Signal Shot Noise Formula

Photo-electron stream is poisson

$$p(n) = (\bar{n})^n \frac{e^{-\bar{n}}}{n!}$$

$\bar{n}$ - average number of photo-electrons counted over a time T (Current $\bar{I} = \frac{e\bar{n}}{T}$)

$$\overline{\Delta^2 n} = \overline{(n-\bar{n})^2} = \bar{n} \quad \text{for a Poisson Distribution}$$

But: $$\overline{\Delta^2 i} = \frac{\overline{\Delta^2 n}}{T^2} \times e^2 = \frac{\bar{n}}{T^2} e^2$$

$$= \Big(\underbrace{\frac{\bar{n}}{T} e}_{\bar{I}}\Big) e \left(\frac{1}{T}\right)$$

If the bandwidth is at the sampling limit $\frac{1}{T} = 2\Delta\nu$ where $\Delta\nu$ is the bandwidth. (Many references use $\Delta\nu$ rather than Δf)

$e = 1.62\times10^{-19}$ C

Thus: $$\boxed{\overline{\Delta^2 i} = 2e\bar{I}\Delta\nu}$$ $\left(\bar{I} = \eta \frac{P_{opt}}{hf} e\right)$

η = quantum efficiency

Example 1 μwatt at 1 μm illuminating a detector with unit quantum efficiency

$\Delta\nu$ = 1 GHz

$$\overline{\Delta^2 i} = 2\times(1.6\times10^{-19}) \times \frac{10^{-6}}{6\times10^{-34}\times3\times10^{14}} \times 1.6\times10^{-19} \times 10^{9}$$

$$\cong 2.84\ 10^{-14}\ (\text{amps})^2$$

or $\sqrt{\overline{\Delta^2 i}} \cong 1.68\times10^{-7}$ amps ← root mean square of the current fluctuations

Shot Noise Limited S/N Ratio

$$K^2 = S/N = \frac{I^2}{\overline{\Delta i^2}}$$ where I = signal induced detector current
$= \eta \frac{P_{opt}}{h\nu} e$
ν = frequency of the optical signal

$$K^2 = S/N = \frac{\left(\frac{P_{opt}}{h\nu} e \eta\right)^2}{2 e^2 \frac{P_{opt}}{h\nu} \eta \Delta\nu} = K^2$$

$$\therefore \frac{P_{opt}}{h\nu} e \eta = e 2 \Delta\nu K^2$$ = shot noise limited photo-current

If $K = 1$ this is two photons / bandwidth for or one photon per sampling time

Called the quantum limit. $P_{opt} = \frac{2h\nu\Delta\nu}{\eta}$ is then called the noise equivalent power ← N.E.P

Points

a) For a receiver - ideally would like to be in the shot noise limit. Thermal noise and amplifier noise degrades the system.

b) Do this by amplifying the signal
   1) optically with Erb. amplifiers
   2) electronically with avalanche diodes

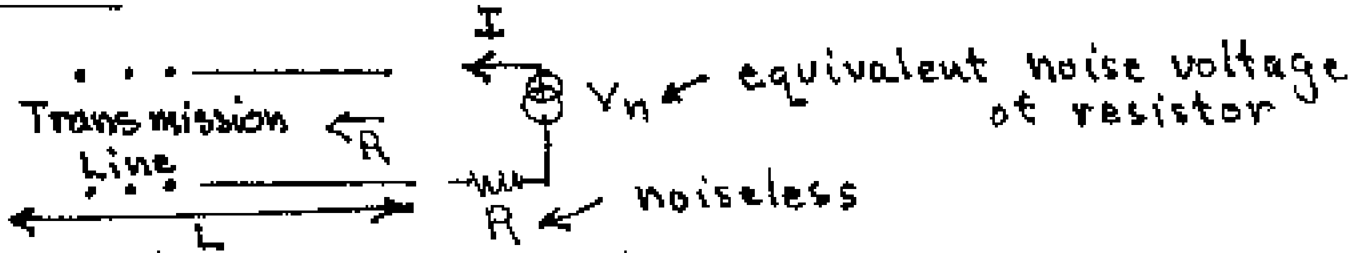
c) One uses $\overline{\Delta i^2} = 2eI\Delta\nu$ to define $I = I_D$, the dark current corresponding to the minimum noise fluctuations. Similarly for $I = I_b$ for noise fluctuations due to background light.

