Coherent Detection

The *heterodyne* and *homodyne* detection techniques that were developed for radio transmission, where they are known as *coherent*, are also used in optical transmission. However, in optical transmission, the term "coherent" indicates that another light source is used as the local oscillator at the receiver.

In the case of digital communications systems, the phase, the frequency, or the amplitude of a carrier signal may be modulated. When the phase is modulated, the method is called *phase-shift keying* (PSK); when the frequency is modulated, it is called *frequency-shift keying* (FSK); when the amplitude is modulated, it is called *amplitude-shift keying* (ASK). Each modulation method has its own advantages and disadvantages. Another method by which the intensity of a light source is modulated and also detected by a photodetector is known as *intensity modulation with direct detection* (IM/DD).

The modulation technique plays an important role in the performance of fiber transmission. For example, coherent techniques improve receiver sensitivity by ~20 dB, and thus longer fibers may be used (an additional 100 km at 1.55 μm). In dense wavelength division multiplexing (DWDM) systems, where many channels are used in the same fiber, the channel spacing with IM/DD is in the order of 100 GHz whereas with coherent techniques it can be as small as 1–10 GHz.

An example of coherent detection of an incoming modulated signal using a local oscillator (i.e., a light source of a frequency in the vicinity of the transmitted source) is shown in Figure III.1. In the absence of a local oscillator with a narrow

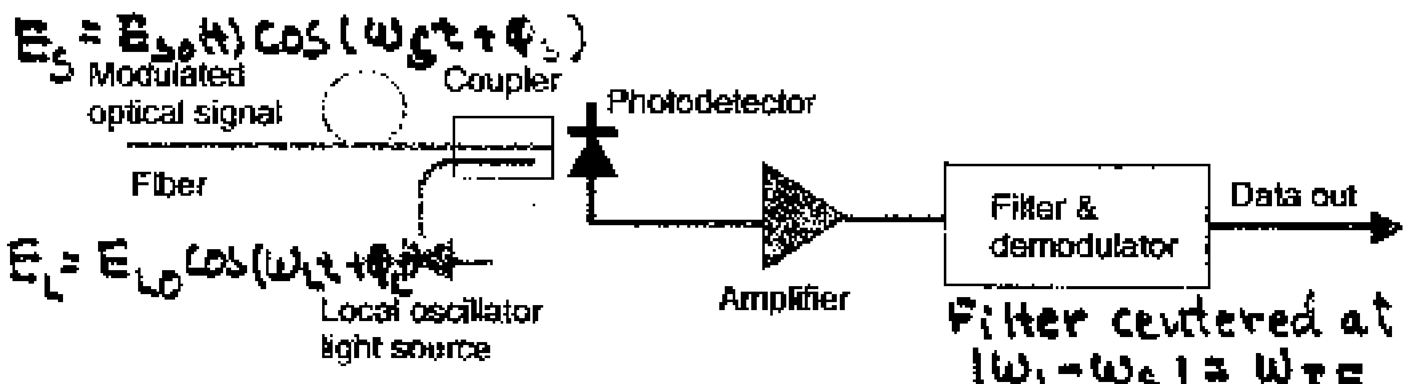


Figure III.1 Coherent detection requires a local oscillator with a narrow spectral width comparable to the source in the incoming signal.

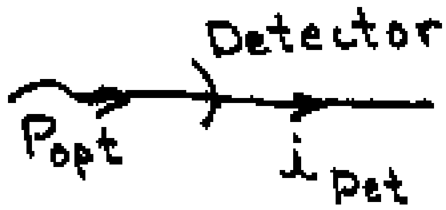
($\omega_L = \omega_S$ - homodyne $\omega_L \neq \omega_S$ - heterodyne)

spectral (or line) width comparable to the source in the incoming signal, however, the coherent detection method is impractical. Local oscillators emit light spontaneously and thus become sources of noise. Therefore, the selection of the local oscillator and its amplitude are important in receiver design. In the case of IM/DD, the incoming signal is directly coupled into the detector, thus eliminating the coupler and the local oscillator.

