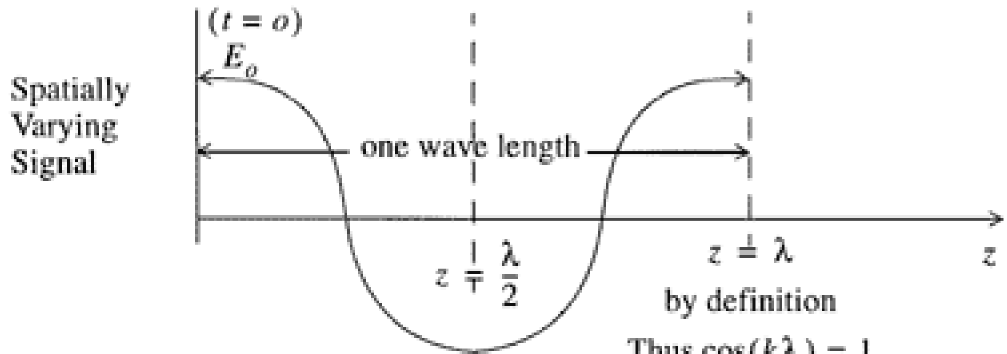
EECS 118 - Basic Comments Regarding Waves
Wavelength, Frequency and Phase Velocity

Basic Properties of Waves

$$\text{Electrical Field Compenent} = E_o\cos(\omega t - kz)$$

$$= \frac{E_o}{2}e^{i(\omega t + kz)} + c.c.$$

$E = E_o\cos(kz)$



by definition

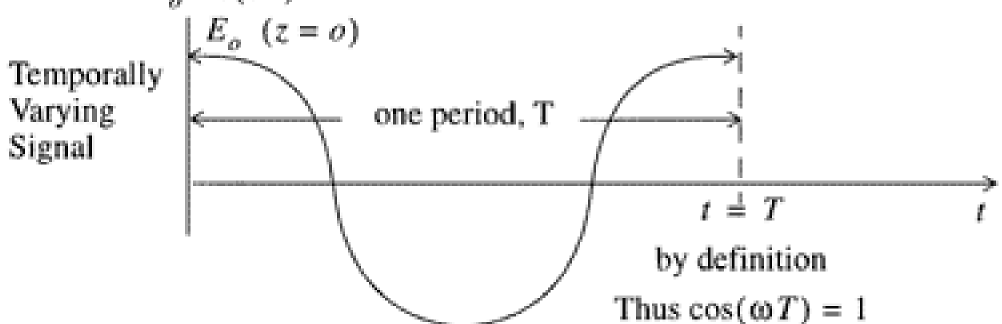
Thus $\cos(k\lambda) = 1$

$k\lambda = 2\pi$ or

$k = \frac{2\pi}{\lambda}$

Number of periods per unit $length = \frac{1}{\lambda} = \frac{k}{2\pi} =$ Spatial frequency

$E = E_o\cos(\omega t)$



by definition

Thus $\cos(\omega T) = 1$

$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$

Number of periods per second $= \frac{1}{T} = f =$ "frequency"

Observations

1) $\frac{\omega}{k} = \frac{2\pi}{T} \times \frac{\lambda}{2\pi} = \frac{\lambda}{T} = \lambda f$

2) $f = \frac{1}{T} =$ no. of cycles per sec

$=$ frequency

phase velocity

3) To sit on the peak of the wave

$\omega t - kz = o$ Thus $\frac{z}{t} = \frac{\omega}{k} = \boxed{\lambda f = c}$

4) c is the speed of light 299,792,248 $\frac{m}{s}$

<u>Theoretical</u> Information Transmission Rate Limit is set by

a) The <u>frequency</u> $= f$ (More accurately the range of available f's $= \Delta f$ = band width. [Units = Hz = sec^{-1}])

b) The <u>direction</u> for a given f. (More accurately the range of available spatial directions)

