Channel Capacity and Channel Capacity Limitations (Noise)

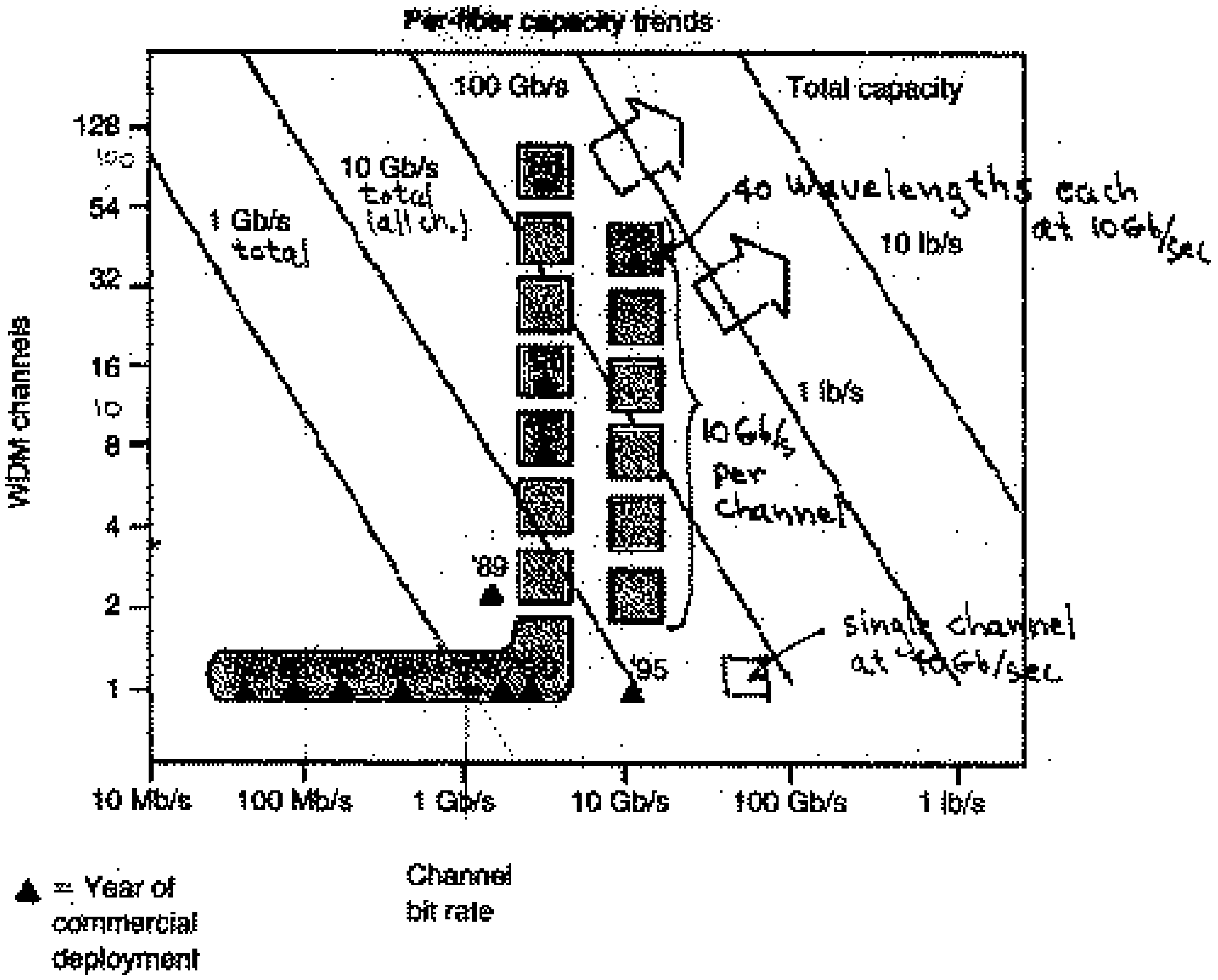


Figure I.1 Per-fiber capacity trends. (From Lucent Technologies, *Bell Labs Technology*, vol. 2 no. 2, Fall 1998, p. 3.)