Thermal Noise of a Resistor

$$v_{th}^2 = R\,4kT\Delta f$$

$$i_{th}^2 = \frac{4kT\Delta f}{R}$$

<u>Proof</u>



<u>Matched</u> Transmission Line

Maximum Power Transfer

$$I = \frac{V_n}{2R} \qquad \left(\text{Note } V_n \text{ is actually } \sqrt{\overline{V_n^2} - (\overline{V_n})^2}\right) \text{ (statistical)}$$

Power Delivered To Trans Line by R $\rightarrow I^2R = \left(\frac{V_n}{2R}\right)^2 R = \frac{V_n^2}{4R}$

= power on the transmission line delivered to the resistor

$= kT\Delta f$

Thus $V_n^2 = 4kTR\Delta f$

Argument for Thermal Power $= kT\Delta f$

By Equipartition of Energy k_BT = (energy per mode) on Trans Line in Thermal Equilibrium

Thus Power $= \frac{k_BT}{L} \cdot \left(\frac{J}{m}\right) * c = \frac{J}{sec}$

Mode Condition For a Transmission Line of Length L

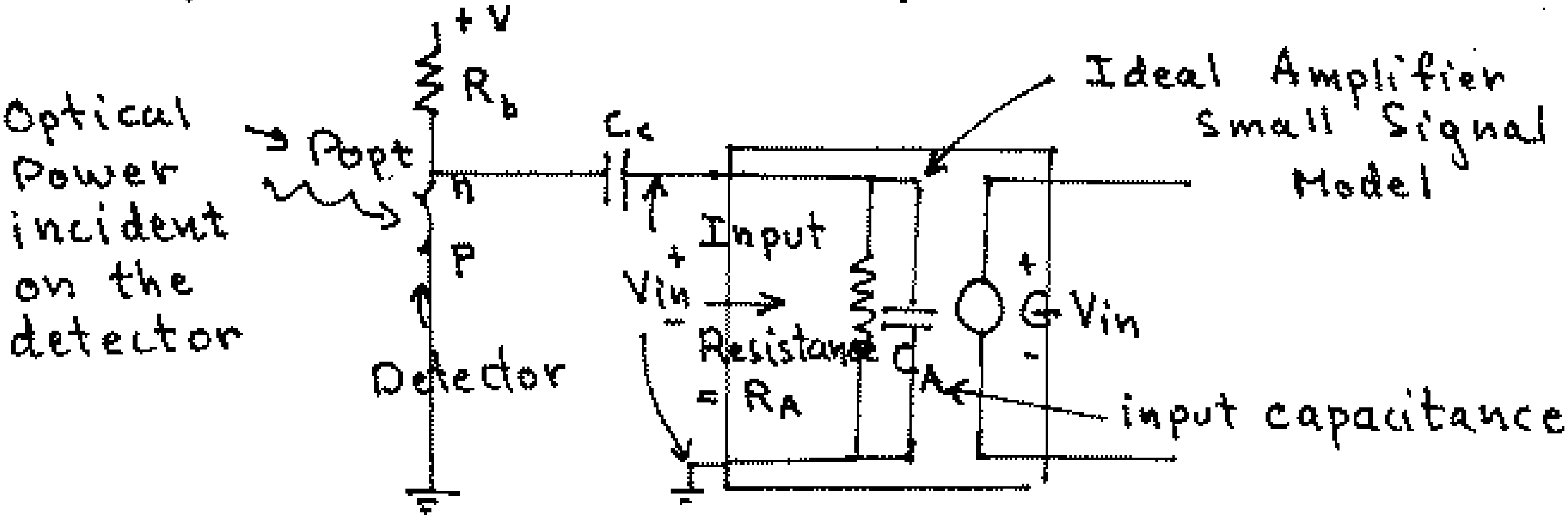
$$KL = m(2\pi)$$

$$\frac{2\pi f}{c} L = m(2\pi) \quad \rightarrow \text{or} \quad L = m\lambda \leftarrow \text{wavelength}$$

