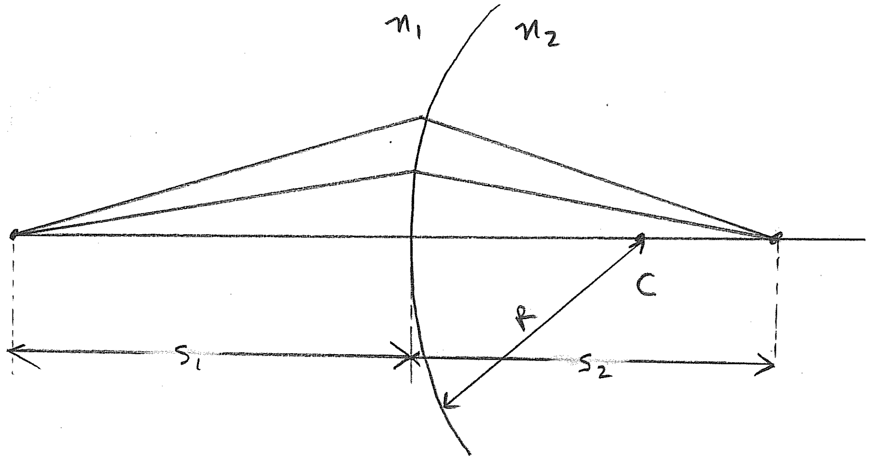
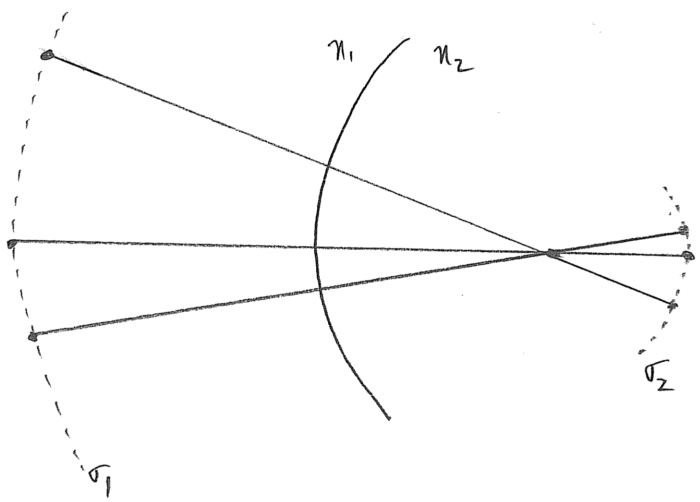
RECALL: Imaging of on-axis point by a single spherical interface between two n's.



we used fermat's principal and the paraxial approximation to derive

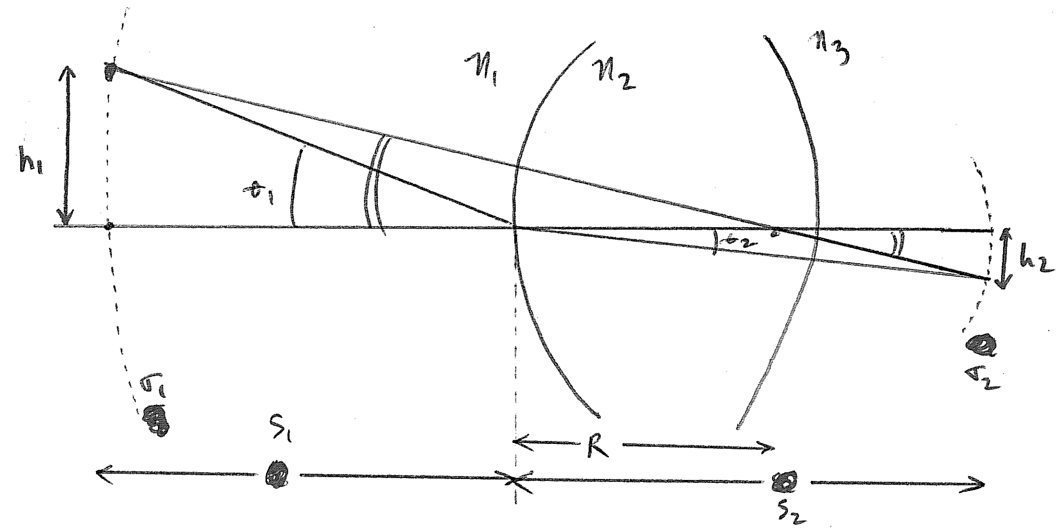
$$\frac{n_1}{s_1} + \frac{n_2}{s_2} = \frac{n_2 - n_1}{R} \qquad (5.8 \text{ Hecht})$$