Basic Channel Capacity Limitations

Binary transmission m bits

Number of messages $M = 2^m$

Transmission at r bits/sec $m = r t_s$

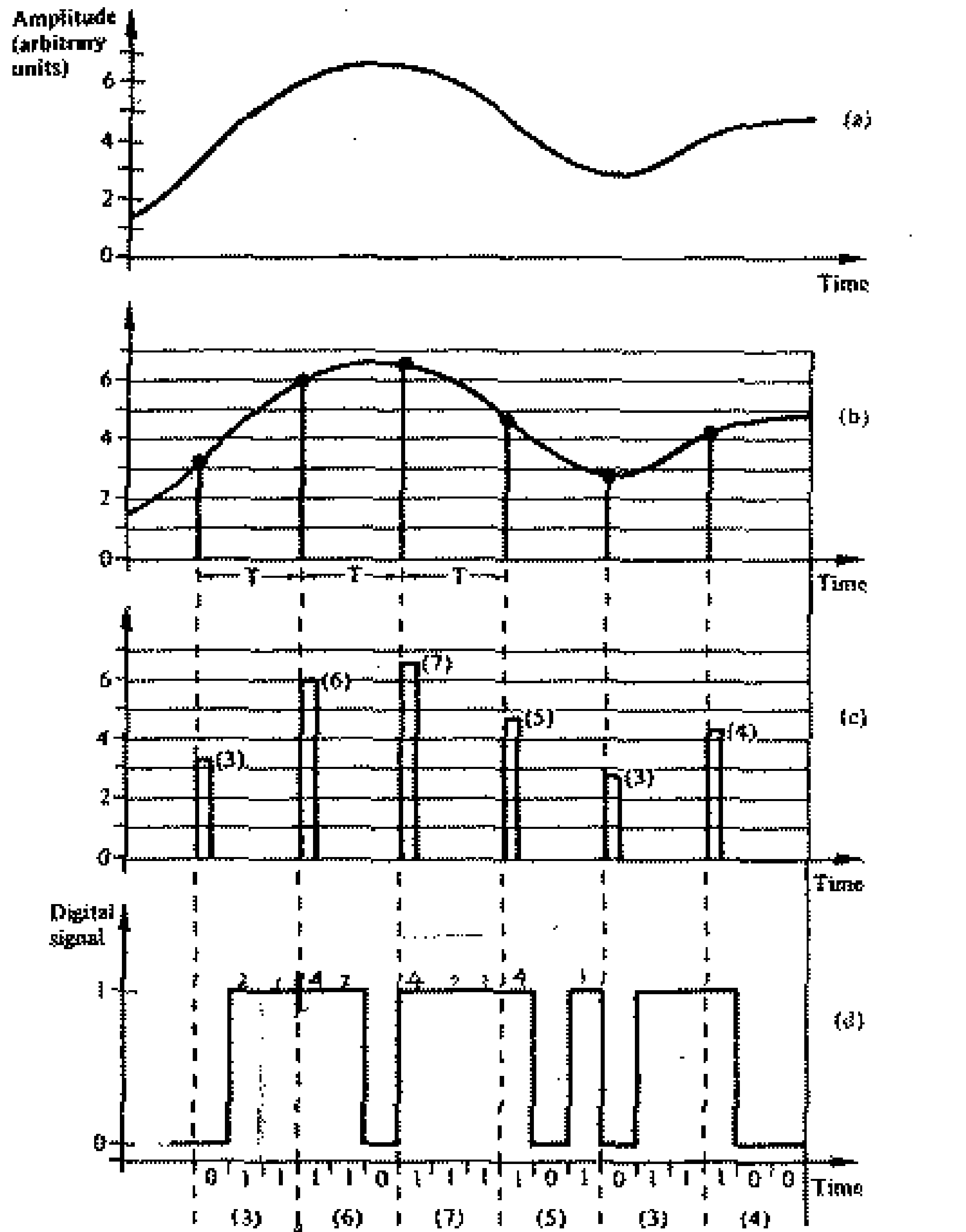
Thus $M = 2^{rt}$ or $\lg_2 M = rt$

Thus $r = \frac{1}{t_s} \ln_2 M$

t_s = <u>sampling time</u> $\left(\frac{1}{t_s} = 2\Delta f\right)$ Δf = bandwidth