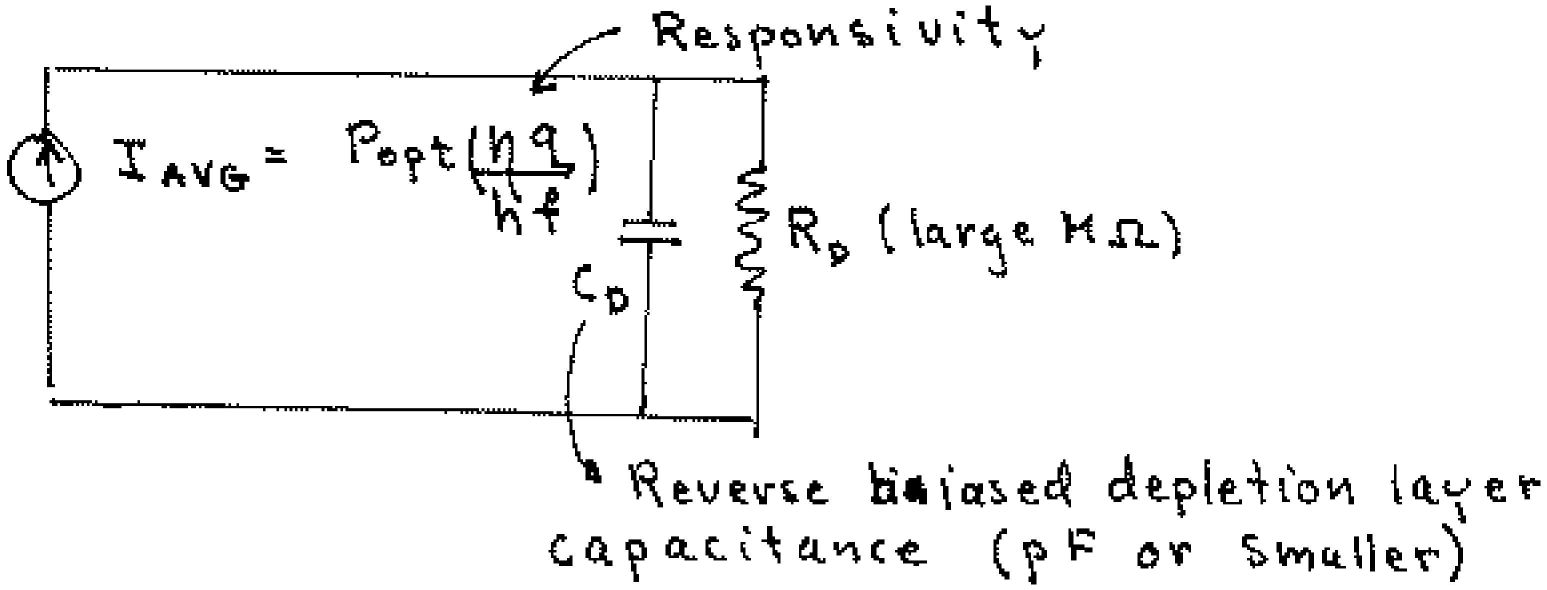
Thus $\frac{\Delta f}{c} L = m$

$\therefore$ Power $= (k_BT)\Delta f \quad$ (L/c cancels) $= \frac{V_n^2}{4R}$

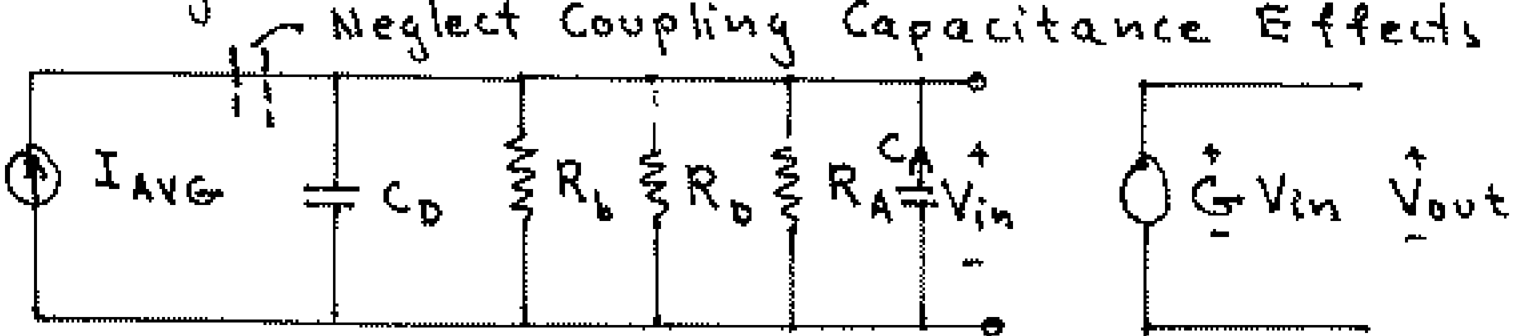
Simple Example of an S/N Ratio Calculation



