# Solutions.

EE118
2/1/04

## Homework # 1

1) a) $x(t,z) = 3\cos(\omega t - kz)$, $\omega = 2\pi f = 6\pi \cdot 10^{13}$

$= 3\cos(6\pi \cdot 10^{13} t - 4\pi \cdot 10^{3} z)$ $\quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{5 \cdot 10^{-4}} = 4\pi \cdot 10^{3}$

Note z in cm

b) $T = \frac{1}{f} \cong 33 \cdot 10^{-15}$ s $\quad \omega = 2\pi f = 6\pi \cdot 10^{13}$ rad/sec

c) Infrared

d) $V_{ph} = \frac{\omega}{k} = \frac{6\pi \cdot 10^{13}}{4\pi \cdot 10^{3}} = 1.5 \times 10^{10}$ cm/s

e) $n = \frac{c}{V_{ph}} = \frac{3 \times 10^{10} \text{ cm/s}}{1.5 \times 10^{10} \text{ cm/s}} = 2$

2) a) Total loss $= 0.5 \frac{dB}{km} \times 100 km + 0.3 + 1 \times 2 = 52.3$ dB

b) $dB = 10 \log_{10} \frac{P_{tran}}{P_{receive}} \Rightarrow 52.3 = 10 \log\left(\frac{1mW}{P_{receive}}\right) \Rightarrow P_{receive} = 5.9 \times 10^{-9}$ W

3) $\Delta t = D \cdot \Delta\lambda \cdot L$

$\Delta\lambda = \frac{\Delta\omega}{\omega_0} \cdot \lambda = \frac{\lambda}{\omega_0 \cdot t}$

$= \left(10 \frac{ps}{nm \cdot km}\right)(0.531)(10 km)$

$= \frac{\lambda}{2\pi\left(\frac{c}{\lambda}\right) \cdot t} = \frac{\lambda^2}{2\pi c t}$

$= 53.1$ ps

$= \frac{(1 \times 10^{-6} m)^2}{2\pi(3 \times 10^{8} m/s)(1 \times 10^{-12} s)} = 5.31 \times 10^{-10}$

$= 0.531$ nm

4) $m x(t) = 0.8 \cos(\omega_1 t)$ $\quad$ Amplitude $= \sqrt{I} = \sqrt{I_0}\,(1 + 0.8\cos\omega_1 t)^{1/2}$

$I = I_0\,(1 + 0.8\cos\omega_1 t)$ $\quad \simeq \sqrt{I_0}\left(1 + \frac{1}{2} 0.8 \cos\omega_1 t - \frac{1}{8} \cdot 0.64 \left(\frac{1 + \cos 2\right.\right.$

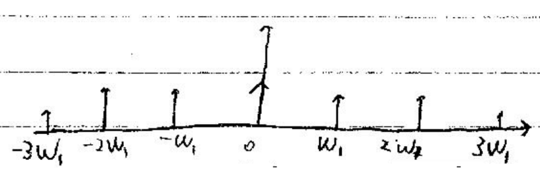
to check for 5% estimation,

$(1 + 0.4\cos\omega_1 t - 0.04 - 0.04\cos 2\omega_1 t)^2$

$= (0.9216 + \ldots)$

try again with three terms $\simeq \sqrt{I_0}\left(1 + \frac{1}{2} 0.8\cos\omega_1 t - \frac{1}{8} 0.64\left(\frac{1 + \cos 2\omega_1 t}{2}\right) + \frac{3}{64}(0.512)(\cos\right.$

this two terms are not enough.



4. Fiber modulates via intensity

$$\therefore \quad m x(t) = 0.8 \cos \omega_m t \rightarrow I \propto (1 + 0.8 \cos \omega_m t) \cdot \cos \omega_c t$$

recovered amplitude modulation

$$E \propto \sqrt{I} \propto (1 + 0.8 \cos \omega_m t)^{1/2} \cdot \cos \omega_c t$$

Expand $(1 + 0.8 \cos \omega_m t)^{1/2}$ using

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \text{H.O.T.}$$

$$(1 + 0.8 \cos \omega_m t)^{1/2} = 1 + \tfrac{1}{2} 0.8 \cos \omega_m t - \tfrac{1}{8} (0.8)^2 \cos^2 \omega_m t$$

$$= 1 + 0.4 \cos \omega_m t - 0.08 \cos^2 \omega_m t$$

$$= 1 + 0.4 \cos \omega_m t - 0.08 \cdot \tfrac{1}{2} (1 + \cos 2\omega_m t)$$

$$= 1 + 0.4 \cos \omega_m t - 0.04 - 0.04 \cos 2\omega_m t$$

$$= 0.96 + 0.4 \cos \omega_m t - 0.04 \cos 2\omega_m t$$

Now recombine with carrier to find amplitude spectrum

$$E \propto (0.96 + 0.4 \cos \omega_m t - 0.04 \cos 2\omega_m t) \cdot \cos \omega_c t$$

$$\propto 0.96 \cos \omega_c t + 0.4 \cos \omega_m t \cos \omega_c t - 0.04 \cos 2\omega_m t \cos \omega_c t$$

$$\propto 0.96 \cos \omega_c t + \tfrac{1}{2} (0.4) \cdot [\cos(\omega_c - \omega_m) t + \cos(\omega_c + \omega_m) t]$$
$$+ \tfrac{1}{2} (0.04) [\cos(\omega_c - 2\omega_m) t + \cos(\omega_c + 2\omega_m) t]$$

$\therefore$ We can see the spectrum of the signal ranges from $\omega_c - 2\omega_m$ to $\omega_c + 2\omega_m$.

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To verify if recovered amplitude matches signal within 5%, look at power series expansion of amplitude, squared, and compare with orig intensi

$$\propto (0.96 + 0.4 \cos \omega_m t - 0.04 \cos 2\omega_m t)^2$$

$$\begin{aligned}\propto\ & 0.9216 + 0.768 \cos \omega_m t - 0.0768 \cos 2\omega_m t \\ & + 0.16 \cos^2 \omega_m t - 0.032 \cos \omega_m t \cos 2\omega_m t \\ & + 0.0016 \cos 2\omega_m^2 t\end{aligned}$$

$$\begin{aligned}\propto\ & \overbrace{0.9216}^{0} + \overbrace{0.768 \cos \omega_m t}^{1} - \overbrace{0.0768 \cos 2\omega_m t}^{2} \\ & + 0.16\left(\tfrac{1}{2}\right)\left(\underbrace{1}_{0} + \overbrace{\cos 2\omega_m t}^{2}\right) - 0.032\left(\tfrac{1}{2}\right)\left(\underbrace{\cos \omega_m t}_{1} + \underbrace{\cos 3\omega_m t}_{3}\right) \\ & + 0.0016\left(\tfrac{1}{2}\right)\left(\underbrace{1}_{0} + \underbrace{\cos 4\omega_m t}_{4}\right)\end{aligned}$$

Collecting same harmonics (we only need DC and first) for comparison

0th : $0.9216 + 0.16\left(\tfrac{1}{2}\right) + 0.0016\left(\tfrac{1}{2}\right) = 1.0024$

1st : $0.768 - 0.032\left(\tfrac{1}{2}\right) = 0.752$

$$\approx 1.0024 + 0.752 \cos \omega_m t$$

| original | vs recovered | |
|---|---|---|
| 1 | 1.0024 | (0.2% difference) |
| 0.8 | 0.752 | (6% difference) |

So the 1st harmonic does not fall to within the 5% specification