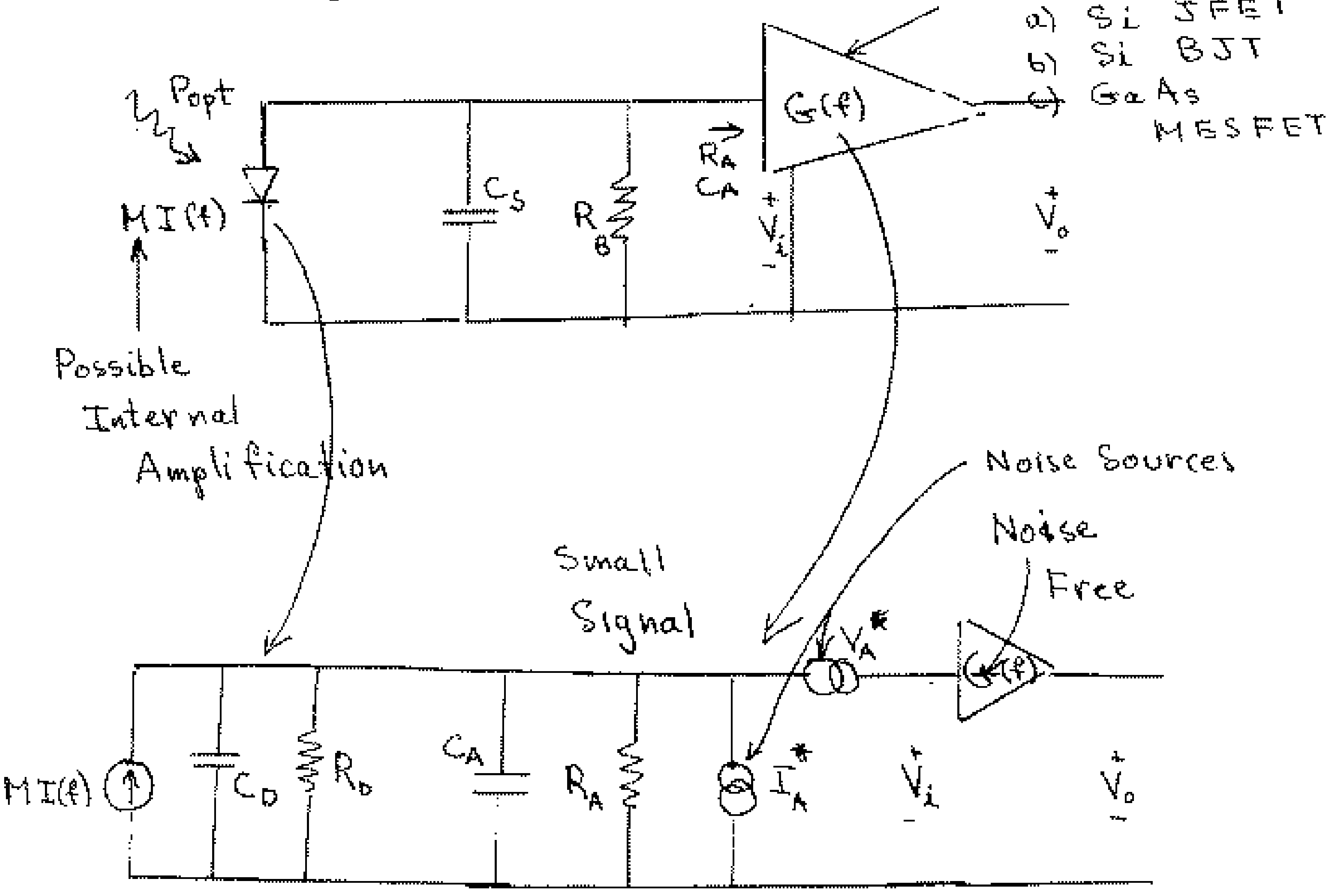
Example Receiver S/N Ratio Calculation



→ low pass filter to reduce noise with minimal "ISI"