WHAT ABOUT IMAGING BIGGER OBJECTS? We can use symmetry here → we can rotate our coordinate system about point C and 5.8 still holds. It turns out that points on spherical object surface σ_1 are perfectly imaged (in paraxial approx) to points on the spherical image surface σ_2



$$\boxed{\begin{aligned} R_{\sigma_1} &= s_1 + R \\ R_{\sigma_2} &= R - s_2 \end{aligned}}$$

So how to determine magnification for spherical refracting surface? — WE ARE NO DEALING w̄ FINITE SIZED OBJECTS



We know s_i b/c we have the Eqn for imaging of spherical refracting surface in paraxial approx.

SNELL'S LAW (small angle approximation)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \tan\theta_1 \approx n_2 \tan\theta_2$$

$$\cancel{n_1 \frac{h_1}{s_1} = n_2 \frac{h_2}{s_2}} \quad \therefore \quad \boxed{\frac{h_2}{h_1} = \frac{-n_1 s_2}{n_2 s_1} = M}$$

$$n_1 \frac{h_1}{s_1} = n_2 \frac{-h_2}{s_2}$$

So ~~object~~ image ~~...~~ is on a curved surface

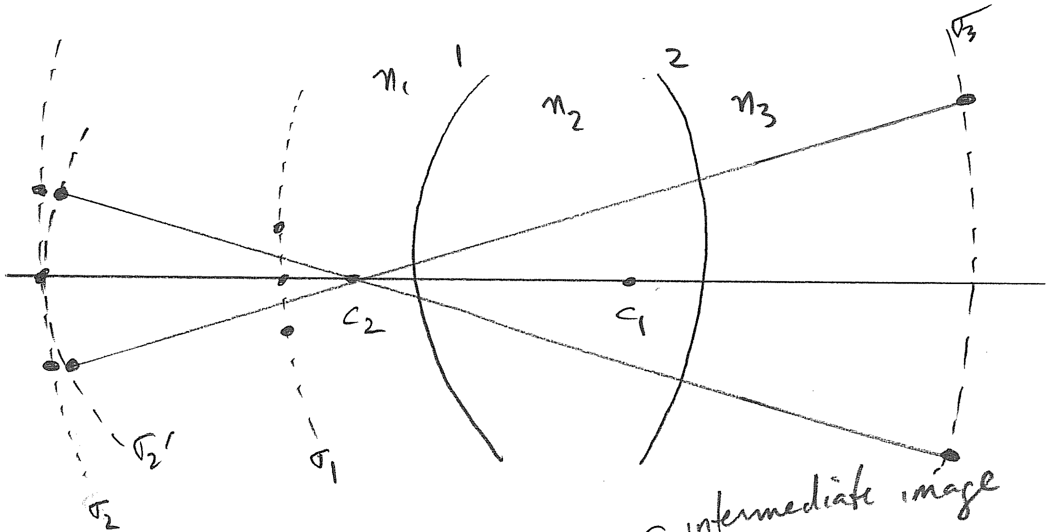
$\boxed{R' = R - s_2}$ and has size $h_2 = h_1 M = h_1 \left[\frac{n_1 s_2}{n_2 s_1}\right]$

~~we then approximate curved intermediate image surface σ_i as σ_i' that has the radius of curvature that is perfectly imaged by sphericsurface 2.~~

$$\cancel{M = M_1 M_2 = \frac{-n_1 s_i}{n_i s_1} \cdot \frac{n_2 s_2}{n_2 s_i} = \frac{s_2}{s_0}}$$

Now we want to start thinking about adding a second refracting surface. ~~The trick is to approximate~~

Consider a real spherical object surface that creates a virtual spherical image surface after surface 1 does imaging



intermediate image

We now approximate spherical surface σ_2 (with radius of curvature $R_{\sigma_2} = -s_2 + R_1$) as surface σ_2' that has the radius of curvature that is perfectly imaged by surface 2. $\left(R_{\sigma_2'} = d + R_2 - s_2\right)$.