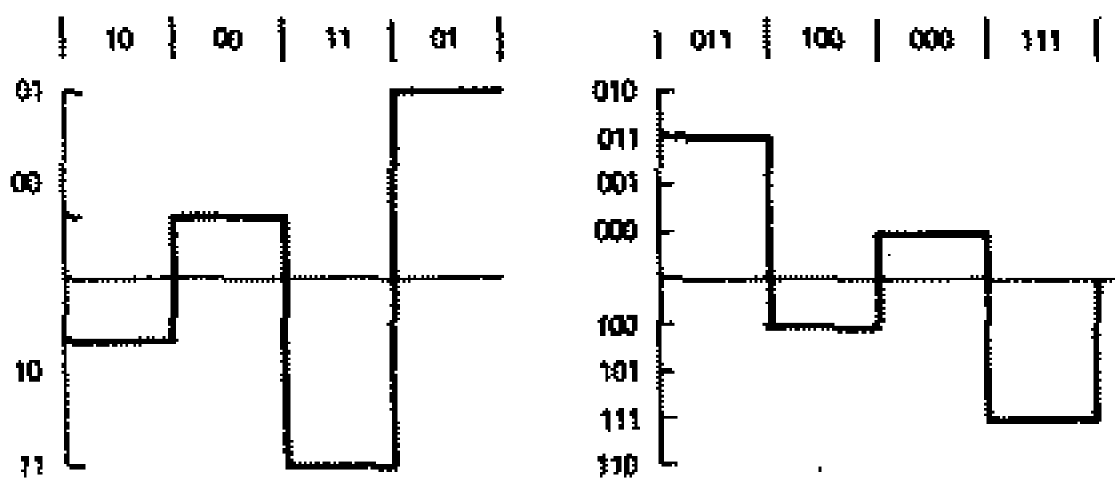
Thus $\boxed{r = 2\Delta f \ln_2 M}$ → This is the bit rate <u>required</u> to send messages at the sampling rate if there are M possible messages. (1 message per sampling time)

Analog to Binary Digital Conversion



Steps in the conversion of an analogue waveform into a binary digital signal: (a) part of an analogue waveform; (b) analogue waveform sampled at time intervals of T; (c) the sampled waveform showing allocation of amplitude to one of $2^3 = 8$ levels; (d) sampled information converted to binary digital form with 3 bits/sample. (NRZ Code)

Analog to Multilevel Digital

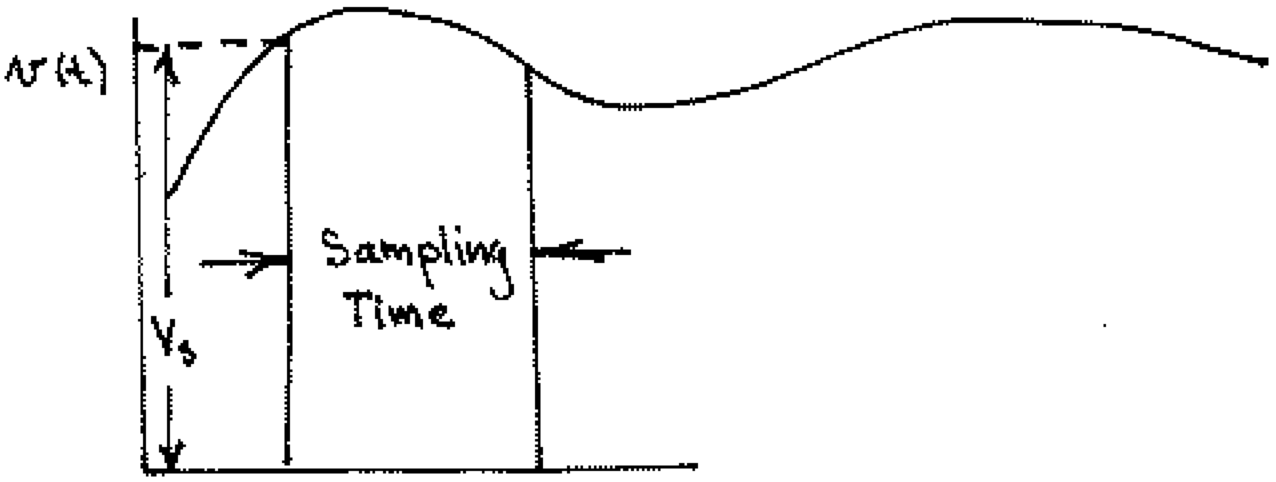


(a) Two Bits Per Signal

(b) Three Bits Per Signal

FIGURE 3-9. Multilevel AM Schemes

M is Limited By the Signal / Noise Ratio



Maximum Number of Messages Determined by Noise
V_N (actually $\sqrt{\overline{V_N^2}}$)

$$M \le \frac{|V_S|}{V_N} \le \frac{|V_{MAX}|}{V_N} = \frac{((V_M)^2 + V_N^2)^{1/2}}{V_N} = \left(1 + \left(\frac{V_M}{V_N}\right)^2\right)^{1/2}$$

