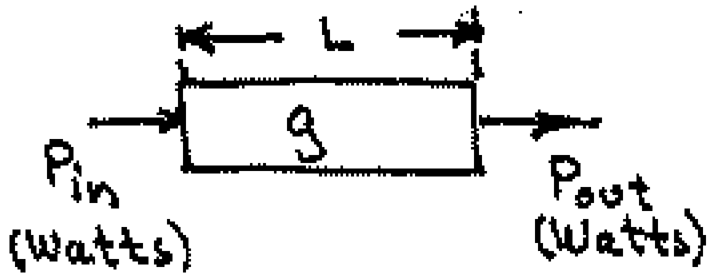
Basic Coupled Mode Equations – Electric Field Gain

Generalization of The Amplifier Gain Equation

①



Basic Equation

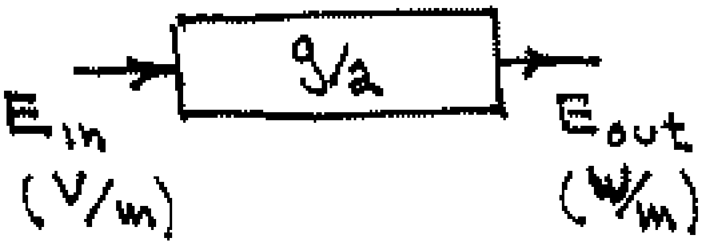
$$\frac{dP}{dz} = g P$$

gain coefficient (m^{-1})

Solution

$$P_{out} = P(L) = P_{in} e^{gL} = P_{in} G$$
$$= P(0) G$$

② Express This in Terms of Field Amplitude To be Able To Treat <u>Phase</u> Sensitive Devices



Basic Equation

$$\frac{dE}{dz} = \frac{g}{2} E$$

$$E_{out} = E_{in} e^{gL}$$

since $P \propto E^2$ this implies ①

③ <u>Furthermore</u> g can be complex. For example

Let $E = \left(\frac{E_0}{2} e^{i\omega t} + c.c.\right)$ and separate into $e^{i\omega t}$ and $e^{-i\omega t}$ terms ($E_0 = |E_0| e^{i\phi_0}$ – complex)

<u>Time average power</u> $P_{AVG} = \frac{1}{2} \frac{A |E_0|^2}{\sqrt{\mu/\epsilon}}$

Basic Equation

$$\frac{dE_0}{dz} = \left(\frac{\tilde{g}}{2}\right) E_0 \quad \text{complex}$$

$$E_{0out} = E_{0in} e^{\frac{\tilde{g}}{2} L} = E_{0in} \tilde{G}$$

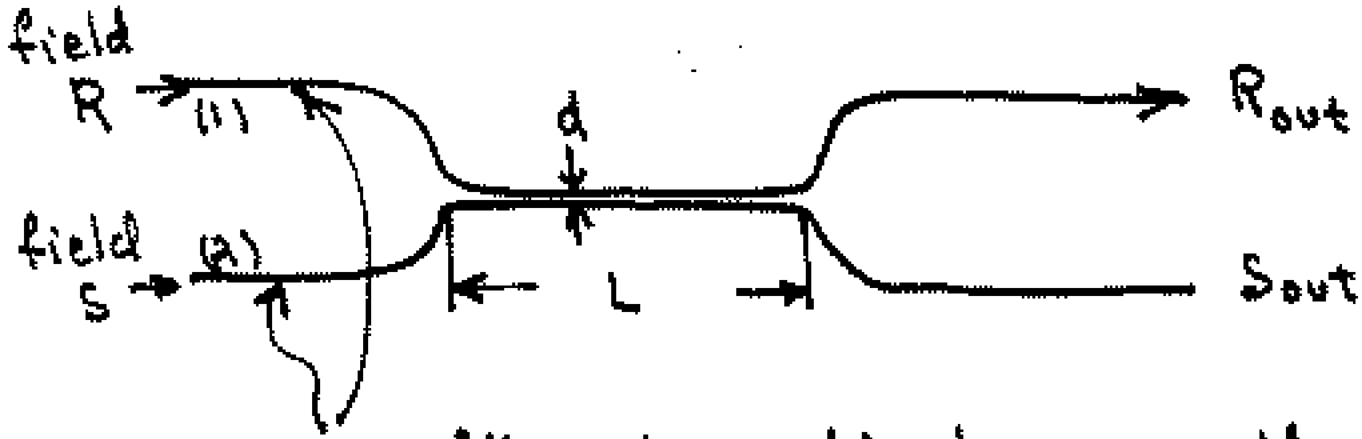
Time Averaged output Power $\propto E_{0out} E_{0out}^*$

Thus $P_{out\,Avg} = P_{in\,Avg} e^{\left(\frac{\tilde{g} + \tilde{g}^*}{2}\right) L} = P_{in\,Avg} e^{gL}$

where $g = Re(\tilde{g})$

Advantage: With $\tilde{g}$ one obtains the phase change as well as amplitude change (amplification)

Coupled Modes – A Model Switch or Directional Coupler



Two fibers (or optical waveguides which come into close proximity (separation d) over length L. The field of one guide (R-say) can penetrate the other guide (guide 2) and influence its field (S-say) vice versa as well. Let the "coupling" be specified by a parameter κ. In analogy with a single gain equation, we now have two coupled equations:

$$① \quad \frac{dR}{dz} = \kappa S + \frac{\tilde{g}}{2} R \quad \text{previous gain terms}$$

$$② \quad \frac{dS}{dz} = \kappa R + \frac{\tilde{g}}{2} S$$

Coupling terms.

