rays and wavefronts, derive the law of reflection and Snell's law. The ray diagram of Fig. 4.54 should be helpful.

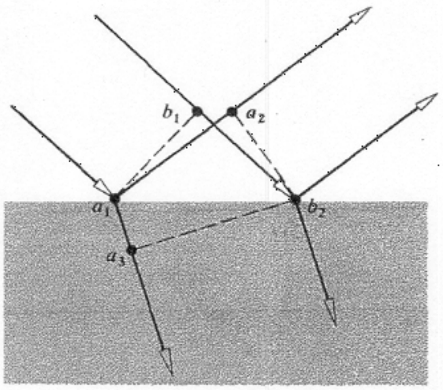


Figure 4.54

4.13 Starting with Snell's law, prove that the vector refraction equation has the form

$$n_t\hat{\mathbf{k}}_t - n_i\hat{\mathbf{k}}_i = (n_t \cos\theta_t - n_i \cos\theta_i)\hat{\mathbf{u}}_n. \qquad [4.8]$$

4.14 Derive a vector expression equivalent to the law of reflection. As before, let the normal go from the incident to the transmitting medium, even though it obviously doesn't really matter.

4.15 In the case of reflection from a planar surface, use Fermat's principle to prove that the incident and reflected rays share a common plane with the normal $\hat{\mathbf{u}}_n$, namely, the plane of incidence.

4.16* Derive the law of reflection, $\theta_i = \theta_r$, by using the calculus to minimize the transit time, as required by Fermat's principle.

4.17* According to the mathematician Hermann Schwarz, there is one triangle that can be inscribed within an acute triangle such that it has a minimal perimeter. Using two planar mirrors, a laserbeam, and Fermat's principle, explain how you can show that this inscribed triangle has its vertices at the points where the altitudes of the acute triangle intersect its corresponding sides.

4.18 Show analytically that a beam entering a planar transparent plate, as in Fig. 4.55, emerges parallel to its initial direction. Derive an expression for the lateral displacement of the beam. Incidentally, the incoming and outgoing rays would be parallel even for a stack of plates of different material.

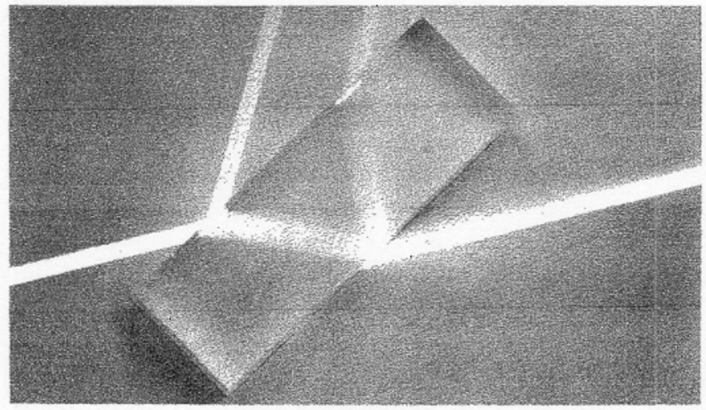


Figure 4.55 (Source unknown.)

4.19* Show that the two rays that enter the system in Fig. 4.56 parallel to each other emerge from it being parallel.

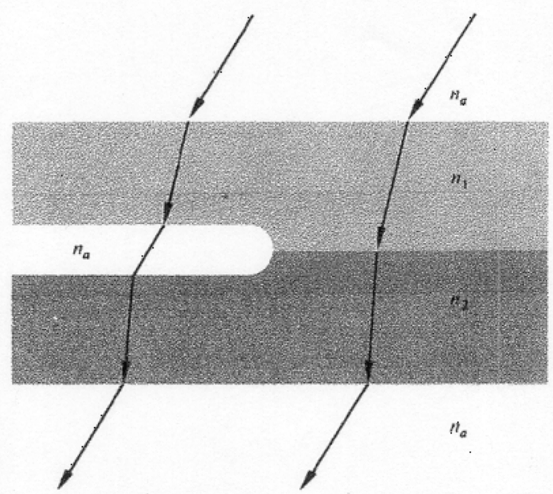
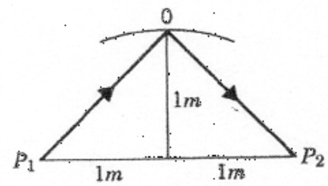


