The instantaneous angular frequency of the pulse is the derivative of the phase and is given by

$$\frac{d}{dt}\left(\omega_0 t + \frac{\kappa}{2}\frac{t^2}{T_0^2}\right) = \omega_0 + \frac{\kappa}{T_0^2}t.$$

We define the *chirp factor* of a Gaussian pulse as T_0^2 times the derivative of its instantaneous angular frequency. Thus the chirp factor of the pulse described by (2.26) is κ. This pulse is said to be *linearly chirped* since the instantaneous angular frequency of the pulse increases or decreases *linearly* with time t, depending on the sign of the chirp factor κ. In other words, the chirp factor κ is a constant, independent of time t, for linearly chirped pulses.

Let $A(z, t)$ denote a chirped Gaussian pulse as a function of time and distance. At $z = 0$,

$$A(0, t) = A_0 e^{-\frac{1+i\kappa}{2}\left(\frac{t}{T_0}\right)^2}. \tag{2.27}$$

If we solve (2.24) for a chirped Gaussian pulse (so the initial condition for this differential equation is that $A(0, t)$ is given by (2.27)), we get

$$A(z, t) = \frac{A_0 T_0}{\sqrt{T_0^2 - i\beta_2 z(1 + i\kappa)}} \exp\left(-\frac{(1 + i\kappa)(t - \beta_1 z)^2}{2\left(T_0^2 - i\beta_2 z(1 + i\kappa)\right)}\right). \tag{2.28}$$

The key point to note here is that $A(z, t)$ is also the envelope of a chirped Gaussian pulse for all $z > 0$, but the width of this pulse increases as z increases if $\beta_2\kappa > 0$. This happens because the parameter governing the pulse width is now $T_z^2 = T_0^2 - i\beta_2 z(1 + i\kappa)$, and the magnitude of T_z monotonically increases with increasing z if $\beta_2\kappa > 0$. A measure of the pulse broadening at distance z is the ratio $|T_z|/T_0$. The analytical expression (2.25) for this ratio can be derived from (2.28).

Broadening of Chirped Gaussian Pulses

Figure 2.7 shows the pulse broadening effect of chromatic dispersion graphically. In these figures, the center or carrier frequency of the pulse, ω_0, has deliberately been shown vastly diminished for the purposes of illustration. We assume β_2 is negative; this is true for standard single-mode fiber in the 1.55 μm band. Figure 2.7(a) shows an unchirped ($\kappa = 0$) Gaussian pulse, and Figure 2.7(b) shows the same pulse after it has propagated a distance $2T_0^2/|\beta_2|$ along the fiber. Figure 2.7(c) shows a chirped Gaussian pulse with $\kappa = -3$, and Figure 2.7(d) shows the same pulse after it has propagated a distance of only $0.4T_0^2/|\beta_2|$ along the fiber. The amount of broadening

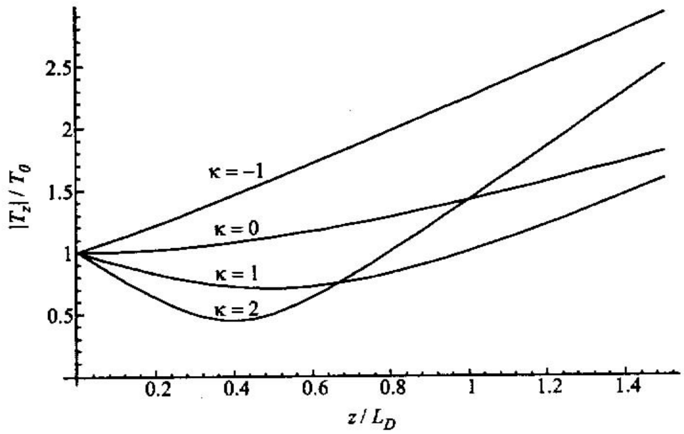

Figure 2.9 Evolution of pulse width as a function of distance (z/L_D) for chirped and unchirped pulses in the presence of dispersion. We assume $\beta_2 < 0$, which is the case for 1.55 μm systems operating over standard single-mode fiber. Note that for positive chirp the pulse width initially decreases but subsequently broadens more rapidly. For systems operating over standard single-mode fiber at 1.55 μm, $L_D \approx$ 1800 km at 2.5 Gb/s, whereas $L_D \approx$ 115 km at 10 Gb/s.

2.3.2 System Limitations

The pulse broadening effect of chromatic dispersion causes the signals in adjacent bit periods to overlap. This phenomenon is called *intersymbol interference* (ISI). We next derive the system limitations imposed by chromatic dispersion for unchirped Gaussian pulses. The results can be extended in a straightforward manner to pulses with chirp.