Diode Circuit Model (Small Signal)



Small Signal Circuit Model is Thus



(Note: For this Simple Example, The Capacitances can be added and the resistances combined

$$C = C_D + C_A \quad , \quad R = R_b \parallel R_D \parallel R_A$$

For Low Frequencies (Not so Low that C_c is important)

$$\text{Signal } V_{out} = G \cdot (I_{AVG} \times R)$$

$$= G \cdot \left(P_{opt}\left(\frac{\eta q}{hf}\right)\right) R$$

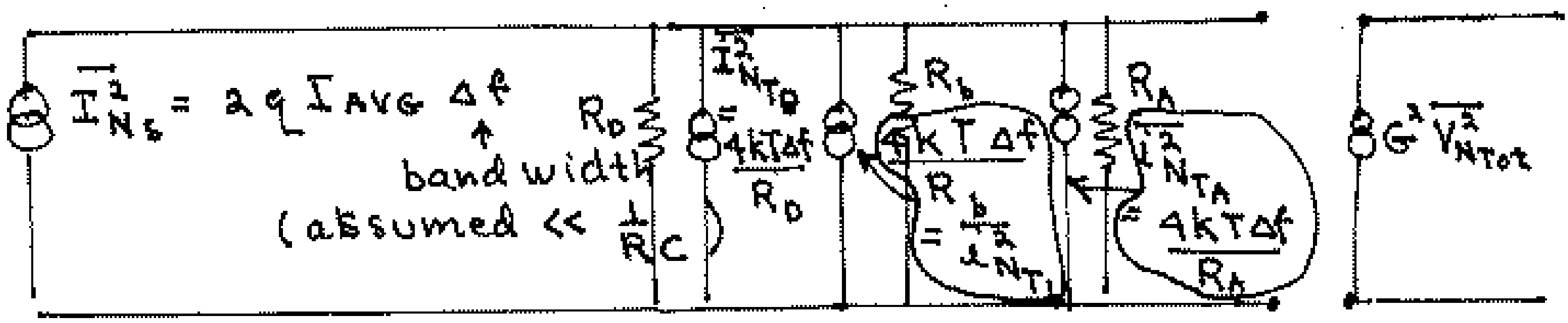
Noise Calculation

Noise Sources (Assuming The Amplifier is Ideal For The Moment)

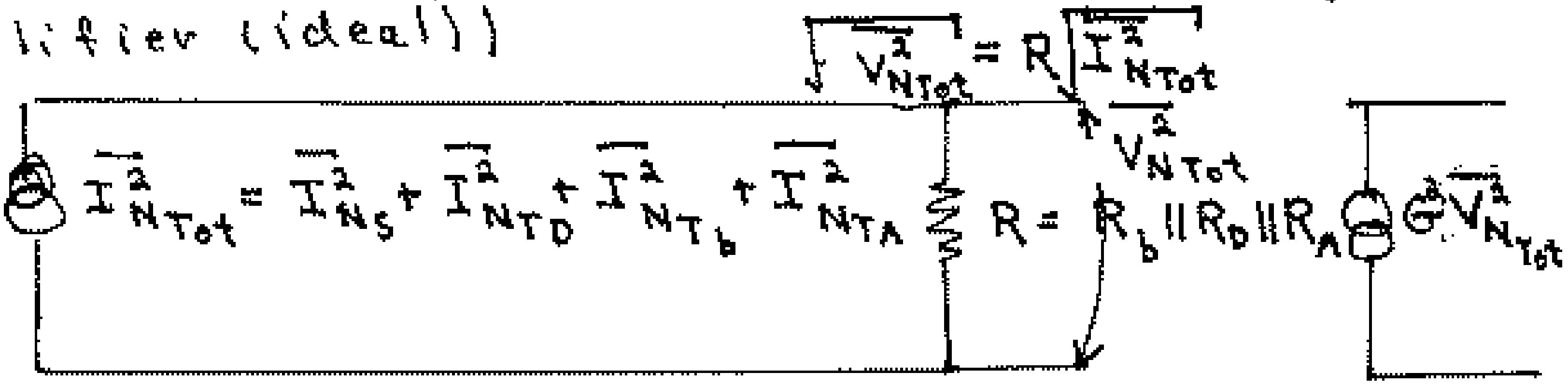
a) Shot Noise Due to $I_{AVG} = \eta \frac{P_{opt}}{hf} q$