* IM/DD - Intensity modulation/Direct Detection

ω_{IF} - intermediate frequency

Basic Expressions Needed for Coherent Detection

① 

$i_{Det} = R P_{opt}$ $\quad R = \frac{q\eta}{hf}$ $\quad \eta$ = quantum efficiency

② P_{opt} in terms of electric field, E

$$P_{opt} = \underset{\substack{\uparrow \\ \text{Detector} \\ \text{Area}}}{A}\, E^2 \Big/ \sqrt{\frac{\mu}{\epsilon}} \qquad \sqrt{\frac{\mu}{\epsilon}} = \frac{1}{n}\sqrt{\frac{\mu}{\epsilon_0}} = \frac{1}{n}(377)\ \Omega\text{'s}$$

Thus $i = R_E E^2$ $\qquad R_E = \frac{Aq\eta}{n\,hf}\sqrt{\frac{\epsilon}{\mu}}$ $\qquad \frac{[\text{amps m}^2]}{\text{Volt}^2}$

For two fields E_s and E_L

$$E^2 = E_s^2 + E_L^2 + 2E_sE_L$$

multiply by $\frac{A}{\sqrt{\mu/\epsilon}}$

$$\frac{A}{\sqrt{\frac{\mu}{\epsilon}}}E^2 = \frac{A}{\sqrt{\frac{\mu}{\epsilon}}}E_s^2 + \frac{A}{\sqrt{\frac{\mu}{\epsilon}}}E_L^2 + 2\sqrt{\frac{A}{\sqrt{\frac{\mu}{\epsilon}}}E_s^2\,\frac{A}{\sqrt{\frac{\mu}{\epsilon}}}E_L^2}$$

or

$$P_{opt}^2 = P_s^2 + P_L^2 + 2\sqrt{P_sP_L}$$

③ For Phasor Representation of The Fields

$$E_s = \frac{|E_{so}|}{2}e^{i\omega_s t + i\phi_s} + c.c. \xrightarrow{\text{Thus}} P_{so} = \frac{1}{2}\frac{A}{\sqrt{\mu/\epsilon}}|E_{so}|^2$$

$$E_L = \frac{|E_{Lo}|}{2}e^{i\omega_L t + i\phi_L} + c.c. \xrightarrow{\text{Thus}} P_{Lo} = \frac{1}{2}\frac{A}{\sqrt{\mu/\epsilon}}|E_{Lo}|^2$$

$$E_s^2 = \frac{|E_{so}|^2}{2} + \boxed{\frac{|E_{so}|^2}{2}\cos(2\omega_s t + 2\phi_s)}$$

$$E_L^2 = \frac{|E_{Lo}|^2}{2} + \boxed{\frac{|E_{Lo}|^2}{2}\cos(2\omega_s t + 2\phi_s)}$$

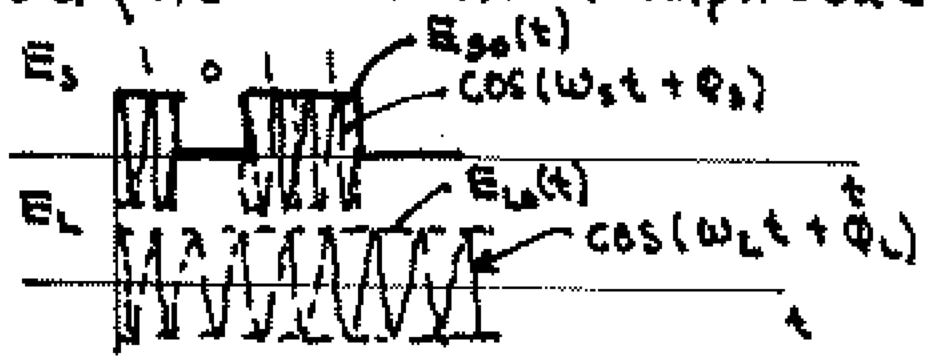
$$E_sE_L = \boxed{\frac{|E_{so}||E_{Lo}|}{2}\cos((\omega_s+\omega_L)t + \phi_s + \phi_L)} + \frac{|E_{so}||E_{Lo}|}{2}\cos((\omega_s-\omega_L)t + \phi_s - \phi_L)$$

These three rapidly oscillating terms cannot cause a detector response and can thus be ignored

④ $$P_{opt}^2\Big|_{\text{time-average}} = P_{so}^2 + P_{Lo}^2 + 2\sqrt{P_{so}P_{Lo}}\cos((\omega_s-\omega_L)t + \phi_s - \phi_L)$$