c) The <u>signal / noise ratio</u>

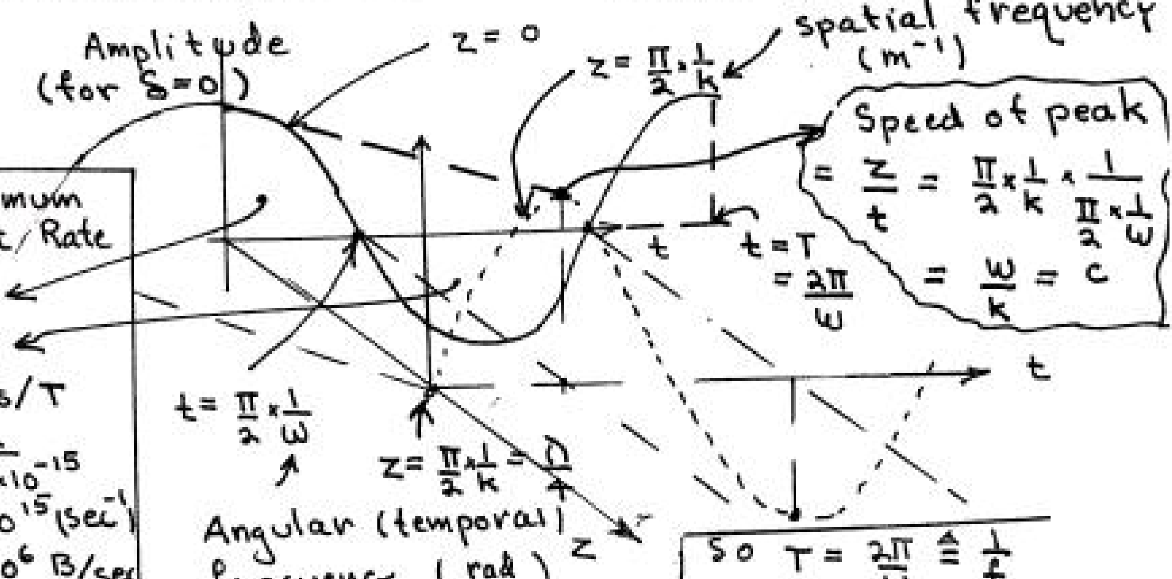
<u>Theoretical</u> Information Transmission Delay Limit is set by the speed of light, c.

<u>Important</u> <u>Practical</u> <u>limitations</u>

a) loss of energy during transmission

b) spreading of the pulse-width (dispersion)

<u>Basic Nature of a Wave</u> $\} \rightarrow A\cos(\phi)$ = Amplitude with $\phi = \omega t - kz + \delta$



Maximum Bit Rate

2bits/T

$\simeq \frac{2}{3 \times 10^{-15}}$

$\simeq 10^{15} (sec^{-1})$

$\simeq 10^{6}$ B/sec

Unfortunately $\Delta f = 10^{6}$ GHz

So $T = \frac{2\pi}{\omega} \triangleq \frac{1}{f}$

$\lambda = \frac{2\pi}{k}$

$k = \omega / c$ or $\lambda f = c$

EECS 118 - Basic Comments Regarding Waves
dB Loss, Dispersion and Acceptance Angle

Three Parameters for Optical Fibers

1) Loss

$$\text{Loss in dB} = -10 \;\; lg_{10} \frac{P_o}{P_i}$$

Power decreases exponentially with distance

$$P_o = P_i e^{-\alpha L}$$ α is loss in nepers/m

$$\begin{aligned}
\text{Thus loss in } db &= -10 \;\; lg_{10}(e^{-\alpha L}) \\
&= -10 \;\; lg_{10}(e^{-\alpha L}) \;\; lg_{10}(e) \\
&= 10 \; (\alpha) \times .4343 \\
&= 4.343\alpha
\end{aligned}$$

2) Dispersion [psec/nm/km] $= D$

$$\boxed{\Delta\tau = D \Delta\lambda L}$$

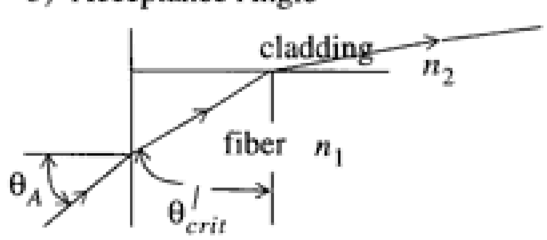
amount of broadening in picoseconds | bandwidth of the signal | fiber length

Example: If Δf of the signal is 10 GH_z what is the broadening per km if D = 10 psec/nm/km

<u>ans</u>

$$\begin{aligned}
&10 \;\; \text{psec} \times (\Delta\lambda) \times 1 km \\
&= 10 \;\; \text{psec} \times \left(\frac{1}{30} nm\right) \times 1 km = \frac{1}{3} \text{psec}
\end{aligned}$$