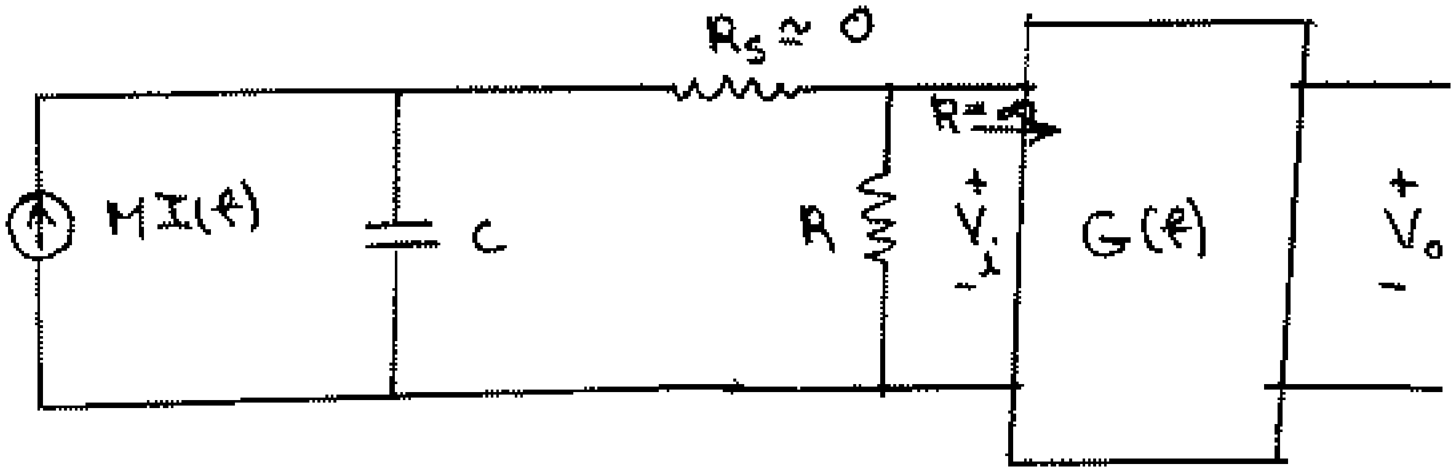
→ clock recovery

Basic Equivalent Circuit



Reference on Noise:
Noise in Receiving System
Raoul Pettai, Wiley (1984)

Example Small Signal Analysis



$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_D} + \frac{1}{R_B}$$

$$C = C_D + \underbrace{C_S}_{\text{stray}} + C_A$$

$$V_i = \frac{R\,M\,I(f)}{1 + j\,2\pi f\,C R}$$

$$V_o = G(f)\,V_i = \frac{G(f)\,R\,M\,I(f)}{1 + j\,2\pi f\,C R}$$

For "Equalized" Amplifier $G(f) = G_o(1 + j2\pi f RC)$

and

a) Output is independent of frequency

$$V_o = G_o M R I$$

b) $\frac{V_o}{I(f)}$ has the same functional relationship as a transimpedance amplifier