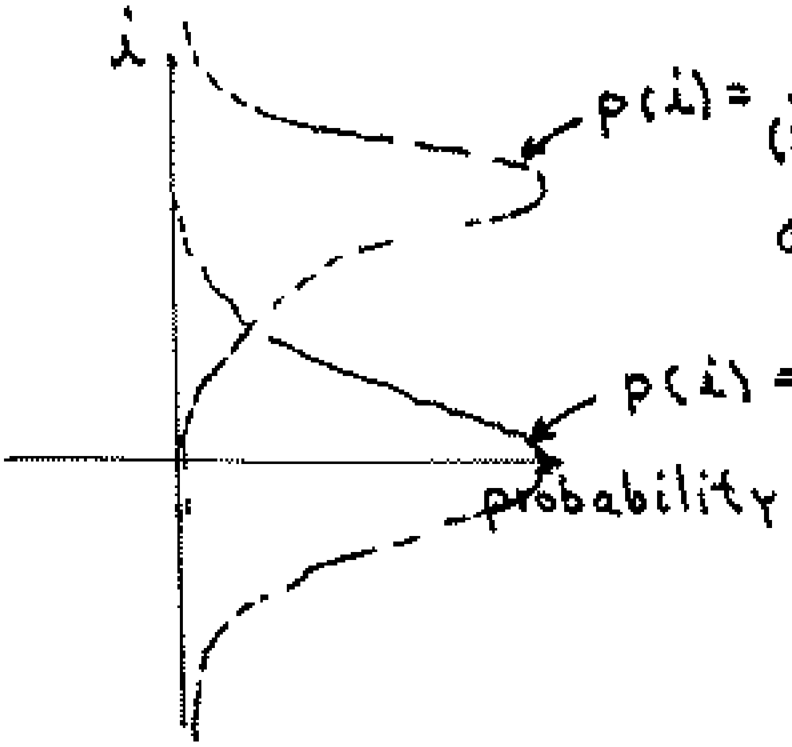
⑤ And $$i_{Det}\Big|_{\text{time-average}} = R\left(P_{so}^2 + P_{Lo}^2 + 2\sqrt{P_{so}P_{Lo}}\cos((\omega_s-\omega_L)t + \phi_s - \phi_L)\right)$$

Example Homodyne A.S.K. (Amplitude Shift Keying) (NRZ) (a)



Assume E_{Lo} sufficient to have noise due solely to shot noise

Assume 1-bit and 0-bit described by Gaussian Distributions (C.L.T)



$$p(i) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-\frac{(i-I_1)^2}{2\sigma^2}}$$

σ due to shot noise from E_{Lo}

$$p(i) = \frac{1}{(2\pi)^{1/2}\sigma} e^{-\frac{(i-I_0)^2}{2\sigma^2}}$$

Can express currents in terms of photo-electron number per bit

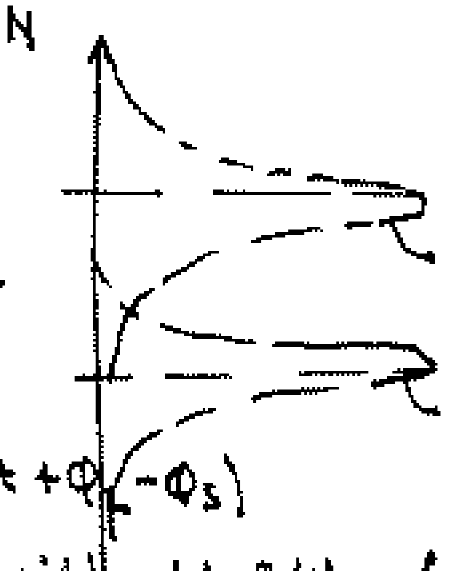
$$\frac{(i-I_1)^2}{2\sigma^2} = \frac{\left(\frac{iT}{e} - \frac{I_1 T}{e}\right)^2}{2\left(\frac{\sigma T}{e}\right)^2} = \frac{(N - N_1)^2}{2 N_{Lo}}$$

$R = \frac{e\eta}{hf}$

$$\left(\frac{\sigma T}{e}\right)^2 = \left(2eRP_{Lo}\frac{\Delta f\, T^2}{e^2}\right) = \left(RP_{Lo}\frac{T}{e}\right) = N_{Lo}$$

$N_{Lo} = \frac{R}{e} P_{Lo} T$

$N = \frac{R}{e}(P_s + P_L + 2\sqrt{P_L P_s} \cdot \cos((\omega_L - \omega_s)t + \phi_L - \phi_s))$



$$p(N) = \frac{1}{(2\pi)^{1/2} N_{Lo}} e^{-\frac{(N-N_1)^2}{2N_{Lo}}}; \quad \int_{-\infty}^{+\infty} p(N)\,dN = 1$$

$$p(N) = \frac{1}{(2\pi)^{1/2} N_{Lo}} e^{-\frac{(N-N_0)^2}{2N_{Lo}}} \quad \int_{-\infty}^{+\infty} p(N)\,dN = 1$$

with $\omega_L = \omega_s$ for homodyne

Example of Homodyne ASK

$$N = \frac{R}{e}\left(P_s + P_L + 2\sqrt{P_L P_s}\cos((\omega_L - \omega_s)t + \phi_L - \phi_s)\right)$$