3) Acceptance Angle



Problem

Show that

$$\sin\theta_A = [n_1^2 - n_2^2]^{1/2}$$

(see next)

Two Basic Wave Calculations

a) Acceptance Angle

$t=0 \Rightarrow \varphi = kz$

distance perpendicular to phase front

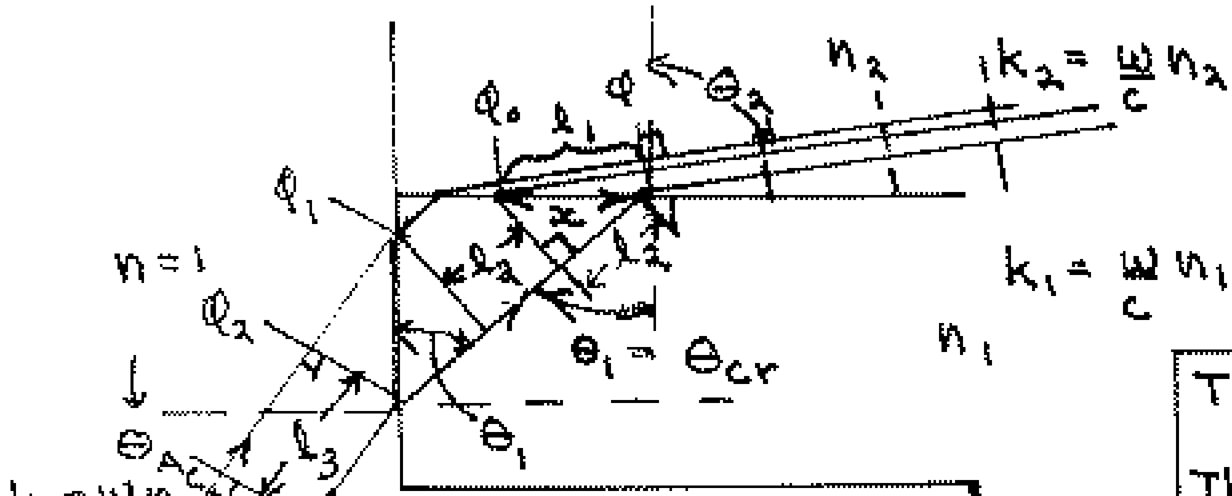
Note:

$\varphi - \varphi_0 = k_2 l_2 =$

$= k_1 l_1$

$l_1 = x \sin\theta_2$

$l_2 = x \sin\theta_1$



Thus $k_2 \sin\theta_2 = k_1 \sin\theta_1$

Thus $n_2 \sin\theta_2 = n_1 \sin\theta_1$ (Snell's "Law")

From Snells Law

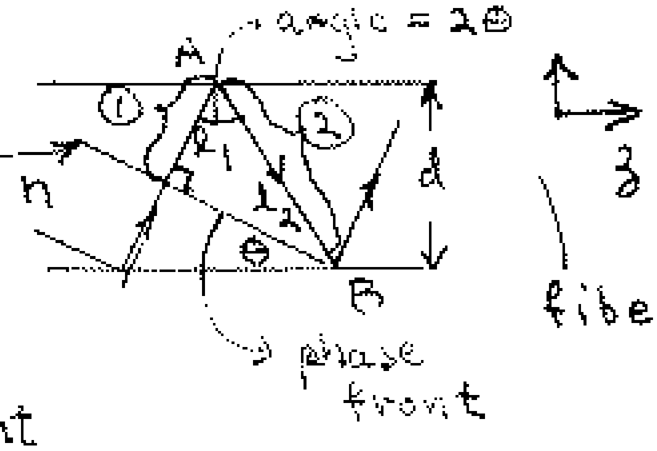
$\sin\theta_{Ac} = n_1 \cos\theta_1 = n_1(1-\sin^2\theta_1)$