Various Noise Sources

$$(I_{sh}^*)^2 = \Delta f\,(2eI\,M^2 F)$$ — Photodiode Shot Noise

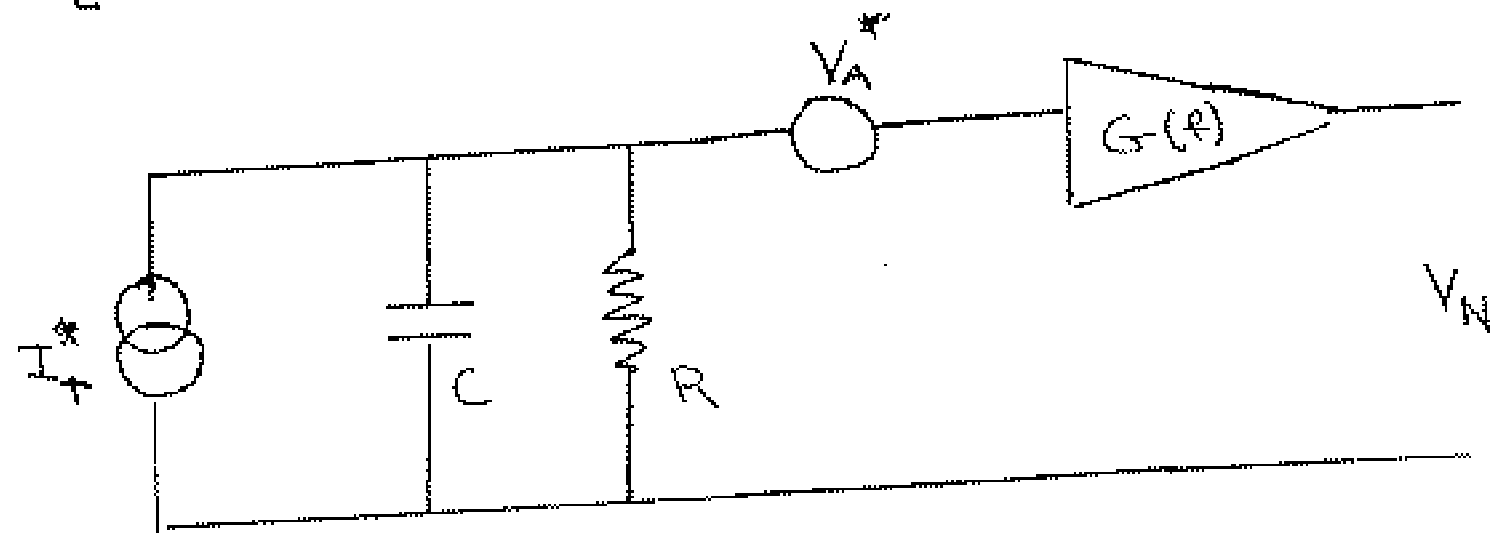
M: current amplification factor

F: Excess Noise Due to M

$$(I_{th}^*)^2 = \Delta f\left(\frac{4kT}{R}\right) = 4kT\left(\frac{1}{R_A} + \frac{1}{R_B} + G_D\right)\Delta f$$

$$G_D = \frac{1}{R_D}$$

Equivalent Noise Circuit:



$$(I_T^*)^2 = (I_{sh}^*)^2 + (I_{th}^*)^2 + (I_A^*)^2$$

$$C = C_S + C_A + C_D$$

$$\frac{1}{R} = \frac{1}{R_D} + \frac{1}{R_A} + \frac{1}{R_B}$$

$$V_N^2 = G_0^2 \int_{0}^{\Delta f} |G(f)|^2 (V_A^*)^2\, df + \int_{0}^{\Delta f} \frac{G^2(f)\, R^2\, (I_T^*)^2}{|1 + j2\pi f C R|^2}\, df$$

$0 \to \Delta f$ (over the band width)

$$V_N^2 = G_o^2 \int_0^{\Delta f} \left[\{ 1 + 4\pi^2 f^2 C^2 R^2 \} (V_A^*)^2 + R^2 (I_T^*)^2 \right] df$$

Thus

$$\frac{V_o}{V_N} = \frac{G_o M I R}{\left[\left(\Delta f + \frac{4\pi^2}{3} (\Delta f)^3 C^2 R^2 \right) (V_A^*)^2 + \Delta f \, R^2 (I_T^*)^2 \right]^{1/2}}$$

$$= K = \frac{I}{\sqrt{\Delta f} \left[\frac{(V_A^*)^2}{M^2} \left(\underbrace{\frac{1}{R^2}}_{(a)} + \underbrace{\frac{4\pi^2}{3} (\Delta f)^2 C^2}_{(b)} \right) + \underbrace{2eIF}_{(c)} + \underbrace{\frac{4kT}{M^2 R}}_{(d)} + \underbrace{\left(\frac{I_A^*}{M}\right)^2}_{(e)} \right]^{1/2}}$$

Notes:

1) S/N increases with M until shot noise dominates
2) Increasing R excellent as long as (a) & (d) significant. If R too large equalization is needed to assure band-width
3) Shot Noise causes K to be signal dependent
4) At high frequencies b) dominates (increases as C^2 – thus important to minimize C)
5) Assumes noise sources uncorrelated

Given K (12 for BER ≈ 10^{-9}) can solve for I

$$I^2 - 2pI - q = 0$$

$p = eFK^2\Delta f$ (c)

$$q = \underbrace{K^2\Delta f (I_A^*)^2}_{(e)} + \underbrace{K^2 \frac{4kT\Delta f}{M^2 R}}_{(d)} + \frac{K^2\Delta f}{M^2}\left[\frac{V_A^{*2}}{R^2}\left(\underbrace{1}_{(a)} + \underbrace{\frac{4\pi^2}{3}(\Delta f)^2 C^2 R^2}_{(b)}\right)\right]$$

<u>Solution:</u>

$$I = p \pm \sqrt{p^2 + q}$$

Relevant sign is + (otherwise noise can → 0)

$$= p\left\{1 + \left(1 + q/p^2\right)^{1/2}\right\}$$

Advantageous to have M to decrease q + << p^2 (but F increases with M)

Note:

Minimum

$I = 2p$ → shot noise limit

$= K^2 2e\Delta f F$

Example Si-APD

$\eta = .75$ $\mathcal{R} = .5$ A/W $F = 6$

$B = 2\Delta f = 1$ MHz $K = 12$

$$I = K^2 e(2\Delta f) F$$
$$= 144 \times 1.6 \times 10^{-19} \times 10^6 \times 6$$
$$= 1.4 \times 10^{-10} \text{ } \underline{\text{amps}}$$

Show that <u>when</u> thermal Noise Dominates and $\Delta f = \frac{1}{2\pi RC}$

$$M^2 \gg \frac{2\pi kTC}{e^2 F^2 K^2}$$

assures the shot noise limit