Let there be no gain (for the moment). Thus $\tilde{g} = 0$. Assume a lossless coupler. Then the total time-average power must not change $\frac{dP}{dz} = 0$

But $P_{AVG} = \frac{1}{2}\frac{A}{\sqrt{\frac{\mu}{\epsilon}}}(RR^* + SS^*)$. Thus Multiply Eq(1) by R^* and Eq(2) by S^* and $(\text{Eq}(1))^* \times R$ and $(\text{Eq}(2))^* \times S$ and add the four resulting Eqtns to obtain:

$$\frac{d}{dz}(RR^* + SS^*) = \kappa(SR^* + RS^*) + \kappa^*(S^*R + R^*S)$$
$$= 0$$

Thus $\kappa = -\kappa^*$ or κ is imaginery (loss-less coupling)

Letting $\kappa = iK$, the basic equations are

$$① \quad \frac{dR}{dz} = iKS \quad ; \quad ② \quad \frac{dS}{dz} = iKR$$

Solutions For The Coupler.

(1) $\frac{dR}{dz} = iKS \qquad \frac{dS}{dz} = iKR$

From (1) $\frac{d^2R}{dz^2} = iK\frac{dS}{dz} = (iK)^2 R = -K^2 R$

Thus $R = A\cos(Kz) + B\sin(Kz)$

$S = C\cos(Kz) + D\sin(Kz)$

But from Eq(1) $-AK = iKD$ and $KB = iKC$

Thus $R = A\cos(Kz) + iC\sin(Kz)$

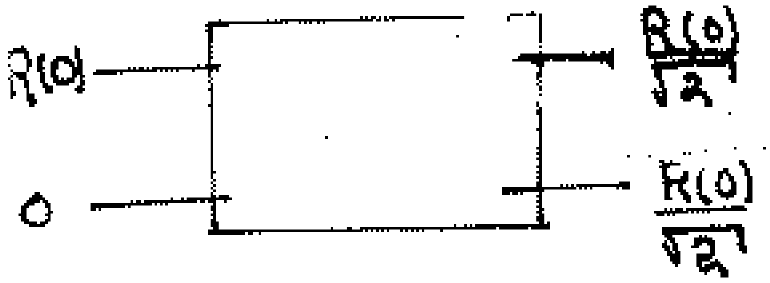
$S = C\cos(Kz) + iA\sin(Kz)$

<u>Example</u>: 1) 3 dB Coupler

Let $(KL) = (\pi/4)$; $C = 0$ (No S-wave at input)

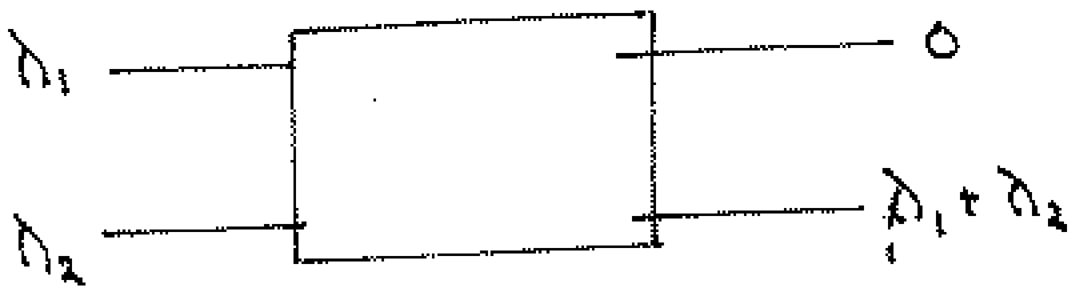
$R(L) = A\frac{1}{\sqrt{2}}$

$S(L) = \underbrace{i}_{\pi/2 \text{ phase shift}} A\frac{1}{\sqrt{2}}$



2) Wavelength Multiplexer - Demultiplexer

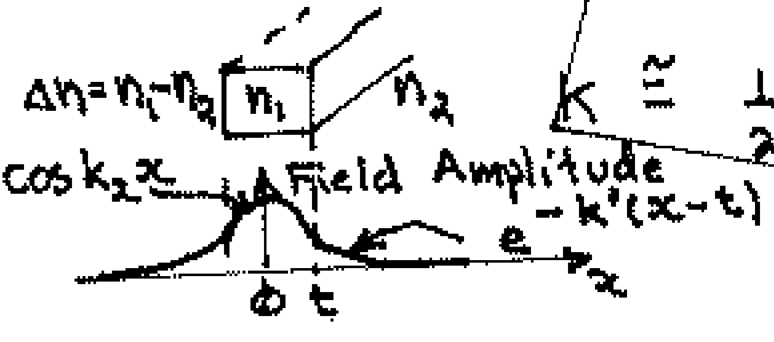
Design coupling so that at λ_1 (wavelength 1) cross-over occurs, while at λ_2 it doesn't.



$K_{\lambda_1}L = \frac{\pi}{2}$

$K_{\lambda_2}L = \pi$

Value of K



For $k' \ll t$ $\quad k' \gg k_x \cong 1/t$

$$K \cong \frac{1}{2}(\Delta n)\left(\frac{2\pi}{\lambda}\right)\left(e^{-k'(d-2t)}\right)$$

$\frac{2\pi}{\lambda} \cong 10^4\ \text{cm}^{-1}$; $e^{-k'(d-2t)}$: $10^{-1} - 10^{-5}$

Coupled Waveguides