$= n_1\left(1 - \left(\frac{n_2}{n_1}\right)^2 \sin^2\theta_2\right)^{1/2}$

$\theta_2 = 90^\circ$ for critical angle

Acceptance Angle → Thus $\sin\theta_c = n_1\left(1 - \left(\frac{n_2}{n_1}\right)^2\right)^{1/2} = (n_1^2 - n_2^2)^{1/2}$

b) Modal Constraint



Total phase change for a complete round-trip

$$\varphi_T = \underbrace{k_2 l_1}_{\substack{\text{phase change going}\\ \text{from ① to ②}}} + \underbrace{k_2 l_2}_{\substack{\text{phase change}\\ \text{from ② to ③}}} + \varphi_{r1} + \varphi_{r2}$$

φ_{r1}: phase change on reflection at A

φ_{r2}: reflection at B

φ (phase) = constant = φ_1 [modulo 2π] : Phase change in going from ① + ②

is $\varphi = k(l_1 + l_2) = k\left(\frac{d}{\cos\theta} - 2d\tan\theta\cdot\sin\theta + \frac{d}{\cos\theta}\right)$

$+ \varphi_A + \varphi_B$

phase changes on reflection at A & B

$= k\, d \cos\theta + \varphi_A + \varphi_B$

$\boxed{\text{Thus for a mode } \varphi = \frac{\omega}{c} n d \cos\theta + \varphi_A + \varphi_B = m(2\pi) \text{ and } \theta \text{ has a fixed value}}$

