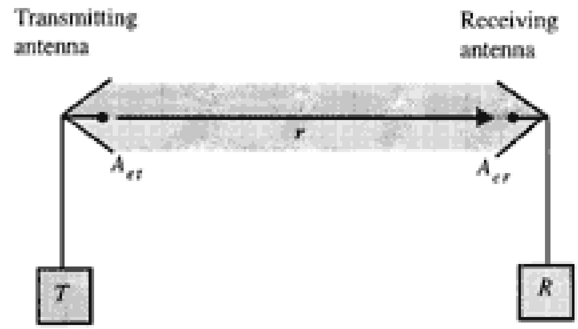
EE118 - Free-Space Communication Link
The Friis Formula

Basic Equations for Free-Space Comm. Links

The Friis formula for a communication link is given by $P_r/P_t = \dfrac{A_{er}A_{et}}{r^2\lambda^2}$ where



P_r is the received power

P_t is the transmitted power

A_{er} is the effective area of the receiving antenna

A_{et} is the effective area of the transmitting antenna

r is the distance between the antennas

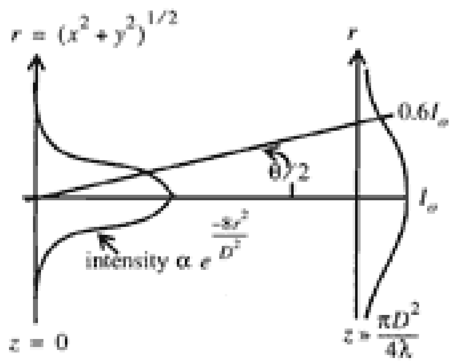
λ is the wavelength.

a) Establish the validity of this expression for P_r/P_t based upon a simple diffraction argument (i.e, uncertainty relation).

For Gaussian $\frac{D}{2}$ is the $\frac{1}{e^2}$ intensity radius at the antenna ($z = 0$).

① Diffraction Angle $\approx \frac{\lambda}{D} \times \frac{2}{\pi}$; $= \theta$ θ is the far field cone angle to .6 times the peak intensity at $\theta = 0$.

② $\Omega_A = 2\pi(1 - \cos\theta) \approx \pi\theta^2 = \pi\left(\frac{\lambda}{D}\right)^2\left(\frac{2}{\pi}\right)^2$ = solid angle of antenna diffraction cone.



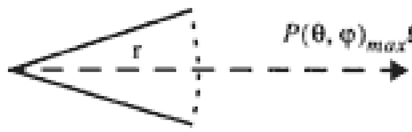
③ $P_t =$ Power radiated into cone (Watts); Intensity at r, $S = \left(\dfrac{P_t}{\pi\theta^2 r^2}\right)(Watts/m^2)$

④ $P_r =$ Received Power $= S \bullet A_{er} = \dfrac{P_t}{\pi\theta^2 r^2} A_{er} = \dfrac{P_t A_{er}}{\pi\left(\frac{\lambda}{D}\right)^2\left(\frac{2}{\pi}\right)^2}$

$$= \frac{P_t\ \pi D^2\ A_{er}}{\lambda^2 4r^2} = \frac{P_t\ A_{et}\ A_{er}}{\lambda^2 r^2}$$

Thus the $\dfrac{\text{signal}}{\text{thermal noise}}$ ratio $= (P_t\, A_{et}\, A_{er})/(\lambda^2 r^2 kT\Delta f)$

b) Argue similarly that if the directivity of an antenna pattern is defined as $D = \dfrac{P(\theta, \varphi)_{max}}{P(\theta, \varphi)_{avg}}$ then $D = \dfrac{4\pi}{\Omega_A}$ where Ω_A is the solid angle of the main beam.

(5) 

$$P(\theta,\varphi)_{max}\Omega_A r^2 = \text{total power} = P(\theta,\varphi)_{avg} \times 4\pi r^2$$

$$D \triangleq \frac{P(\theta,\varphi)_{max}}{P(\theta,\varphi)_{avg}} = \frac{4\pi}{\Omega_A} \triangleq \frac{\text{Gain}}{k} \quad k = 1 \text{ if lossless}$$

c) Using a diffraction argument show that in general $D = 4\pi \frac{A_e}{\lambda^2}$

(6) $$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\pi\theta^2} = \frac{4\pi^2}{4\pi(\lambda/D)^2} = \frac{4\pi^2(D/2)^2}{\lambda^2} = 4\pi\frac{A_e}{\lambda^2}$$

$$A_e = \pi\left(\frac{D}{2}\right)^2$$

(Note: For radar, treat A_{er} in (4) as a cross-section for isotropic scattering, defined as σ, and apply (4) a second time for the return path. Thus let P_{t_r} be the radar return power. This is given by $P_{t_r} = \left(\frac{P_t A_{et}\sigma}{\lambda^2 r^2}\right)$;

For isotropic scattering the effective transmitting area is $A_{et_r} = \frac{\lambda^2}{4\pi}$ (D = 1) ; then applying (4) above

the radar signal received $= \frac{P_{t_r} A_{et_r}}{\lambda^2 r^2} A_{et} = P_t (A_{et})^2 \sigma / 4\pi\lambda^2 r^4$)

Assumes transmitter antenna = receiver antenna