Consider a fiber of length L. From (2.25), the width of the output pulse is given by

$$T_L = \sqrt{T_0^2 + \left(\frac{\beta_2 L}{T_0}\right)^2}.$$

This is the half-width of the pulse at the $1/e$-intensity point. A different, and more commonly used, measure of the width of a pulse is its *root-mean square (rms) width* T^{rms}. For a pulse, $A(t)$, this is defined as

$$T^{\mathrm{rms}} = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 |A(t)|^2 \, dt}{\int_{-\infty}^{\infty} |A(t)|^2 \, dt}}. \tag{2.29}$$

Problems

2.1 Show that the numerical aperture NA of a step-index multimode fiber is

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2}.$$

2.2 A standard multimode glass fiber has a core diameter of 50 μm and cladding refractive index of 1.45. If it is to have a limiting modal dispersion δT of 10 ns/km, find its numerical aperture. Also calculate the maximum bit rate for transmission over a distance of 20 km.

2.3 Derive equation (2.10) for the evolution of the magnetic field vector $\tilde{\mathrm{H}}$.

2.4 Derive an expression for the cutoff wavelength $\lambda_{\mathrm{cutoff}}$ of a step-index fiber with core radius a, core refractive index n_1, and cladding refractive index n_2. Calculate the cutoff wavelength of a fiber with core radius $a = 4$ μm and $\Delta = 0.003$.

2.5 Consider a step-index fiber with a core radius of 4 μm and a cladding refractive index of 1.45.

(a) For what range of values of the core refractive index will the fiber be single moded for all wavelengths in the 1.2–1.6 μm range?

(b) What is the value of the core refractive index for which the V parameter is 2.0 at $\lambda = 1.55$ μm? What is the propagation constant of the single mode supported by the fiber for this value of the core refractive index?

2.6 Assume that, in the manufacture of single-mode fiber, the tolerance in the core radius a is $\pm 5\%$ and the tolerance in the normalized refractive index difference Δ is $\pm 10\%$, from their respective nominal values. If the nominal value of Δ is specified to be 0.005, what is the largest nominal value that you can specify for a while ensuring that the resulting fiber will be single moded for $\lambda > 1.2$ μm even in the presence of the worst-case (but within the specified tolerances) deviations of a and Δ from their nominal values? Assume that the refractive index of the core is 1.5.

2.7 In a reference frame moving with the pulse, the basic propagation equation that governs pulse evolution inside a dispersive fiber is

$$\frac{\partial A}{\partial z} + \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial t^2} = 0,$$

where $A(z,t)$ is the pulse envelope. If $A(0,t) = A_0 \exp(-t^2/2T_0^2)$ for some constants A_0 and T_0, solve this propagation equation to find an expression for $A(z,t)$.
Note: You may use the following result without proof:

$$\int_{-\infty}^{\infty} \exp(-(x-m)^2/2\alpha)\, dx = \sqrt{2\pi\alpha}$$

for all *complex* m and α provided $\Re(\alpha) > 0$.
Hint: Consider the Fourier transform $\tilde{A}(z,\omega)$ of $A(z,t)$.

2.8 Starting from (2.28), derive the expression (2.25) for the width T_z of a chirped Gaussian pulse with initial width T_0 after it has propagated a distance z.

2.9 Show that an unchirped Gaussian pulse launched at $z = 0$ remains Gaussian for all z but acquires a distance-dependent chirp factor

$$\kappa(z) = \frac{\mathrm{sgn}(\beta_2) z/L_D}{1 + (z/L_D)^2}.$$

2.10 Show that the rms width of a Gaussian pulse whose half-width at the $1/e$-intensity point is T_0 is given by $T_0/\sqrt{2}$.

2.11 For a narrow but chirped Gaussian pulse with chirp factor $\kappa = -6$, calculate the chromatic dispersion limit at a bit rate of 1 Gb/s, in the 1.55 μm band, for a penalty of 2 dB. Compare this with the chromatic dispersion limit for unchirped pulses plotted in Figure 2.10.

2.12 Consider a chirped Gaussian pulse for which the product $\kappa\beta_2$ is negative that is launched at $z = 0$. Let $\kappa = 5$.

(a) For what value of z (as a multiple of L_D) does the launched pulse attain its minimum width?

(b) For what value of z is the width of the pulse equal to that of an unchirped pulse, for the same value of z? (Assume the chirped and unchirped pulses have the same initial pulse width.)

2.13 Show that the optimum choice of the pulse width of an unchirped Gaussian pulse (with narrow spectral width) that minimizes the pulse broadening effects of chromatic dispersion over a fiber of length L is

$$T_0^{\mathrm{opt}} = \sqrt{\beta_2 L}.$$

2.14 In discussing the chromatic dispersion penalty, the Bellcore standard for SONET systems [Bel95] specifies the spectral width of a pulse, for *single-longitudinal mode* (SLM) lasers, as its 20-dB spectral width divided by 6.07. We will study these lasers in Section 3.5.1. Show that for SLM lasers whose spectra have a Gaussian profile, this is equivalent to the rms spectral width.

2.15 Show that in the case of four-wave mixing, the nonlinear polarization is given by (2.54)–(2.58).

2.16 You want to design a soliton communication system at 1.55 μm at which wavelength the fiber has $\beta_2 = -2$ ps^2/km and $\gamma = 1$/W-km. The peak power of the pulses you can generate is limited to 50 mW. If you must use fundamental solitons and the bit period must be at least 10 times the full width at half-maximum (T_{FWHM}) of the soliton pulses, what is the largest bit rate you can use?