Figure 4.56

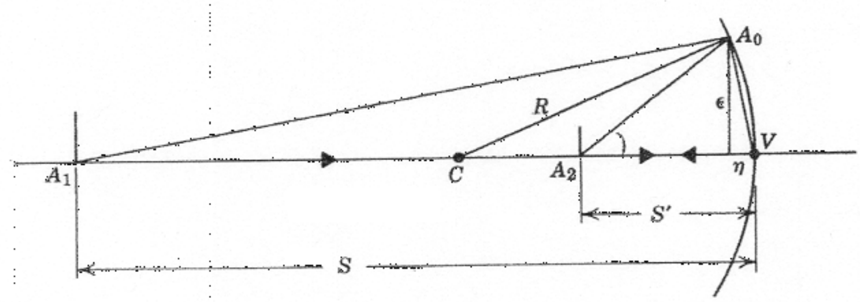
4.20 Discuss the results of Problem 4.18 in the light of Fermat's principle, that is, how does the relative index n_{21} affect things? To see the lateral displacement, look

Klein_Hecht_Probs.pdf

2.2.5 You want to place a toroidal mirror at O to obtain approximate imaging of P_1 onto P_2. What should be the values of the two principal radii of curvature? (One principal curve lies in the plane of the figure, and the other lies in a perpendicular plane.)



2.2.6 Consider reflection from a spherical surface with center of curvature at C, radius of curvature R. The surface cuts the plane of the paper in the arc A_0V. A_1VA_2 is an exact ray, and $A_1A_0A_2$ is a virtual ray γ.



a. Show that the condition for approximate image formation, namely,

$$\text{O.P.L.}(\gamma) - \text{O.P.L.(ray)} \sim \epsilon^3 \text{ (or higher power)} \quad (1)$$

is satisfied provided that

$$\frac{1}{S} + \frac{1}{S'} = \frac{2}{R} \quad (2)$$

and find the expression for the coefficient of the leading term on the RHS of Eq. (1).

b. When $|A_2V| < S'$, with S' satisfying Eq. (2), is the optical path length of the ray A_1VA_2 a maximum or a minimum?

1.33 to 1.32. Why is the change in n so much smaller than the corresponding change in K_e?

3.26 Show that for substances of low density, such as gases, which have a single resonant frequency ω_0, the index of refraction is given by

$$n \approx 1 + \frac{Nq_e^2}{2\epsilon_0 m_e(\omega_0^2 - \omega^2)}.$$

3.27* *In the next chapter, Eq. (4.47), we'll see that a substance reflects radiant energy appreciably when its index differs most from the medium in which it is embedded.*

a) The dielectric constant of ice measured at microwave frequencies is roughly 1, whereas that for water is about 80 times greater—why?
b) How is it that a radar beam easily passes through ice but is considerably reflected when encountering a dense rain?

3.28 The equation for a driven damped oscillator is

$$m_e\ddot{x} + m_e\gamma\dot{x} + m_e\omega_0^2 x = q_e E(t).$$

a) Explain the significance of each term.
b) Let $E = E_0 e^{i\omega t}$ and $x = x_0 e^{i(\omega t - \alpha)}$, where E_0 and x_0 are real quantities. Substitute into the above expression and show that

$$x_0 = \frac{q_e E_0}{m_e} \frac{1}{[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2]^{1/2}}.$$

c) Derive an expression for the phase lag, α, and discuss how α varies as ω goes from $\omega \ll \omega_0$ to $\omega = \omega_0$ to $\omega \gg \omega_0$.

