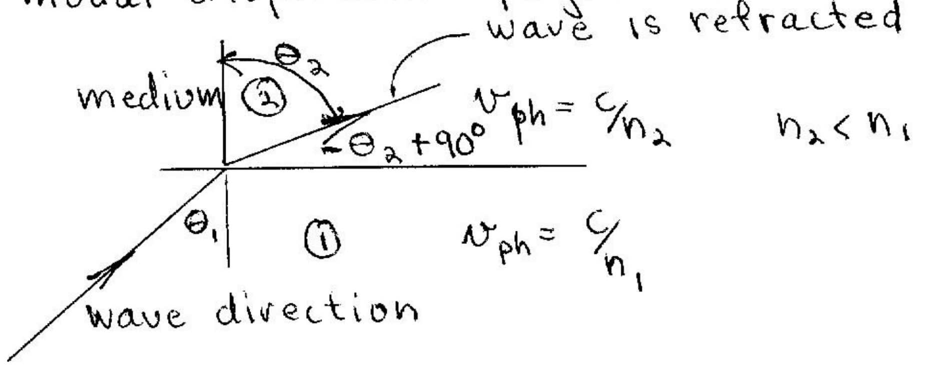
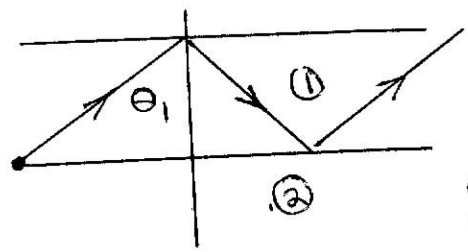
Such group velocity dispersion can be come important in an optical fiber when numerous "channels" (wavelengths) are used or the band width becomes extremely large.

Note on modal dispersion - pages 27 and 28



Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (EECS 117)
(projection of the index along the boundary is the same in medium ① and medium ②)

<u>Mode</u> (simplified)



Two important aspects for θ_1

1) It is discrete (a given value for a particular mode)

2) <u>Just</u> at cut-off the beam in medium ② is skimming along the surface (θ_2 is $\simeq 90°$) and "far from cut-off" $\theta_1 \simeq 90°$

Thus in a multimode fiber cut-off (near) modes have a speed c/n_2 and far above cut-off c/n_1

For a length L the difference in travel time is

$$\underline{\text{approximately}}\ \frac{L(n_1 - n_2)}{c} = \frac{L(n_1 - n_2)}{n_1}\frac{n_1}{c} = L\Delta\frac{n_1}{c} \triangleq \delta T$$

which is Expression (2.2) for $n_1 \simeq n_2$ which is usually the case. Here Δ is defined as $\left(\frac{n_1 - n_2}{n_1}\right)$

Bit Rate B implies Bit spacing of $\frac{1}{B}$

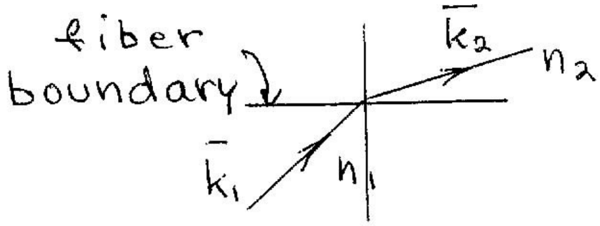
The spreading of each bit should be $\leq \frac{1}{2B}$ for inter-symbol interference <u>not</u> to be a problem.

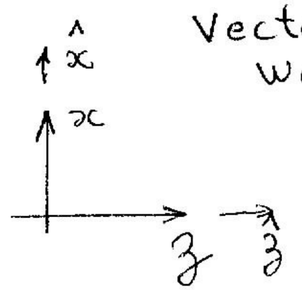
Thus $\delta T = L \Delta \frac{n_1}{c} \leq \frac{1}{2B}$ which is Eq. (2.2) giving the limitation on

$BL \leq \frac{1}{2} \frac{c}{n_1 \Delta}$ for a multimode fiber.

The mode condition

Vector picture for wave propagation





$$\bar{k}_1 = k_{1x}\hat{x} + k_{1z}\hat{z} \qquad k_1 = (k_{1x}^2 + k_{1z}^2)^{1/2} = \frac{\omega n_1}{c} \quad (1)$$

$$\bar{k}_2 = i k'_{2x}\hat{x} + k_{1z}\hat{z} \qquad k_2 = (-(k'_{2x})^2 + k_{1z}^2)^{1/2} = \omega \frac{n_2}{c} \quad (2)$$

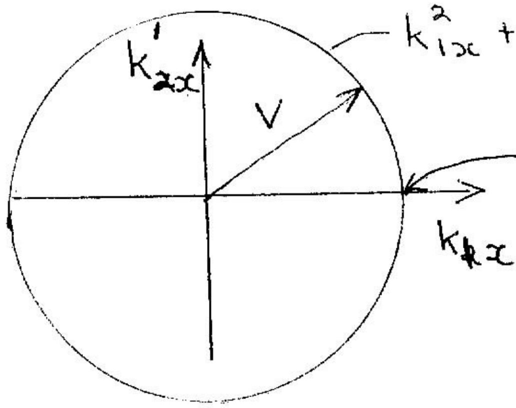
From (1) and (2) $k_1^2 - k_2^2 = \frac{\omega^2}{c^2}(n_1^2 - n_2^2) = k_{1x}^2 + (k'_{2x})^2$

$= V^2/a^2$

fiber radius

Plot k_{1x} versus k'_{2x}

$k_{1x}^2 + (k'_{2x})^2 = \frac{V^2}{a^2} = \left(\frac{2\pi}{\lambda}\right)^2$



V must be ≤ 2.405 for a single mode

The chirped Gaussian Pulse (page 47) of text.