m: integer

Optical Fiber Characteristics

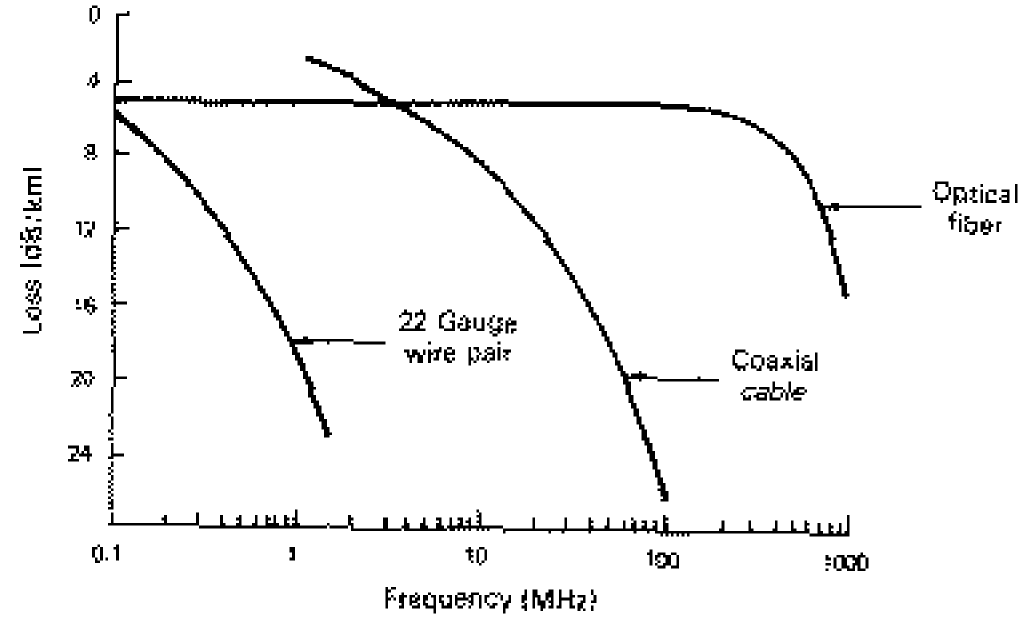


FIGURE 4-5. Comparison of Optical Fiber, Coaxial Cable, and Wire Pair *[STAL85]*

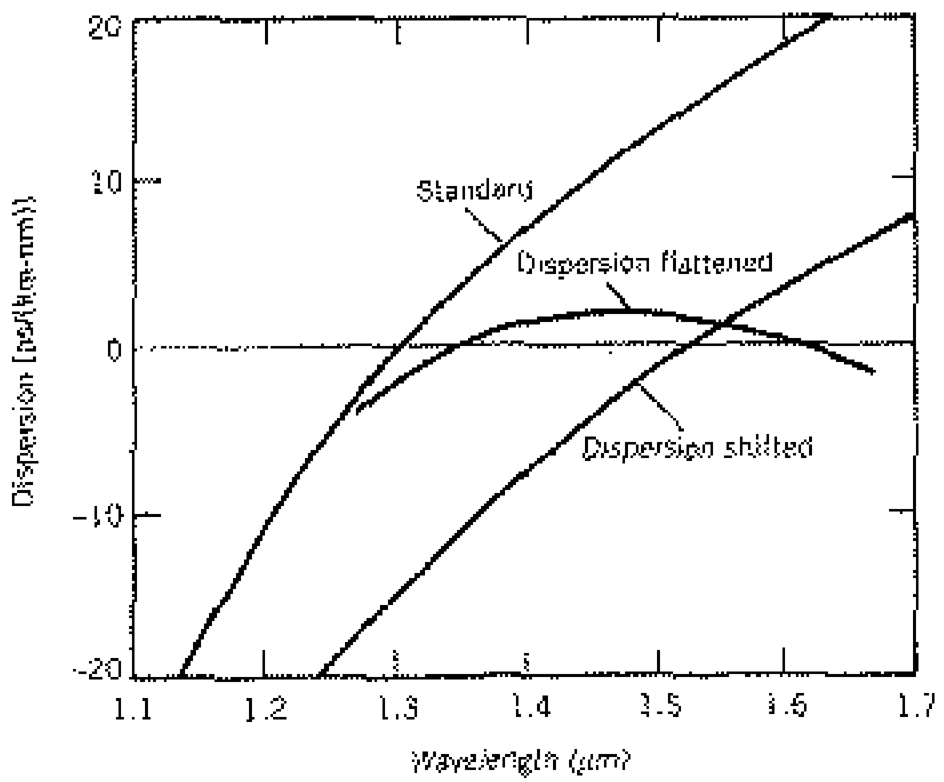


Figure 2.11 Typical wavelength dependence of the dispersion parameter D for standard, dispersion-shifted, and dispersion-flattened fibers.

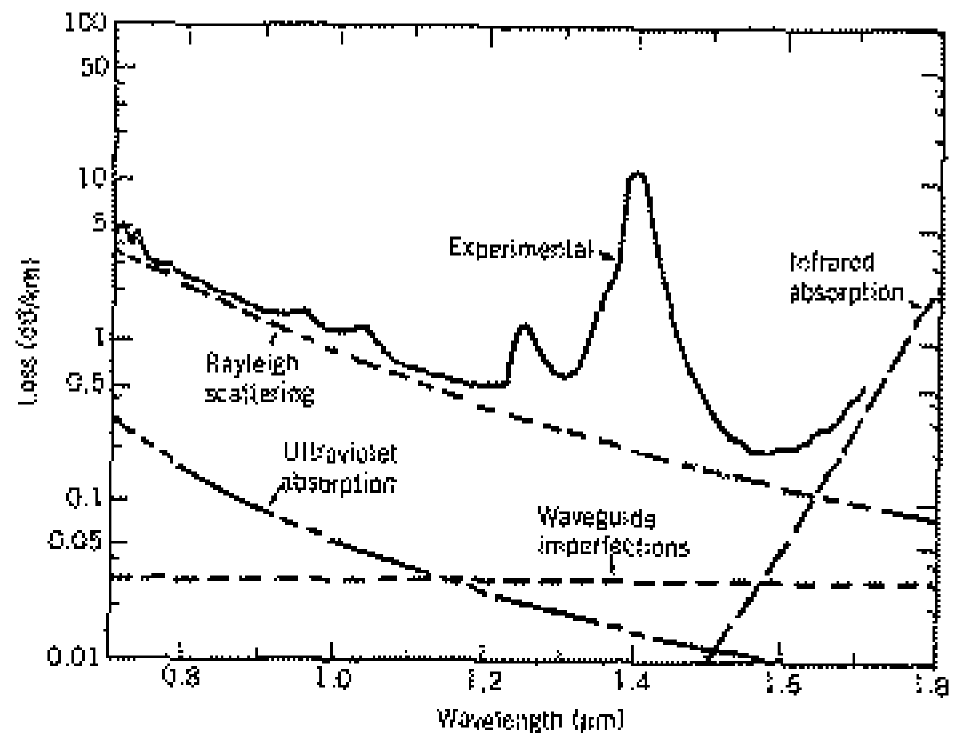


Figure 2.15 *Spectral loss profile of a single-mode fiber. Wavelength dependence* of fiber loss for several fundamental loss mechanisms is also shown. (After Ref. [11]. ©1979 IEE. Reprinted with permission.)