b) Thermal Noise Due to the Resistors

Noise Equivalent Circuit



This can be simply written as (including the amplifier (ideal))



(For those who have had EECS 120, this model represents a Power Spectral Density Analysis

$$\left[\ \underline{\text{Power}} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} i^2(t)\, R\, dt = \lim_{T\to\infty} \frac{R}{2\pi T}\int_{-\infty}^{+\infty} |I(\omega)|^2 d\omega \right.$$

with $I(\omega) = F.T.(i(t))$ and $d\omega = 2\pi df$])

Then
$$\frac{S}{N} = \frac{|V_{out}|^2}{G^2 \overline{V^2_{NTot}}} = \frac{P_{opt}^2 \left(\frac{\eta q}{hf}\right) \cancel{R^2} \cancel{G^2}}{\overline{I^2_{NTot}} \cdot \cancel{R^2} \cancel{G^2}}$$

$$K^2 = \frac{S}{N} = \frac{P_{opt}^2 \left(\frac{\eta q}{hf}\right)}{2q P_{opt} \frac{\eta q}{hf} \Delta f + 4kT\Delta f \left(\frac{1}{R_A} + \frac{1}{R_D} + \frac{1}{R_b}\right)}$$

Amplifier Noise

Noise Figure

In a receiver, the photodetector is followed by a front-end amplifier. Components within the front-end amplifier, such as the transistor, also contribute to the thermal noise. This noise contribution ~~is usually~~ can be stated by giving the *noise figure* of the front-end amplifier. The *noise figure* of a front-end amplifier specifies the factor by which the thermal noise present at the input of the amplifier is enhanced at its output. We will denote this quantity by F_n. Thus the thermal noise contribution of the receiver has variance

$$\sigma^2_{\text{thermal}} = \frac{4k_B T}{R_L} F_n B_e \tag{4.2}$$

input resistance of the amplifier

electrical bandwidth (Δf)

when the front-end amplifier noise contribution is included. Typical values of F_n are 3–5 dB. (Recall $10 \lg \dfrac{\sigma^2_{\text{thermal}}}{\sigma^2_{\text{thermal for } F_n = 1,\ T = 290^\circ}} \triangleq F_n$ in dB)

[Thus assuming $T = 290^\circ$ for operating temperature a F_n of $3\text{ dB} = 10 \lg(F_n)$; Therefore $F_n = 2$]

[Note: A noiseless Amplifier has $F_n = 1$ (the input resistor has noise)]

There are at least two other ways of specifying amplifier noise

Noise Temperature of the Amplifier

$$\sigma^2_{\text{thermal}} = \frac{4k_B T}{R_L} B_e + \frac{4k_B}{R_L} T_A B_e$$