Thus $r = 2\Delta f \ln_2\left(1 + \frac{V_M^2}{V_N^2}\right)^{1/2} = \Delta f \ln_2\left(1 + \frac{V_M^2}{V_N^2}\right)$

$$\simeq 0.332 \; \underset{\text{Hz}}{\Delta f} \; \underset{\text{dB}}{X}$$

$$X = 20 \log_{10} (V_S / V_N)$$

Example:

625 line color TV video signal $\Delta f \simeq 5.5$ MHz

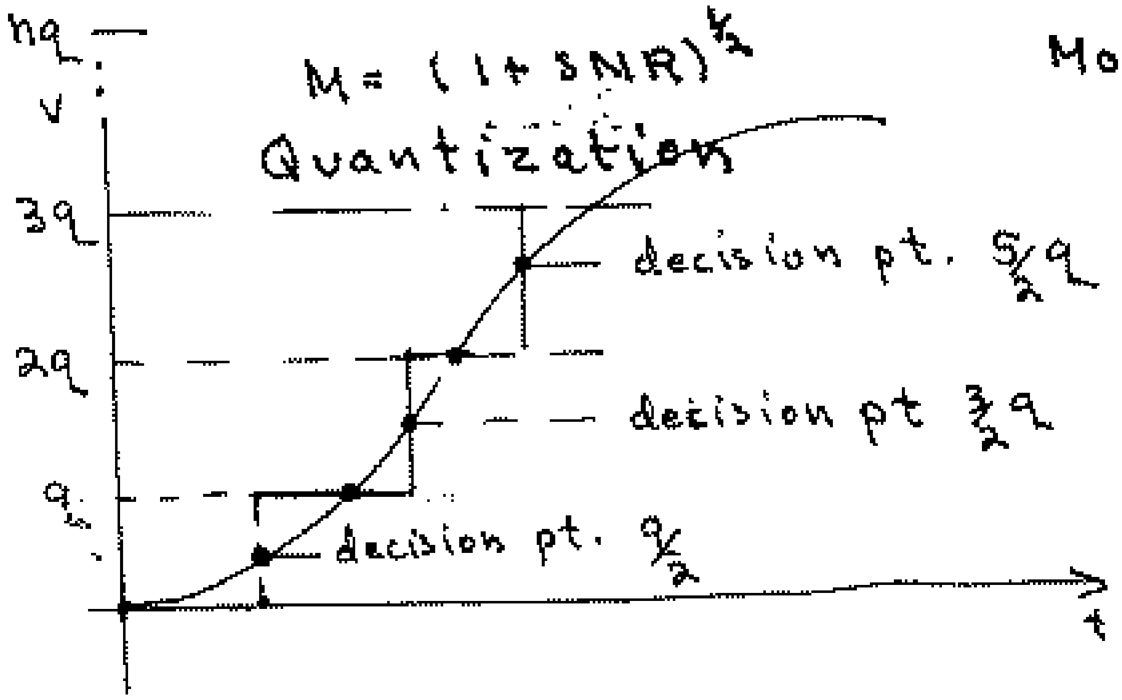
$X \simeq 50$ dB $\rightarrow M = (1 + S/N)^{1/2} = (1 + 10^5)^{1/2} \simeq 300$

Thus there are 300 possible quantization levels to be able to transmit which level at a time t within the maximum sampling time the bit rate must be $\Delta f \ln_2 (1 + S/N)$

$r \simeq 0.332 \times 50 \times 5.5 \times 10^6 = 91$ Mb/sec $\rightarrow$

Total No of Bits in 100 Min $= 91 \times 10^6 \times 6000$
$= 550$ Gbits

Generally a Sampling frequency $\simeq 17.7$ MHz is ideal
Δf then 8.8 MHz and $r \simeq 145$ Mb/sec
with 8 bits / sample! (145/17.7 =



