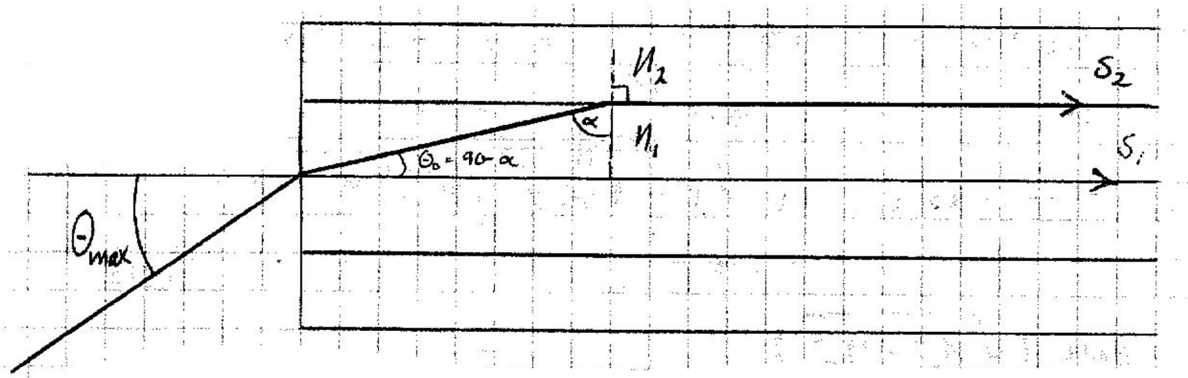
#1



a) The maximum difference in used along path S_2 and s_1, must be less that $\delta T = 10$ ns/km

$$S_1 = v_1 t_1 \Rightarrow t_1 = \frac{S_1}{v_1} = \frac{1\,km}{\frac{c}{n_1}} = \frac{n_1}{c}\cdot km$$

$$S_2 = v_2 t_2 \Rightarrow t_2 = \frac{S_2}{v_2} = \frac{1\,km}{\frac{c}{n_2}} = \frac{n_2}{c}\, km$$

$$t_1 - t_2 = 10\,ns = \frac{n_1}{c}\cdot km - \frac{n_2}{c}\cdot km$$

$$n_1 = 10\,ns \cdot \frac{c}{km} + n_2 = 10\cdot 10^{-9}\,s \cdot \frac{3\cdot 10^8}{10^3} + 1{,}45 = 1{,}453$$

$$\underline{\underline{n_1 = 1{,}453}}$$

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b) Snell's law :- Calculation of The Acceptance Angle (Θ_{max})

First consider boundary between core and cladding

$$n_1 \sin\alpha = n_2 \Rightarrow \sin\alpha = \frac{n_2}{n_1}$$

$$\sin\Theta_{max} = n_1 \sin\Theta_0 = n_1 \cos\alpha$$

$$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

Snell's Law at entrance to fiber

$$\sin\Theta_{max} = n_1\cos\alpha = n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \sqrt{n_1^2 - n_2^2}$$

Thuse

$$\underline{\Theta_{max} = \sin^{-1}\left(\sqrt{n_1^2-n_2^2}\right)} \simeq \sin^{-1}\left(\sqrt{(n_1-n_2)}\sqrt{n_1+n_2}\right)$$

$$\simeq \sin^{-1}\left(\sqrt{\frac{n_1-n_2}{n_1}}\sqrt{(n_1+n_2)n_1}\right)$$

$$\simeq \sin^{-1}\left(\sqrt{\Delta}\sqrt{2}\,n_1\right)$$

c) If multi-mode dispersion is dominant a pulse after having travelled 20km is

200 ns wide. In order to separate to bits each bit must atleast be 200 ns apart. This gives the maximum bit rate [MBR].

$$MBR = \frac{1}{200\cdot 10^{-9}}\ \text{bit/s} = 5\ \text{Mbit/s}$$

$$\underline{\underline{MBR = 5\ \text{Mbit/s}}}$$

#2

We went through the cut-off calculation of a slab wave-guide in class. For a cylindrical guide with cladding, it is

$$\frac{2\pi}{\lambda_{co}} \cdot a \cdot \sqrt{n_1^2 - n_2} < 2{,}405 \text{ , for singelmode fiber}$$

Thus

$$\underline{\underline{\lambda_{co} > \frac{400\pi}{481} \cdot a \cdot \sqrt{n_1^2 - n_2^2}}}$$

$$\lambda_{co} = \frac{400\pi}{481} \cdot a \cdot \sqrt{n_1 - n_1 (1 - 2\Delta + \Delta^2)} \text{ , } \Delta = \frac{n_1 - n_2}{n_1} \Rightarrow n_2 = n_1 (1-\Delta)$$