$$= \frac{R}{e}\left(P_s + P_L + 2\sqrt{P_L P_s}\right)$$

$$= N_s + N_L + 2\sqrt{N_s N_L}$$

For B.E.R = 10^{-9}

from $\frac{1}{2}\mathrm{erfc}\left(\frac{I_1 - I_0}{2\sqrt{2}\sigma}\right)$

$$\frac{I_1 - I_0}{2\sqrt{2}\sigma} = \frac{N_1 - N_0}{2\sqrt{2}\sqrt{N_{L0}}} = \frac{12}{2\sqrt{2}}$$

$$N_1 = \underbrace{\cancel{N_s}}_{\text{small}} + N_L + 2\sqrt{N_s N_L}$$

$$N_0 = N_L = N_{L0}$$

$$\therefore \quad N_1 - N_0 = 2\sqrt{N_s N_L}$$

So $\frac{\sqrt{N_s N_{L0}}}{\sqrt{2}\sqrt{N_{L0}}} = \frac{12}{2\sqrt{2}} \Rightarrow N_s = 36$ photo-electrons.

Example of Homodyne P.S.K.

$$N_1 = N_s + N_L + 2\sqrt{N_s N_L}$$

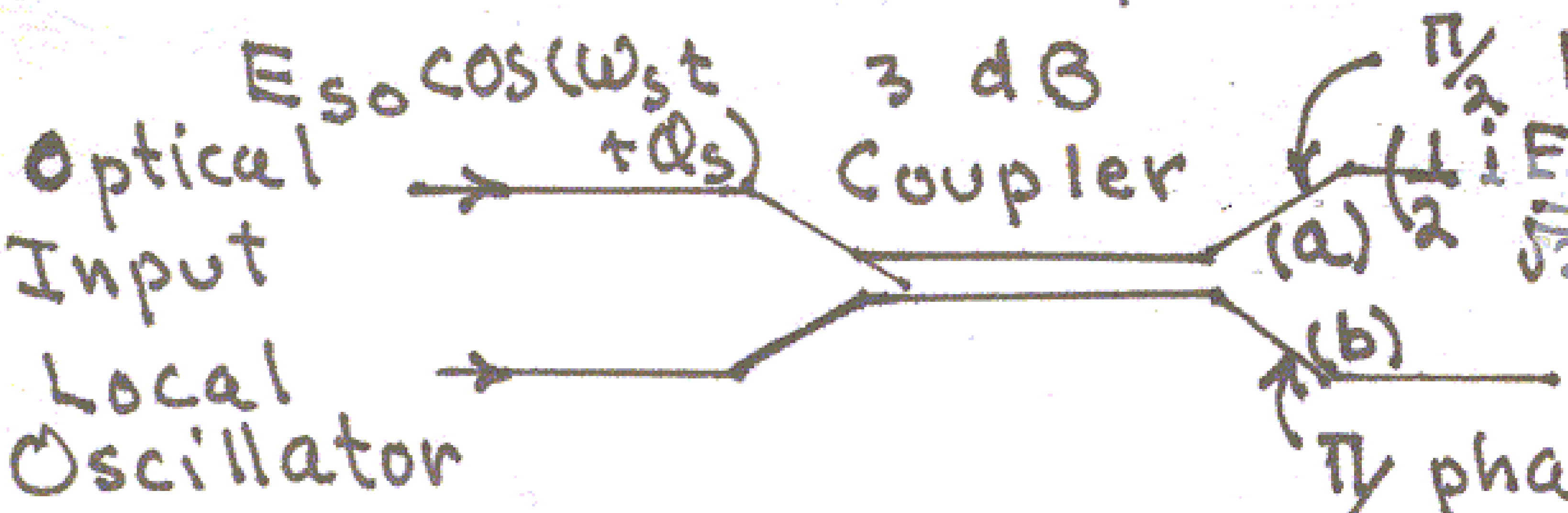
$$N_0 = N_s + N_L - 2\sqrt{N_s N_L}$$

$$\therefore \quad N_1 - N_0 = 4\sqrt{N_s N_L}$$

So $\frac{4\sqrt{N_s N_L}}{\sqrt{N_L}} = 12$

And Thus $N_s = 9$ photons/bit

3) Balanced Heterodyne Detector



$\pi/2$ phase shift for L.O.