More Rigorous Proof That $M = (1+SNR)^{1/2}$ where $SNR = \frac{S}{N}$

$p(V) = \frac{1}{K}$ $\qquad$ K = No of levels

Signal a.c.

$$(V - V|_{average})^2|_{average} = V^2|_{average} - (V|_{Average})^2$$

$$V^2|_{average} = \sum_0^K \underbrace{\frac{1}{K}}_{p(V)} (nq)^2 = \frac{q^2}{K} \cdot \frac{K(K+1)(2K+1)}{6}$$

$$V|_{average} = \sum_0^K \frac{1}{K}(nq) = \frac{q}{K}\,\frac{K(K+1)}{2}$$

Thus

$$V^2|_{Average} - (V|_{average})^2 = q^2(K+1)\left(\left(\tfrac{1}{3}-\tfrac{1}{4}\right)K + \tfrac{1}{6} - \tfrac{1}{4}\right)$$
$$= q^2\frac{(K+1)}{12}(K-1)$$
$$= \frac{q^2}{12}(K^2-1)$$

Probability of Error $p(\varepsilon_q) = \frac{1}{q} \quad \left(\int_{-q/2}^{+q/2} p(\varepsilon_q) = 1\right)$

$$\overline{\varepsilon_q^2} = \int_{-q/2}^{+q/2} \varepsilon_q^2\, p(\varepsilon_q)\, d\varepsilon_q = \frac{q^2}{12}$$

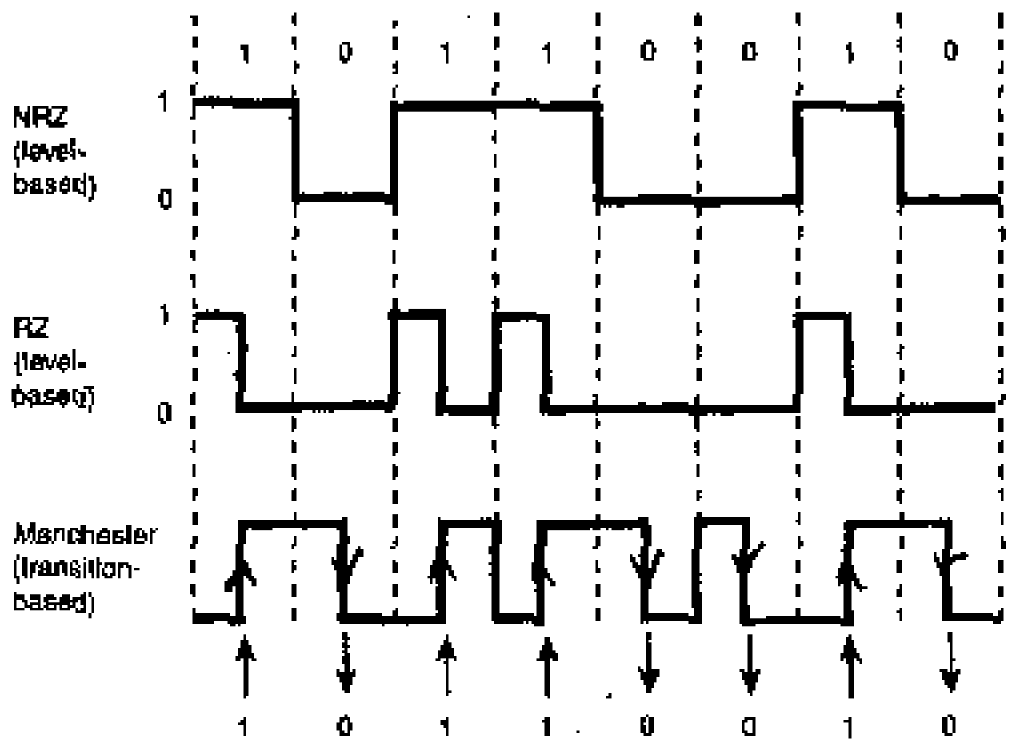
$$\therefore \quad SNR = \frac{V^2|_{Average} - (V|_{Average})^2}{\overline{\varepsilon_q^2}} = (K^2-1)$$

$$\therefore \quad K = M = (1 + SNR)^{1/2}$$

5.8 *Line Coding* (Various Coding Schemes)

A correctly designed transmission symbol set and transmission encoding procedure can compensate for non-ideal channel response, aid in clock recovery, and ensure robust error detection. Figure 5.25 shows three common alternative binary codes, non-return-to-zero (NRZ), return-to-zero (RZ), and Manchester, that can be used for baseband encoding prior to transmission. NRZ and RZ are level-based (the bit value is represented by the signal level), and Manchester is transition-based (the bit value is represented by the transition direction). The three look similar but present different frequency characteristics after they have been through the transmitter's modulator and pulse-shaping filter.