WDM Spectrum for 43 Channels 100 MHz Spacing Centered at 193.1 THz

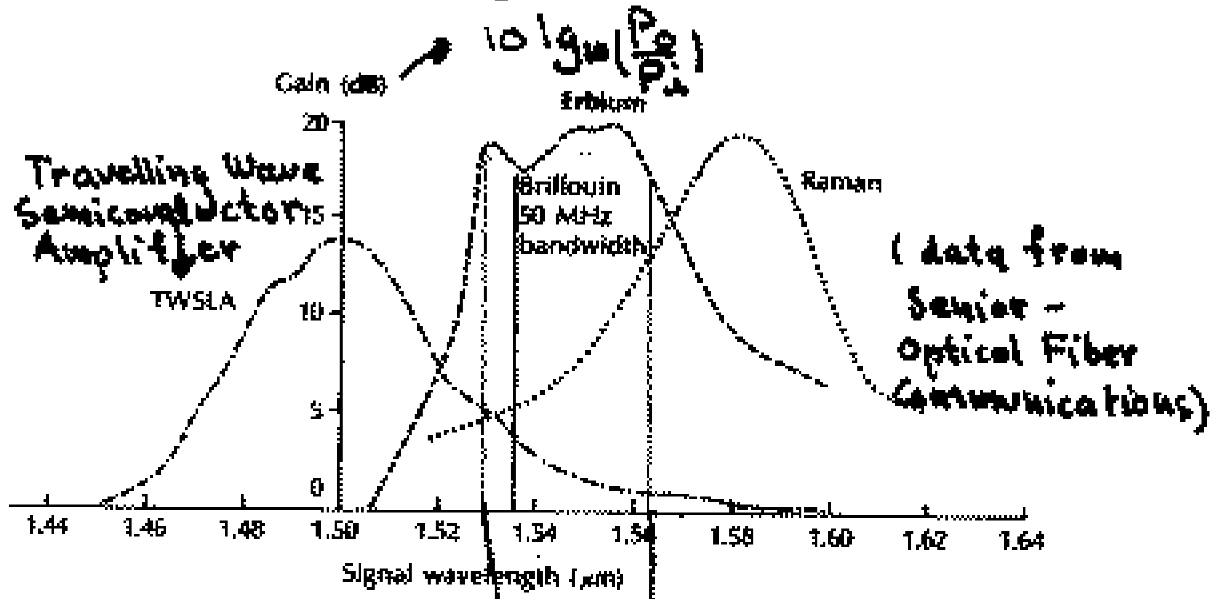


Figure 10.1 Optical amplifier gain characteristics. Reproduced with permission from P. Cochrane, *Br. Telecom. Technol. J.*, 8 (2), p. 1, 1990.