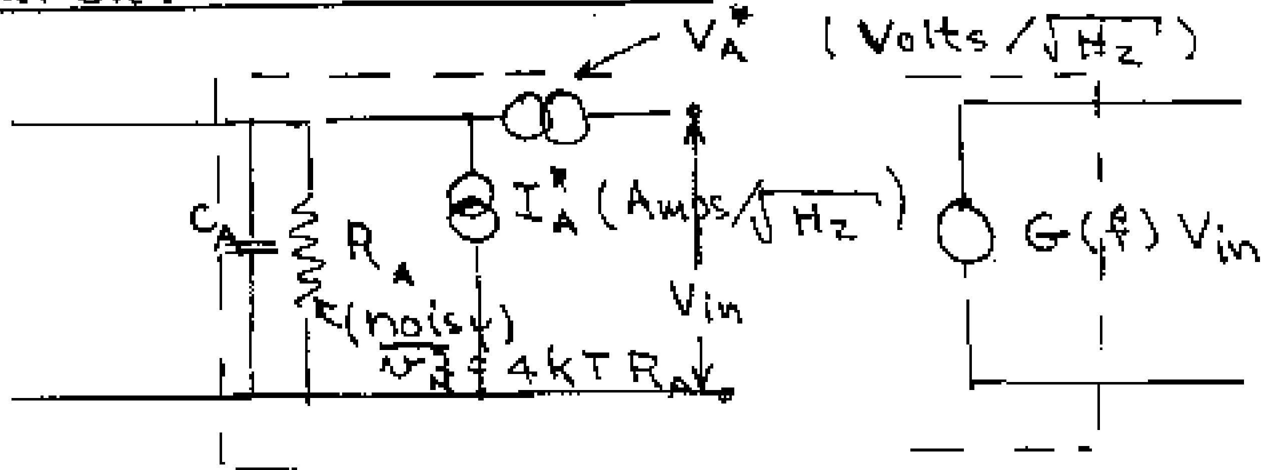
By definition, the effective noise temperature of the amplifier

Equating this to the above expression one observes that

$$F_n = 1 + \left(\frac{T_A}{T}\right)$$

(Thus $F_n = 2$ [3dB] is equivalent to an amplifier with $T_A = T = 290^\circ$)

Equivalent Noise Sources



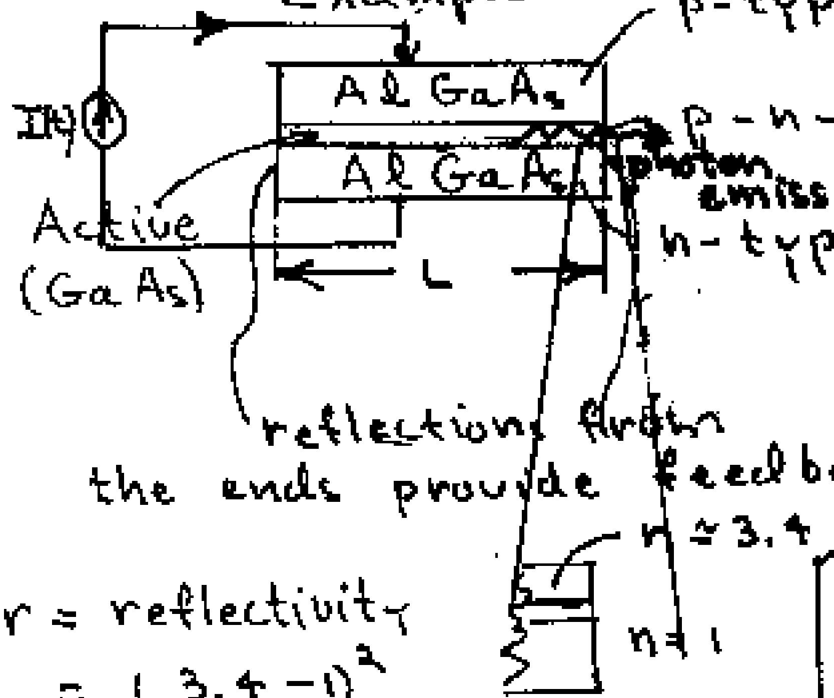
GaAs-MESFET

$I_A^* \simeq .1\,pA/\sqrt{Hz}$

$V_A^* \simeq 1\,nV/\sqrt{Hz}$

Ideal Power-Out Versus Current Characteristic for a Laser Oscillator
(Reflectivity and Internal Loss, External and Internal Quantum Efficiency, and Radiative Recombination)

Basic Laser Principle — p-n Junction (Forward Biased)