Spherical surface σ_2' is imaged by surface 2 to a final spherical image surface σ_3 according to:

$$\frac{n_2}{\underbrace{d - s_2}_{s_2'}} + \frac{n_3}{s_3} = \frac{n_3 - n_2}{R_2} \qquad M_2 = \frac{-n_2 s_3}{n_3 s_2'}$$

mag associated w imaging from surface 2

$$M_{TOTAL} = M_1 M_2 = \left[\frac{n_1 s_2}{n_2 s_1}\right]\left[-\frac{n_2 s_3}{n_3 s_2'}\right]$$

$s_2' = s_2$ when $d \longrightarrow 0$

$$M = \frac{n_1}{n_3}\frac{s_3}{s_1}$$

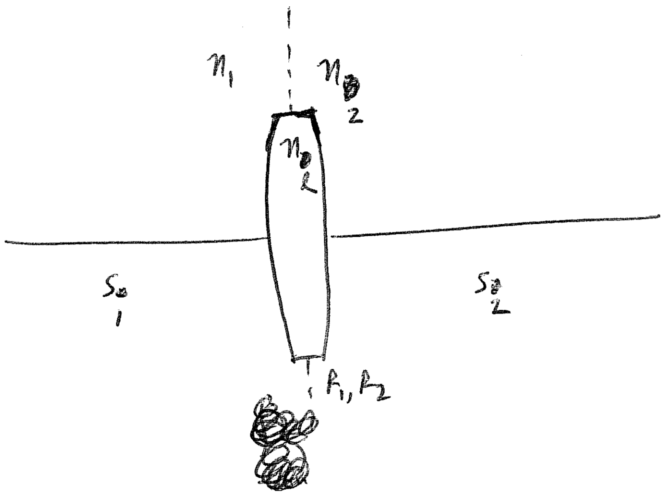
$s_3 =$ distance from surface 2 to σ_3

$s_1 =$ distance from surface 1 to σ_1

Then we go so far as to say curved surfaces σ_3 & σ_1 can be approximated as planar (in paraxial approx)

Then we go so far as to say all curved surfaces are actually planar.

Now we can start thinking about <u>LENSES</u>

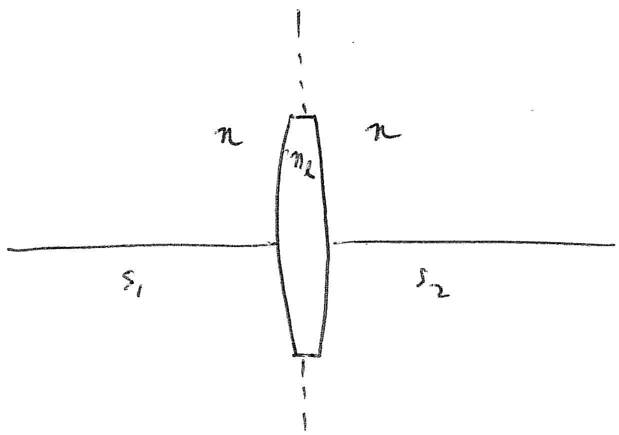


where

$$\boxed{M \cong \frac{n_1}{n_2}\cdot\frac{s_2}{s_1}}$$

$$\boxed{\frac{n_1}{s_1}+\frac{n_2}{s_2} = n_\ell\left(\frac{1}{R_1}-\frac{1}{R_2}\right) - \left(\frac{n_1}{R_1}-\frac{n_2}{R_2}\right)}$$

IN GENERAL
(neglecting lens thickness)



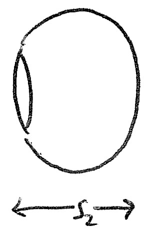
$$\boxed{M \cong \frac{s_2}{s_1}}$$

$$\frac{1}{s_1}+\frac{1}{s_2} = \underbrace{\frac{n_\ell - n}{n}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}_{\frac{1}{f_n}}$$

f_n = focal length "in" material with index n.

OPTICAL SYSTEMS:

(1) Human Eye - Unique system because s_2 (image dist) is always fixed at ~ 1" or 2.5 cm.