Figure 5.25 *Alternative Binary Line Codes*



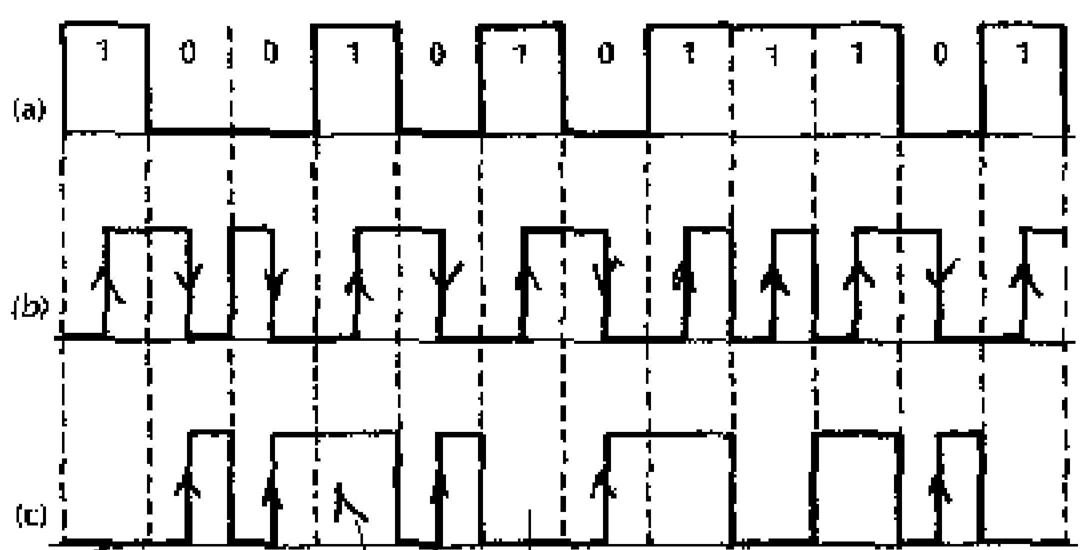


Figure 11.39 Examples of binary 1B2B codes used in optical fiber communications: (a) unencoded NRZ data; (b) biphase or Manchester encoding; (c) coded mark inversion (CMI) encoding.

alternating 00 and 11 for a
01 for a

Baud Rate at 1 Bit Rate

$$r \approx 2\Delta f \; lg_2 M$$

No of levels (Limited By SNR)

band-width

For a channel look at transmission through an ideal filter. (following page).

Thus the <u>baud</u> rate can be $2\Delta f$. (assuming dynamic range is sufficient for signal)

If the Number of levels $M = 2^{\ell}$; then

$$\boxed{r \approx \ell(2\Delta f)} \approx \text{Bit Rate} \approx \ell \,(\text{Baud Rate})$$

Limitation : Noise (example - assume $\Delta f \to \infty$)

$$R_\infty \approx \lim_{\Delta f \to \infty} \Delta f \; lg_2\left(1 + \frac{S}{\underbrace{N_0 \Delta f}}\right)$$

"white" Noise Proportional To Δf

$$\approx \lim_{\Delta f \to \infty} \Delta f \; lg_e\left(1 + \frac{S}{N_0 \Delta f}\right) lg_2(e)$$

$$\simeq \frac{S}{N_0} lg_2(e) = 1.44\left(\frac{S}{N_0}\right)$$

Example a) Thermal Noise $N_0 = kT$

$$\therefore S = R_\infty \frac{N_0}{1.44} \approx R_\infty \frac{kT}{1.44} \approx \frac{(.026) \times 1.6 \times 10^{-19}}{1.44} R_\infty$$