(a): $\left(\frac{1}{\sqrt{2}} i \frac{E_{Lo}}{2} e^{i(\omega_L t + \varphi_L)} + \frac{1}{\sqrt{2}} \frac{E_{so}}{2} e^{i(\varphi_s + \omega_s t)} + c.c.\right)$

$\pi/2$ phase shift for E_{so}

(b): $\left(\frac{1}{\sqrt{2}} i E_{so} e^{i(\omega_s t + \varphi_s)} + \frac{1}{\sqrt{2}} E_{Lo} e^{i(\varphi + \omega t)} + c.c.\right)$

$E_{Lo}\cos(\omega_L t + \varphi_L)$

$= \left(\frac{E_{Lo}}{2} e^{i(\omega_L t + \varphi_s)} + c.c.\right)$

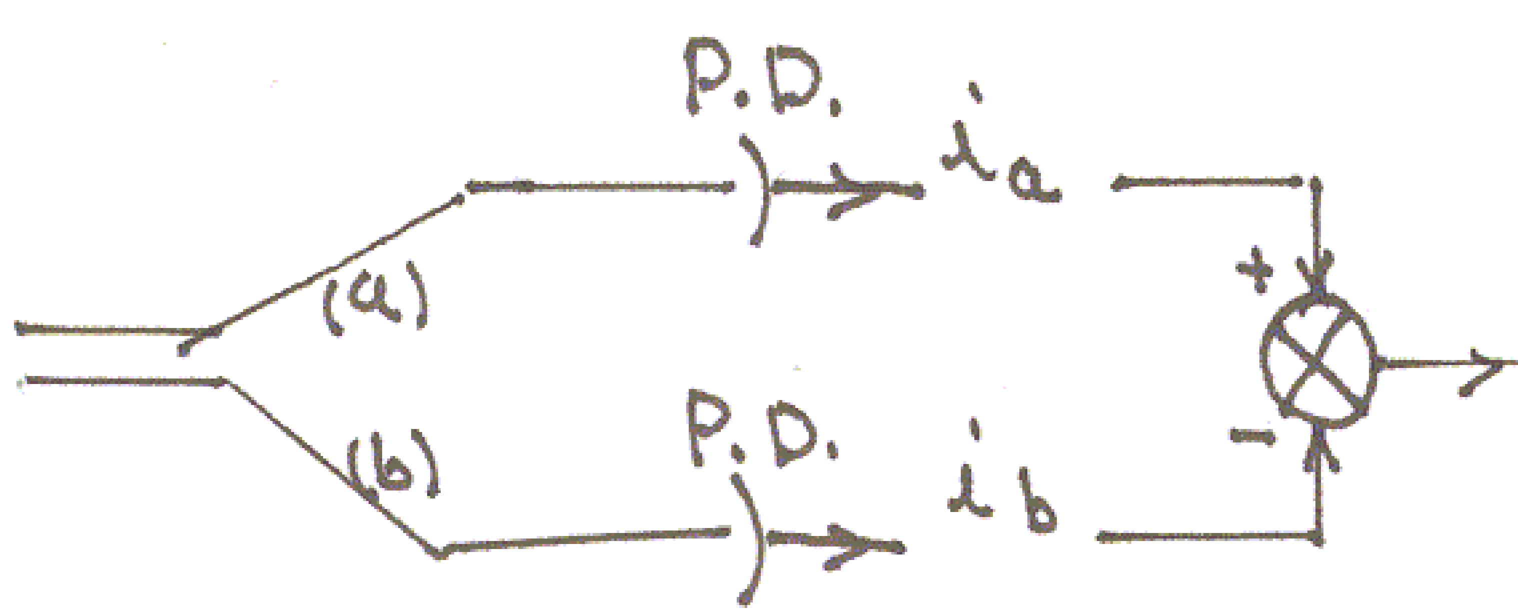
coming from the $\frac{1}{\sqrt{2}}$

$$i_a = \frac{1}{2} R\left(P_{so} + P_{Lo} + 2\sqrt{P_{so}P_{Lo}}\,(-\sin(\omega_L t + \varphi_L))\cos(\omega_s t + \varphi_s)\right)$$

$$-\left(\sin((\omega_L - \omega_s)t + \varphi_L - \varphi_s) + \sin((\omega_L + \omega_s)t + \varphi_L - \varphi_s)\right)$$

only relevant term

$$i_b = \frac{1}{2} R\left(P_{so} + P_{Lo} + 2\sqrt{P_{so}P_{Lo}}\,(-\sin(\omega_s t + \varphi)\cos(\omega_L t + \varphi_L)\right)$$



$$i_o = -R\sqrt{P_{so}P_{Lo}}\,\sin((\omega_L - \omega_s)t + \varphi_L - \varphi_s)$$

Thus the current is doubled and the (electrical) power is quadrupled (a 6 dB enhancement)

In addition the intensity noise of the local oscillator is cancelled by the subtraction!