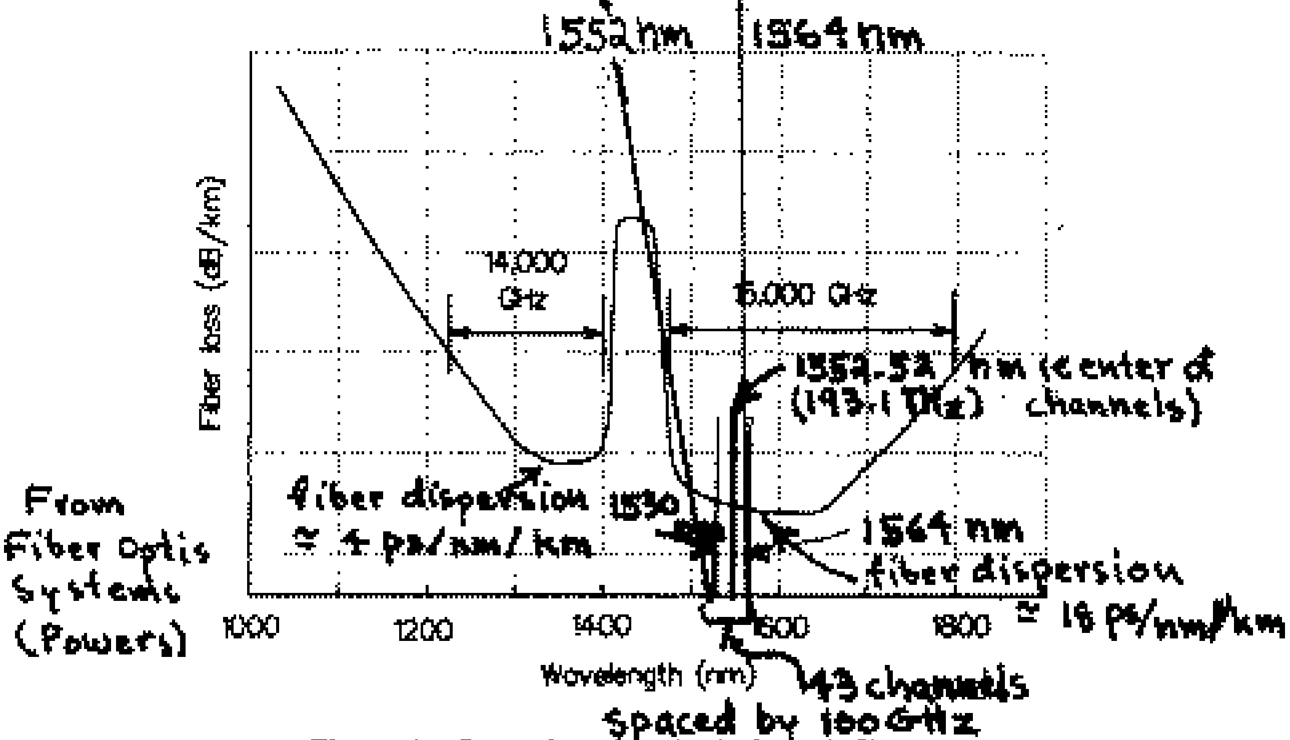
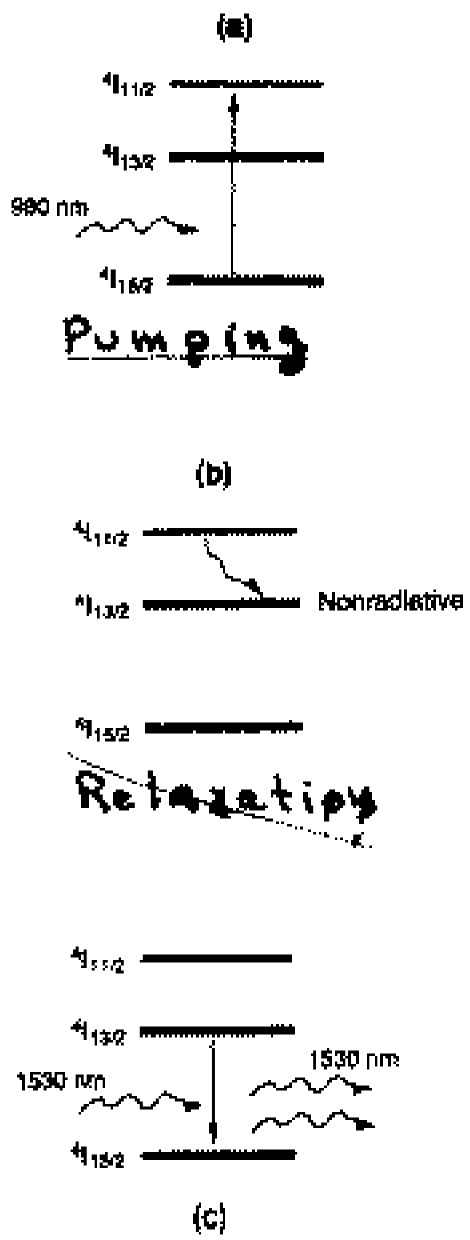


Figure 9.4 Spectral windows in single-mode fibers.

Energy Levels



Amplification

minimum-dispersion wavelength up to the 1500 nm window or by decreasing linewidth.

For long-distance high-data-rate links, then, the usable windows are from 1250 to 1[...] from 1450 to 1600 nm, allowing a combined optical window of 250 nm.

Example: Calculate the width of the 1300 nm window in nm.