Pulses transmitted along an optical fiber have several important characteristics

a) A "carrier frequency", ω_0

b) A possible frequency sweep or "chirp"

$$\frac{d(\text{frequency})}{dt} = \text{Const} \triangleq \frac{\kappa}{(T_0)^2} \quad (\text{sec}^{-2})$$

$\kappa \triangleq$ chirp parameter ; T_0 = pulse width

Integrating, setting the constant = ω_0

$$\text{frequency} = \frac{\kappa}{(T_0)^2} t + \omega_0 = \frac{d}{dt}\text{Phase}$$

Integrating once again setting the constant = 0

$$\text{phase} = \omega_0 t + \frac{1}{2}\frac{\kappa}{(T_0)^2} t^2$$

c) Pulse profile - often assumed Gaussian

$e^{-\frac{1}{2}t^2/T_0^2}$ times an amplitude, A_0

a) b) + c) yield a pulse of the form

$$\underset{\text{Amplitude}}{A_0}\; \underset{\text{Profile}}{e^{-\frac{1}{2}t^2/T_0^2}} \cos\Big(\underset{\text{Carrier}}{\omega_0 t} + \underset{\text{Chirp}}{\frac{\kappa}{2}\left(\frac{t}{T_0}\right)^2}\Big)$$

This can be expressed as

$$\text{Real Part of } \left(A_0 e^{-\frac{1}{2}t^2/T_0^2} e^{+j\frac{\kappa}{2}t^2/T_0^2} e^{i\omega_0 t}\right)$$

The amplitude, pulse width, and chirp can all change as a consequence of modulation detection, or propagation

Sec 2.3.2 Pulse Width Evolution

For any two independent processes (for example initial Pulse Width and Fiber group velocity dispersion), the final (pulse width)^2 = (initial pulse width)^2 + (pulse broadening width due to group velocity dispersion)^2

T_0 = initial pulse width

$$\Delta T = L \Delta\omega \frac{d^2k}{d\omega^2} = L\Delta\omega\,\beta_2 \qquad \left(\beta_2 = \frac{d^2k}{d\omega^2} \text{ in the text}\right)$$

But for band-width limited pulses $\Delta\omega\, T_0 = 1$

Thus

$$T_L^2 = (\text{final pulse width})^2 = \left(T_0^2 + \left(\frac{\beta_2 L}{T_0}\right)^2\right)$$

(For this to hold the formal definition of T_0 is given by Eq. (2.29))

Note the optimum $\frac{dT_L}{dT_0} = 0 \Rightarrow T_0 = \sqrt{\beta_2 L}$

and $T_L = \sqrt{2}\, T_0$ for this situation.

But the bit rate $B \simeq \frac{1}{T_L}$ at most.

Thus $B < \frac{1}{T_L} = \frac{1}{\sqrt{2}\sqrt{\beta_2 L}}$ — Eq (2.30) [We set $\epsilon\sqrt{2} = 1$ since this is an inequality)

From the discussion on group velocity dispersion $D = \frac{\lambda}{c}\frac{d^2n}{d\lambda^2} = \frac{d^2k}{d\omega^2}\frac{\omega}{\lambda} = \beta_2 2\pi \frac{f}{\lambda} = 2\pi\beta_2\frac{c}{\lambda^2}$

Thus one can also write the bit rate limitation as

$$B < \frac{1}{\sqrt{2}}\frac{\sqrt{2\pi c}}{\sqrt{DL}\,\lambda} \quad - \text{Eq. (2.32) page 52}$$

Note that for a situation for which the channel is long (L large) or the broadening is dominated by the spectral width of the source (LED for instance (pulses are then not bandwidth limited))

use $\Delta T = L\Delta\omega\frac{d^2k}{d\omega^2} = L\Delta\omega\beta_2$

Then $T_L \cong L\Delta\omega\beta_2$ (Book uses W_0 for bandwidth)

page 54. Finally using $\beta_2 = \frac{\lambda^2}{2\pi c}D$ and

$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\omega}{\omega}$, $\frac{1}{T_L} = B$, one obtains the limits

$$B < \frac{1}{L\omega\Delta\lambda}\frac{\lambda\, 2\pi c}{\lambda^2 D} = \frac{1}{L\Delta\lambda D}$$

which is Eq (2.34); $L\Delta\lambda D B \lesssim 1$