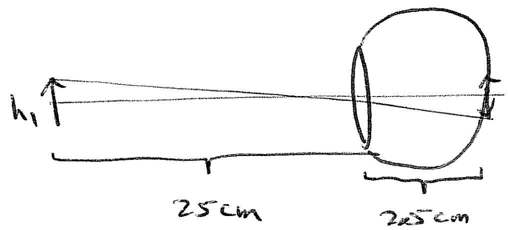


As you move objects closer to eye the imaging equation always holds so

what is happening? Your muscles actually "squish" (or "relax") your eye changing the surface curvature of the lens

so $\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$ holds (← this is changing w/ s_1)

NEAR POINT: The smallest s_1 you can have that your muscles can actually accommodate for. $\cong$ 25 cm in most people.



So $M_{MAX} = \frac{s_2}{s_1} = \frac{2.5\text{ cm}}{25\text{ cm}} \approx \frac{1}{10}$

This is the max magnification the unaided human eye can have. Object height = $h_1 M$.

THE "EYEPIECE" → A lens placed ~~right~~ in front of your eye; real object is placed inside it's focus & an enlarged virtual image of the real object is created OUTSIDE the near point so your eye can accomodate and actually image this enlarged virtual object to your retina.

How big is image on back of your eye?

There are two "stages" now:

(1) Eyepiece images real object h_1 to virtual image w/ height h_2 ~~just~~ somewhere outside near point of eye

(2) Eye images virtual object (height h_2) ~~to~~ to retina with a magnification less than M_{max}

So what is height of image on EYE?

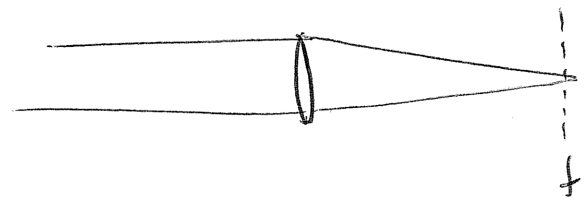
$$h_{eye} = h_o M_1 \cdot M_2$$

Although M_2 ~~is~~ $< M_{max}$, M_1 is bigger by a larger fraction than M_2 is decreased so h_{eye} is bigger than it could have been ~~as~~ with the eye alone and the object at the nearpoint of the

TELESCOPE: (Keplerian design)

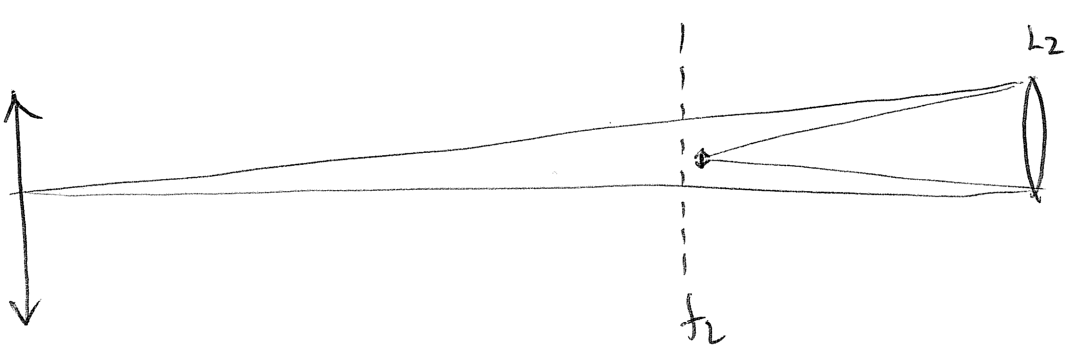
Two lenses separated by the sum of their focal lengths.