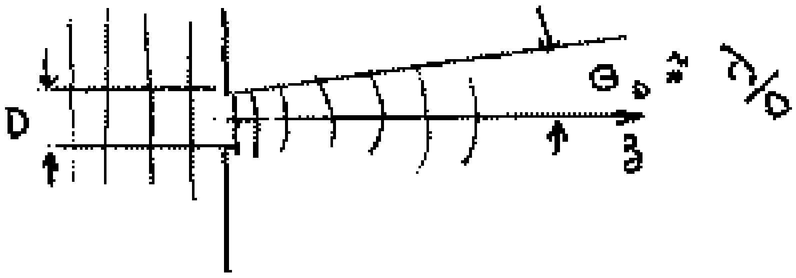
Solution: The frequency and wavelength are interrelated by $\nu = c/\lambda$; so,

$$\begin{aligned}
\frac{d\nu}{d\lambda} &= -\frac{c}{\lambda^2} \\
|\Delta\nu| &= \frac{c}{\lambda^2}\Delta\lambda \\
|\Delta\lambda| &= \frac{\lambda^2}{c}|\Delta\nu| = \frac{(1300\times10^{-9})^2(14,000\times10^9)}{3.0\times10^8} \\
&= 7.89\times10^{-8}\ \text{m} = 78.9\ \text{nm}.
\end{aligned}$$

EE118 - Basic Comments Regarding Waves
Diffraction, Focal Spot Diameter and Depth of Focus of a Lens

Basic Lens Formulae

b) Diffraction



Energy $= \hbar\omega$

Momentum $= \hbar k_x$

$$e^{i(\omega t - k_x x) - i k_z z}$$

Form of the wave

Uncertainty $(\Delta x)\Delta(k_x \hbar) \approx \hbar/2$

Qualitative But Expresses the Physical Aspects of Diffraction

$$\Delta x \, \Delta\left(\frac{k_x}{k_z}\right) = \frac{1}{2k_z}$$

$$k_z \approx \frac{\omega}{c} = 2\pi \frac{f}{c} = \frac{2\pi}{\lambda}$$

(Δk_x small so $k_z \approx \omega/c$)

$$\frac{2\Delta k_x}{k_z} \approx \theta_0$$

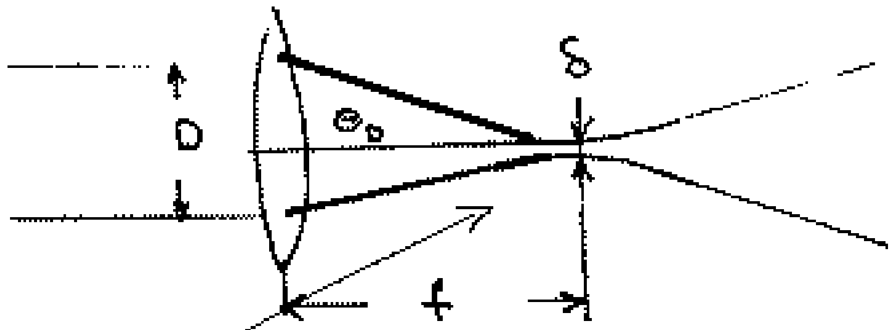
$$\therefore \boxed{\theta_0 \approx \frac{\lambda}{2\pi\Delta x} = \frac{\lambda}{\pi w_0}}$$

where $2\Delta x = w_0 = \frac{D}{2}$

Rigorous → For Gaussian beam $\theta_0 \approx \frac{2}{\pi}\frac{\lambda}{D}$

Diameter to $\frac{1}{e}$ of electric field. ($= 2w_0$ radius)

c) Focussing: a) Size of the focal region:



$$\frac{\lambda}{\delta} \times f \approx \frac{D}{2}$$

$$\therefore \delta \simeq 2\lambda\left(\frac{f}{D}\right)$$

F-number

Example: Take $\frac{f}{D} = 1.4 = F$ $\quad 1\,\mu m = \lambda$

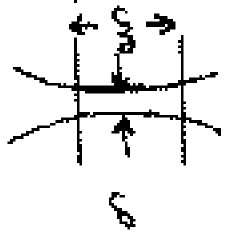
$=$ "F number"

$\delta \approx 2\times10^{-4} \times 1.4$

$\approx$ 3 µm

Thus Note
$\delta \cong 2F\lambda$
$\delta \cong \lambda$

b) length of the focal region



$$\left(\frac{\lambda}{\delta}\right) \times \frac{\xi}{2} = \delta \quad \text{Definition}$$

$$\therefore \xi \cong 2\delta^2/\lambda$$

≈ 18 µm for the above

Gaussian beam of radius w_0

z_0 = conf. param

$$\boxed{2z_0 = \left(\frac{\pi}{2}\right)\frac{(2w_0)^2 n}{\lambda}}$$