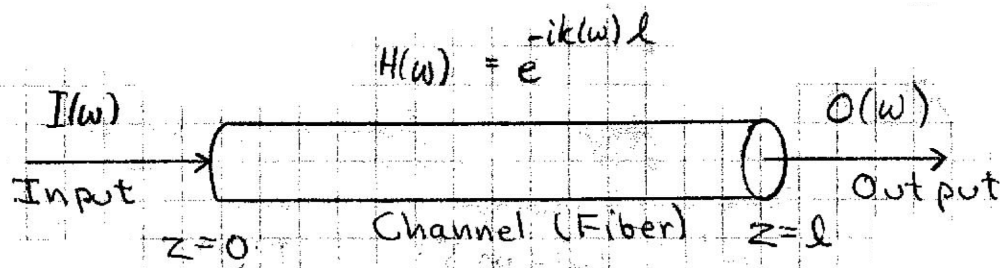
$$= \frac{400\pi}{481} \cdot a \cdot n_1 \sqrt{\Delta (2-\Delta)} \approx \frac{400\pi}{481} \cdot a \cdot n_1 \cdot \sqrt{2\Delta}$$

$$\lambda_{co} = \frac{400\pi}{481} \cdot 4 \cdot 10^{-6} m \cdot \sqrt{2 \cdot 0{,}003} \cdot n_1 = 8{,}1 \cdot 10^{-7} n_1 \text{ [m]}$$

$$\underline{\underline{\lambda_{co} = 8{,}1 \cdot 10^{-7} \cdot n_1 \text{ [m]}}}$$

#3

a)



$$I(z=0,t) = A_0 e^{-\frac{1}{2}(t/T_0)^2} \cdot e^{i\omega_0 t}$$

$$I(\omega) = A\,\mathcal{F}\{e^{-\frac{1}{2}(t/T_0)^2}\} * \mathcal{F}\{e^{i\omega_0 t}\}$$

$$= A_0 W(\omega - \omega_0)$$ where W is the transform of the Gaussian pulse

$$W(\omega) = \int_{-\infty}^{\infty} e^{-\frac{1}{2}(t/T_0)^2} e^{-i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-A_1 t^2 + B_1 t}\, dt; \quad A_1 = \frac{1}{2T_0^2}, \quad B_1 = -i\omega$$

$$-A_1 t^2 + Bt = -A_1\left(t^2 - \frac{B}{A_1}t\right) = -A_1\left[\left(t - \frac{1}{2}\left(\frac{B_1}{A_1}\right)\right)^2 + \frac{1}{4}\left(\frac{B}{A_1}\right)^2\right]$$

$$= \int_{-\infty}^{\infty} e^{-A_1\left[\left(t-\frac{1}{2}\left(\frac{B_1}{A_1}\right)\right)^2 + \frac{1}{4}\left(\frac{B_1}{A_1}\right)^2\right]}\, dt$$

$$= e^{\frac{B_1^2}{4A_1}} \int_{-\infty}^{\infty} e^{-A_1\left(t - \frac{1}{2}\left(\frac{B_1}{A_1}\right)\right)^2}\, dt$$

$$= e^{\frac{B_1^2}{4A_1}} \int_{-\infty}^{\infty} e^{-u^2} \frac{du}{\sqrt{A_1}} = \sqrt{\frac{\pi}{A_1}}\, e^{\frac{B_1^2}{4A_1}}$$

$A_1 = \frac{1}{2T_0^2} \qquad B = -i\omega$

$$W(\omega) = T_0\sqrt{2\pi}\; e^{-\frac{1}{2}T_0^2\omega^2}$$

$$W(\omega-\omega_0) = \sqrt{2\pi}\cdot T_0\cdot e^{-\frac{1}{2}T_0^2(\omega-\omega_0)^2} = \frac{I(\omega)}{A_0}$$

$$O(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} I(\omega)\cdot H(\omega)\cdot e^{i\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A_0\sqrt{2\pi}\cdot T_0\, e^{-\frac{1}{2}T_0^2(\omega-\omega_0)^2}\cdot e^{-ik(\omega)l}\cdot e^{i\omega t}\,d\omega$$

$$= \frac{A_0T_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}T_0^2(\omega-\omega_0)^2}\, e^{-ik(\omega)l}\, e^{i\omega t}\,d\omega$$