(1) Object is very far away from first lens thus it creates a tiny (M = small) image just outside of its focal length & just inside the focal point of the second lens.



$$M_{STAGE\,1} = -\frac{S_i}{S_o} \approx -\frac{f_1}{\text{BIG \#}}$$ and it is negative

(2) Now a <u>really tiny</u> object is placed just inside L_2's focus so L_2 creates a virtual image of this small real ~~fig~~ image at a huge negative distance with a huge positive magnification so S_i is close to $-\infty$



$$M_{STAGE\,2} = \frac{S_i}{S_o} = \frac{\text{BIG NEG \#}}{f_2}$$; $M_{TOTAL} =$ [illegible] #

$$m_{TOTAL} = \frac{-f_1}{BIG\#} \cdot \frac{-BIG\ NEG\#}{f_2} \cong \frac{-f_1}{f_2}$$

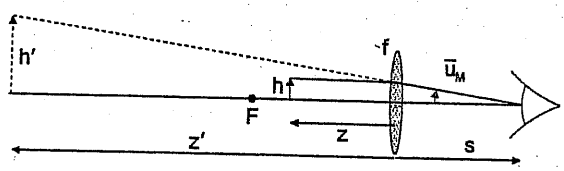
so the virtual object looks like it originated in the same place as the real object (which is just really far from the observer) and it is larger by a factor of $\frac{f_1}{f_2}$. This is how a telescope works → pretty simple, huh.

NOTE: s_{o_1} is never truly ∞
s_{i_2} is never truly $-\infty$ so

to get real sizes need to be careful to do the actual imaging of each lens separately.

Magnifiers

As an object is brought closer to the eye, the size of the image on the retina increases and the object appears larger. The largest image magnification possible with the unaided eye occurs when the object is placed at the near point of the eye, by convention 250 mm or 10 in from the eye. A magnifier is a single lens that provides an enlarged erect virtual image of a nearby object for visual observation. The object must be placed inside the front focal point of the magnifier.



The magnifying power MP is defined as (stop at the eye):

$$MP = \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}}$$

$$MP = \frac{\bar{u}_M}{\bar{u}_U} \qquad d_{NP} = -250 \text{ mm}$$ *near point distance*

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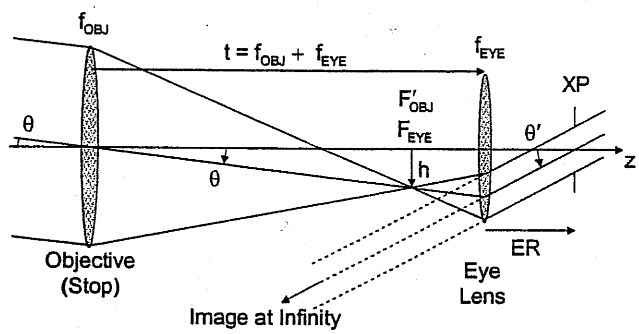
Eye is usually the stop, so we can call this ray the chief ray.

Keplerian Telescope

A Keplerian telescope or astronomical telescope consists of two positive lenses separated by the sum of the focal lengths. The system stop is usually at or near the objective lens.



This telescope can be considered to be a combination of an objective plus a magnifier. The objective creates an aerial image (a real image in the air) at the common focal point that is magnified by the eye lens and presented to the relaxed eye at infinity.

$$h = \theta f_{OBJ} \qquad h = -\theta' f_{EYE} \qquad MP = \frac{\theta'}{\theta} = -\frac{f_{OBJ}}{f_{EYE}}$$

The image presented to the eye is inverted and reverted (rotated by 180° or "upside down"). The MP of a Keplerian telescope is negative.

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Exit Pupil and Eye Relief

The XP should be made larger or smaller than the pupil of the eye so that vignetting does not occur with head or eye motion. This compensates for eye rotation as the rotation point of the eye is not at the EP of the eye. The pupil translates with eye rotation.

A close match of the instrument XP and the eye EP requires precise alignment of the two pupils. Small displacements will change the light level in the image. This is true even if the eye is at the XP.

The human eye pupil diameter varies from 2-8 mm, with a diameter of about 4 mm under ordinary lighting conditions.

When the XP of the instrument overfills the EP of the eye, the eye becomes the system stop. Larger instruments tend to have large XPs, while compact instruments may have small XP diameters (1-1.5 mm).

Sufficient eye relief should be provided to allow the eye to access the XP. Hand-held instruments should have 15-20 mm of eye relief. Microscopes may have as little as 2-3 mm of eye relief. Other systems should have a very long eye relief. For example, a riflescope needs a large ER to avoid problems due to kickback.