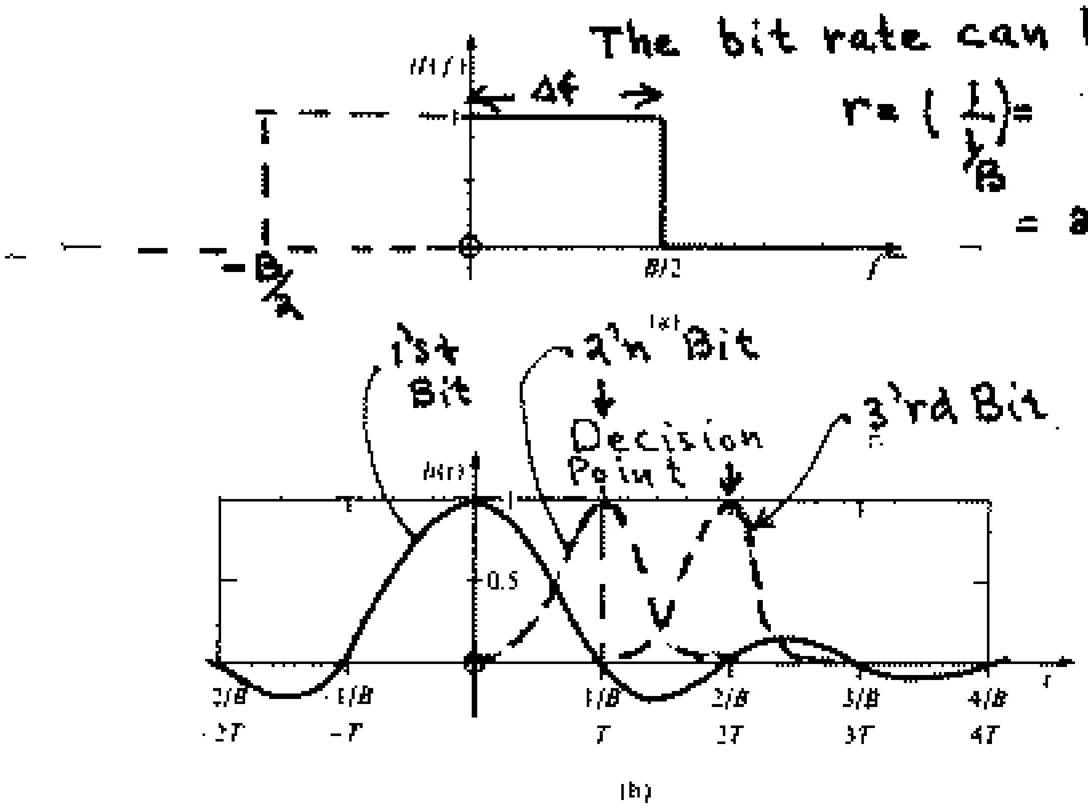
If one wants 50 GB/sec This is $\simeq$ <u>.10 nwatt</u>

b) Quantum Noise $N_0 = hf$

$$S \approx R_\infty \frac{hf}{1.44} \simeq R_\infty \frac{1.6 \times 10^{-19}}{1.44} J$$

(For 50 GB/sec) $\simeq 5 \times 10^{-9}$ watts $\simeq$ <u>5 nwatt</u>

The Ideal Channel and The Shannon Limitation



The bit rate can be

$$r = \left(\frac{1}{1/B}\right) = B = 2\Delta f$$

Fig. 15.4 Ideal low-pass filter:
(a) frequency response $H(f) = 1$ for $0 < f < B/2$, $= 0$ for $B/2 < f$

(b) impulse response $h(t) = \dfrac{\sin \pi t B}{\pi t B}$

$(2\Delta f) \lg_2 M$

which is zero at all times $\pm nT$, where $T = 1/B$, as shown in Fig. 15.4.

The Limiting Channel Capacity is (ideal)

$$\begin{aligned} r = \ell\,(2\Delta f) &= \lg_2(M)\,(2\Delta f) \\ &= \lg_2\left(1 + S/N\right)^{1/2}(2\Delta f) \\ &= \Delta f \lg_2\left(1 + S/N\right) \end{aligned}$$

Since the number of Levels which can be discerned for a signal transmitted through the channel is $(1 + S/N)^{1/2}$

(Note 1: The advantages of eliminating extraneous bits [Compression] becomes apparent)

(Note 2: The higher the S/N ratio, the more levels which can be used in the coding and hence the higher the bit rate, but the baud rate is $2\Delta f$