Expand $k(\omega)$ about ω_0

$$k(\omega) = k(\omega_0) + \left.\frac{dk}{d\omega}\right|_{\omega_0}(\omega-\omega_0) + \frac{1}{2}\left.\frac{d^2k}{d\omega}\right|_{\omega_0}(\omega-\omega_0)^2$$

$$= \beta_0 + \beta_1(\omega-\omega_0) + \frac{1}{2}\beta_2(\omega-\omega_0)^2$$

$$O(\omega) = \frac{A_0T_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}T_0^2(\omega-\omega_0)^2 - i\beta_0 l - i\beta_1(\omega-\omega_0)l - i\frac{\beta_2}{2}(\omega-\omega_0)^2 l + i(\omega-\omega_0)t + i\omega_0 t}\,d\omega$$

$$= \frac{A_0T_0}{\sqrt{2\pi}}\, e^{i(\omega_0 t-\beta_0 l)}\int_{-\infty}^{\infty} e^{-\left(\frac{1}{2}T_0^2 + i\frac{\beta_2}{2}l\right)u^2 + i(t-\beta_1 l)u}\,du$$; where $u = \omega-\omega_0$

$A_2 = \frac{1}{2}\left(T_0^2 + i\beta_2 l\right) \qquad B_2 = i(t-\beta_1 l)$

$$O(\omega) = \frac{A_0 T_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-A_2 \omega^2 + B_2 u} \, du$$

$$= \frac{A_0 T_0}{\sqrt{2\pi}} \cdot e^{i(\omega_0 t - \beta_0 l)} \cdot \sqrt{\frac{\pi}{A_2}} \cdot e^{\frac{B_2}{4A_2}}$$

$$= \frac{A_0 T_0}{\sqrt{2\pi}} \cdot \sqrt{\frac{\pi}{1/2 (T_0^2 + i\beta_2 l)}} \cdot e^{i(\omega_0 t - \beta_0 l)} \cdot e^{-\frac{(t-\beta_1 l)^2}{2(T_0^2 + i\beta_2 l)}}$$

$$\underline{\underline{O(\omega) = \frac{A_0 T_0}{\sqrt{(T_0^2 + i\beta_2 l)}} \cdot e^{-\frac{(t-\beta_1 l)^2}{2(T_0^2 + i\beta_2 l)}} \cdot e^{i(\omega_0 t - \beta_0 l)}}}$$

b)

The ~~slope~~ width of the puls is given by the real part of the following.

$$-\frac{(t-\beta_1 l)^2}{2(T_0^2 + i\beta_2 l)} = -\frac{1}{2} \cdot \frac{(t-\beta_1 l)^2 (T_0^2 + i\beta_2 l)}{(T_0^4 + (\beta_2 l)^2)}$$

Real Part is :

$$-\frac{1}{2} \cdot \frac{T_0^2 (t-\beta_1 l)^2}{(T_0^4 + (\beta_2 l)^2)} = -\frac{1}{2} \frac{(t-\beta_1 l)^2}{T_0^2 + \left(\frac{\beta_2 l}{T_0}\right)^2}$$

We see that the width of the ~~slope~~ pulse envelope ~~is~~ is

$$\underline{\underline{(T(l))^2 = T_0^2 + \left(\frac{\beta_2 l}{T_0}\right)^2 = T_0^2 + (D\Delta\lambda l)^2}} \quad \text{q. e. d}$$

(Note that when $l=0$, one extracts the origional pulse as a check on the algebra)

As discussed in class $\mathrm{Im}\left(-\frac{(t-\beta_1 l)^2}{2(T_0^2 + i\beta_2 l)}\right) = -\frac{1}{2} \frac{(t-\beta_1 l)^2}{T_0^4 + (\beta_2 l)^2} \beta_2 l = \frac{\Omega}{2}(t-\beta_1 l)^2$ chirp parameter

Problem No. 4

$$T(z) = T_0^2 + (\beta_2 z)^2 \cdot T_0^{-2}$$

$$\frac{dT(z)}{dz} = 2T_0 - 2(\beta_2 z)^2 \cdot T_0^{-3} = 0$$

$$T_0^4 = (\beta_2 L_D)^2 \Rightarrow T_0 = \sqrt{|\beta_2| z}$$

$$T(L_D)^2_{min} = |\beta_2| L_D + (\beta_2 L_D)^2 \cdot (|\beta_2| L_D)^{-1}$$

$$= 2|\beta_2| L_D$$

Example: For $|\beta_2| = 2$ psec^2/km and $l = L_D = 20$ km

$$T(L_D)_{min} = \sqrt{2|\beta_2| L_D} = \sqrt{2 \cdot 0,2 \text{ ps}^2/\text{km} \cdot 20 \text{ km}}$$

$$= 2,828 \cdot 10^{-12} \text{ s}$$

Note: I did ~~as~~ not expect you to know the soliton part of the problem. (Coordination between the 1'st ed and 2'nd ed of the Book!)