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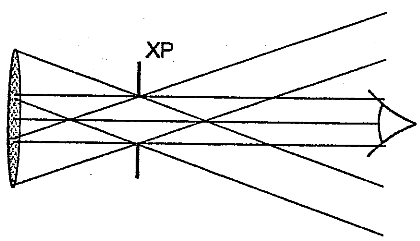
Exit Pupil and The Eye

The EP of the eye should be placed at the XP of the telescope to properly couple the two optical systems. If the eye is not at the XP, vignetting can result:

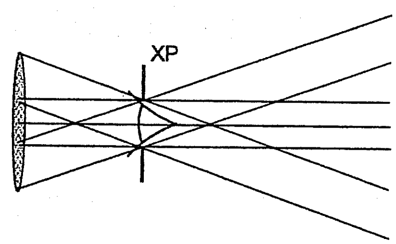
Vignetting occuring @ the eye lens



Ray bundles are shown for different FOVs.

The eye will see only on-axis (or near-axis) object points.

If the eye is displaced laterally, portions of the off-axis field are seen.



When the eye is in the XP, the entire FOV of the telescope is seen.

The eye can rotate to look around within the FOV.

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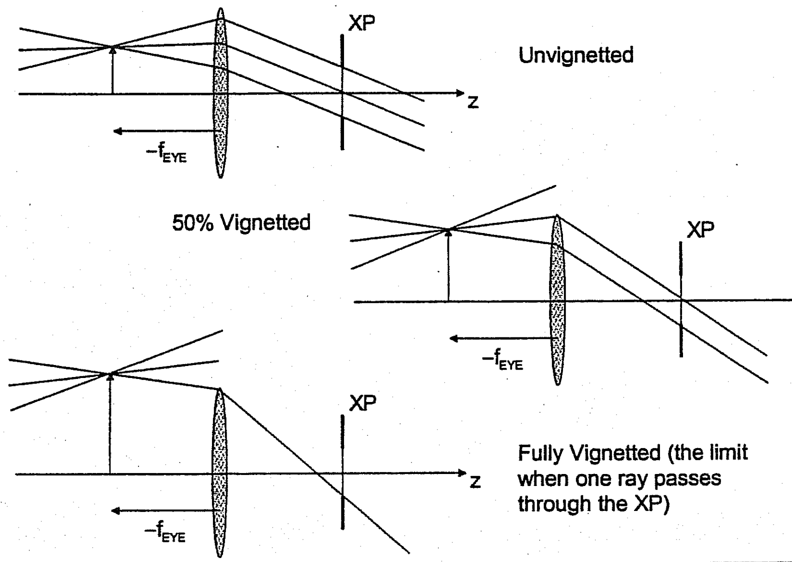
Telescopes – Field of View

The FOV of the Keplerian telescope is limited by vignetting at the eye lens. As the FOV or intermediate image height increases, the ray bundle is clipped by the eye lens.



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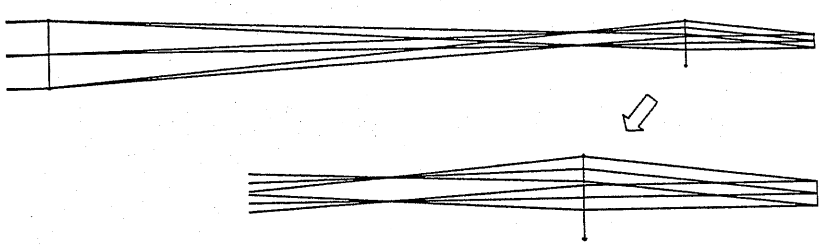
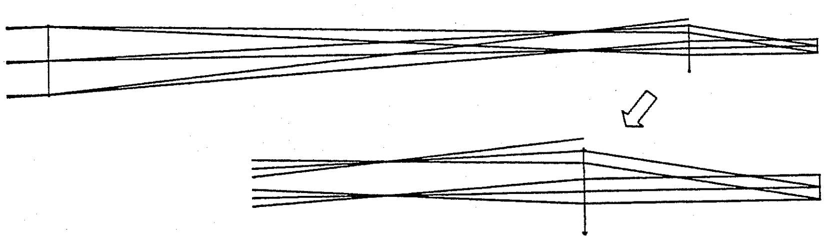
Vignetting Example – 5X Keplerian

Unvignetted: $\bar{U} = 1.34°$



Half Vignetted: $\bar{U} = 1.91°$



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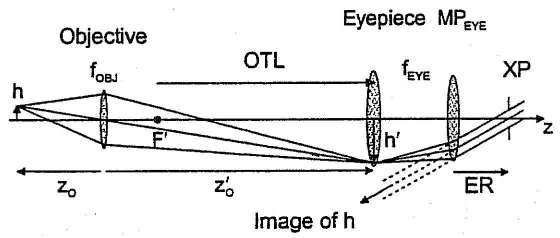
Microscopes

The compound microscope is a sophisticated magnifier which presents an enlarged image of nearby object to the eye. It consists of an objective plus an eyepiece or ocular.