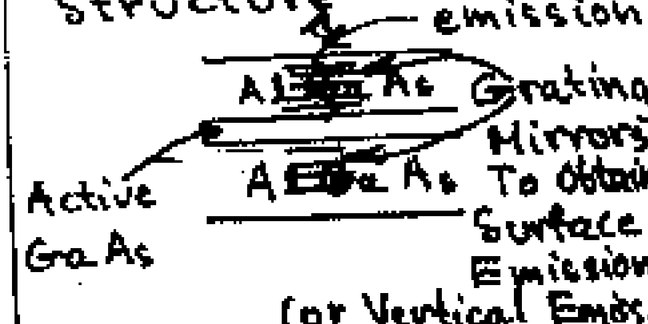


r = reflectivity

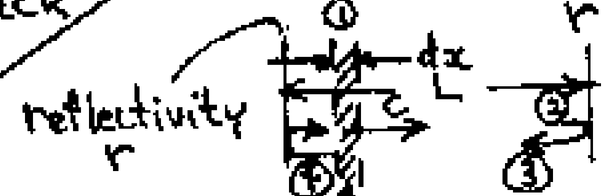
$$= \frac{(3.4-1)^2}{(3.4+1)^2}$$

↳ n

Alternative "Surface Structure"



Equivalent loss/length, α_m — reflectivity r



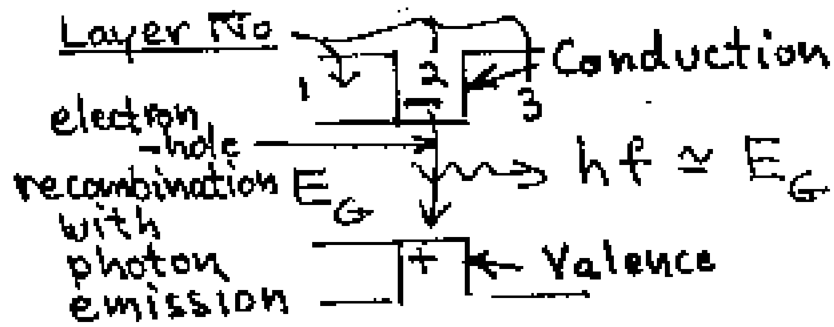
Track dx during one round trip around the cavity as it travels at the speed of light (① to ② to ③ to ④ and back to ①). The energy left in dx is then (initial energy) $\times r \times r$. Write this as (initial energy) $\times e^{-\alpha_m 2L}$

Thus $e^{-\alpha_m 2L} = r^2$ or

$$\boxed{\alpha_m = -\frac{1}{L}\ln(r) = \frac{1}{2L}\ln\left(\frac{1}{r^2}\right)}$$

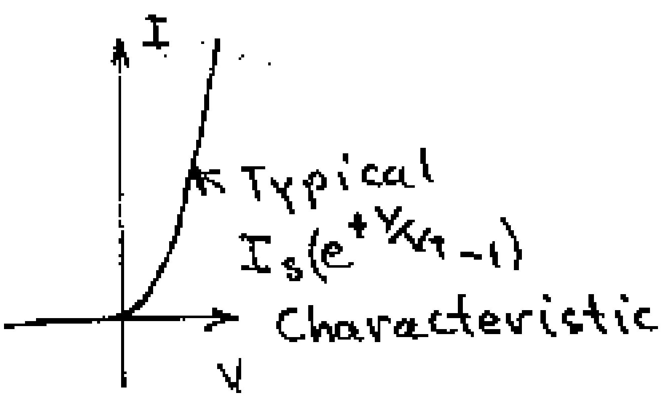
units are $(\text{length})^{-1}$

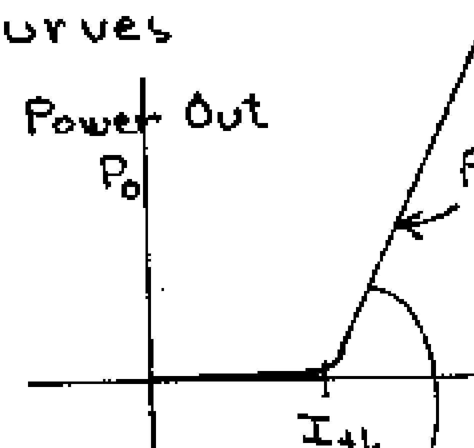
Recombination in Layer 2



Note: For 1.3 – 1.5 μm Need InGaAsP

Current-Voltage And Optical Power Out Versus Current Curves





Internal quantum efficiency

$$P_o = \eta \frac{\alpha_m}{\alpha_m+\alpha}\frac{(I-I_{th})hf}{e}$$

α = internal loss coefficient (cm^{-1})

Slope $\triangleq$ external quantum eff.

$$= \eta \frac{\alpha_m}{\alpha_m+\alpha}\frac{hf}{e}$$