The visual magnification is the product of the lateral magnification of the objective and the MP of the eyepiece.

$$m_{OBJ} = \frac{z'_O}{z_O} \qquad MP_{EYE} = \frac{250\text{mm}}{f_{EYE}}$$

$$m_V = m_{OBJ} MP_{EYE} = \frac{z'_O}{z_O}\frac{250\text{mm}}{f_{EYE}}$$

Note that the visual magnification is negative (z_O is negative).

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Optical Tube Length and NA

The optical tube length OTL of a microscope is defined as the distance from the rear focal point of the objective to the front focal point of the eyepiece (intermediate image). Standard values for the OTL are 160 mm and 215 mm. A relaxed eye (image at infinity) is assumed. The OTL is a Newtonian image distance:

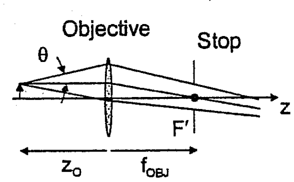
$$m_{OBJ} = -\frac{OTL}{f_{OBJ}} \qquad \frac{z'}{f_E} = -m$$

Newtonian Equation

$$m_V = -\frac{OTL}{f_{OBJ}}\frac{250\text{mm}}{f_{EYE}}$$

The NA of a microscope objective is defined in object space by the half-angle of the accepted input ray bundle. Along with the objective magnification, the NA is inscribed on the objective barrel.

Microscope objectives are often telecentric in object space. The stop is placed at the rear focal point of the objective so that the magnification does not change with object defocus.



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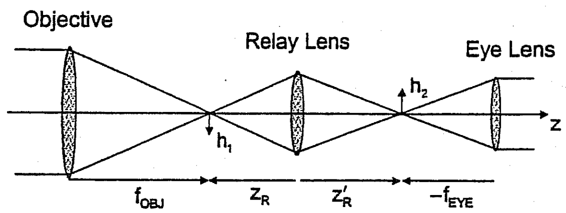
14-3

Relay Lens

A relay lens inserted between the objective and the eye lens of the Keplerian telescope can correct the image orientation. The intermediate image formed by the objective is relayed to a second intermediate image. The second intermediate image is placed at the front focal point of the eyepiece. The re-imaging causes an image rotation of 180°, correcting the image orientation.



The net MP of the relayed Keplerian telescope is positive. The system MP equals the product of the magnification of the relay and the MP of the original Keplerian telescope.

$$m_R = \frac{z'_R}{z_R} \qquad MP = m_R MP_K = -\frac{z'_R}{z_R}\frac{f_{OBJ}}{f_{EYE}}$$

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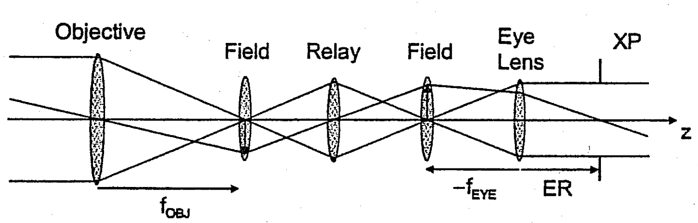
Relay Lenses and Field Lenses

Field lenses can also be added at the intermediate images. A common arrangement is for each field lens to image the pupil into the following relay lens. All of the light collected by the objective is transferred down the optical system. The final field lens is part of the eyepiece.



The first field lens images the objective aperture into the relay aperture, transferring the entrance pupil to the relay lens. If the relay lens diameter is larger than this image, all of the light from an object point collected by the objective will be transferred to the second image. For an optical system used with the eye, the final field lens will not image the relay lens aperture into the eye lens, but rather to the exit pupil with the appropriate eye relief.

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