3.29 Fuchsin is a strong (aniline) dye, which in solution with alcohol has a deep red color. It appears red because it absorbs the green component of the spectrum. (As you might expect, the surfaces of crystals of fuchsin reflect green light rather strongly.) Imagine that you have a thin-walled hollow prism filled with this solution. What will the spectrum look like for incident white light? By the way, anomalous dispersion was first observed in about 1840 by Fox Talbot, and the effect was christened in 1862 by Le Roux. His work was promptly forgotten, only to be rediscovered eight years later by C. Christiansen.

3.30 Imagine that we have a nonabsorbing glass plate of index n and thickness Δy, which stands between a source S and an observer P.

a) If the unobstructed wave (without the plate present) is $E_u = E_0 \exp i\omega(t - y/c)$, show that with the plate in place the observer sees a wave

$$E_p = E_0 \exp i\omega[t - (n-1)\,\Delta y/c - y/c].$$

b) Show that if either $n \approx 1$ or Δy is very small, then

$$E_p = E_u + \frac{\omega(n-1)\,\Delta y}{c} E_u e^{-i\pi/2}.$$

The second term on the right may be envisioned as the field arising from the oscillators in the plate.

3.31* Take Eq. (3.70) and check out the units to make sure that they agree on both sides.

3.32 The resonant frequency of lead glass is in the UV fairly near the visible, whereas that for fused silica is far into the UV. Use the dispersion equation to make a rough sketch of n versus ω for the visible region of the spectrum.

3.33 Augustin Louis Cauchy (1789–1857) determined an empirical equation for $n(\lambda)$ for substances that are transparent in the visible. His expression corresponded to the power series relation

$$n = C_1 + C_2/\lambda^2 + C_3/\lambda^4 + \cdots,$$

where the C's are all constants. In light of Fig. 3.27, what is the physical significance of C_1?

3.34 Referring to the previous problem, realize that there is a region between each pair of absorption bands for which the Cauchy equation (with a new set of constants) works fairly well. Examine Fig. 3.26: what can you say about the various values of C_1 as ω decreases across the whole spectrum? Dropping all but the first two terms, use Fig. 3.27 to determine approximate values for C_1 and C_2 for borosilicate crown glass in the visible.

and since $\lambda_i = \lambda_r$, $\theta_i = \theta_r$. It is interesting to note that

$$n_{ti} = \frac{p_t}{p_i}, \tag{4.93}$$

and so if $n_{ti} > 1$, $p_t > p_i$. Experiments dating back as far as 1850, to those of Foucault, have shown that when $n_{ti} > 1$ the speed of propagation is actually reduced in the transmitting media, even though the momentum apparently increases.†

Do keep in mind that we have been dealing with a very simple representation that leaves much to be desired. For example, it says nothing about the atomic structure of the media or about the probability that a photon will traverse a given path. Even though this treatment is obviously simplistic, it is appealing pedagogically (see Chapter 13).

† This suggests an increase in the photon's effective mass. See F. R. Tangherlini, "On Snell's Law and the Gravitational Deflection of Light." *Am. J. Phys.* **36**, 1001 (1968). Take a *cautious* look at R. A. Houstoun, "Nature of Light." *J. Opt. Soc. Am.* **55**, 1186 (1965).

PROBLEMS

4.1 Calculate the transmission angle for a ray incident in air at 30° on a block of crown glass ($n_g = 1.52$).

4.2* A ray of yellow light from a sodium discharge lamp falls on the surface of a diamond in air at 45°. If at that frequency $n_d = 2.42$, compute the angular deviation suffered upon transmission.

4.3 Use Huygens's construction to create a wavefront diagram showing the form a spherical wave will have after reflection from a planar surface, as in the ripple tank photos of Fig. 4.50. Draw the ray diagram as well.

4.4* Given an interface between water ($n_w = 1.33$) and glass ($n_g = 1.50$), compute the transmission angle for a beam incident in the water at 45°. If the transmitted beam is reversed so that it impinges on the interface, show that $\theta_t = 45°$.

4.5 A beam of 12-cm planar microwaves strikes the surface of a dielectric at 45°. If $n_{ti} = \frac{4}{3}$, compute (a) the wavelength in the transmitting medium, and (b) the angle θ_t.

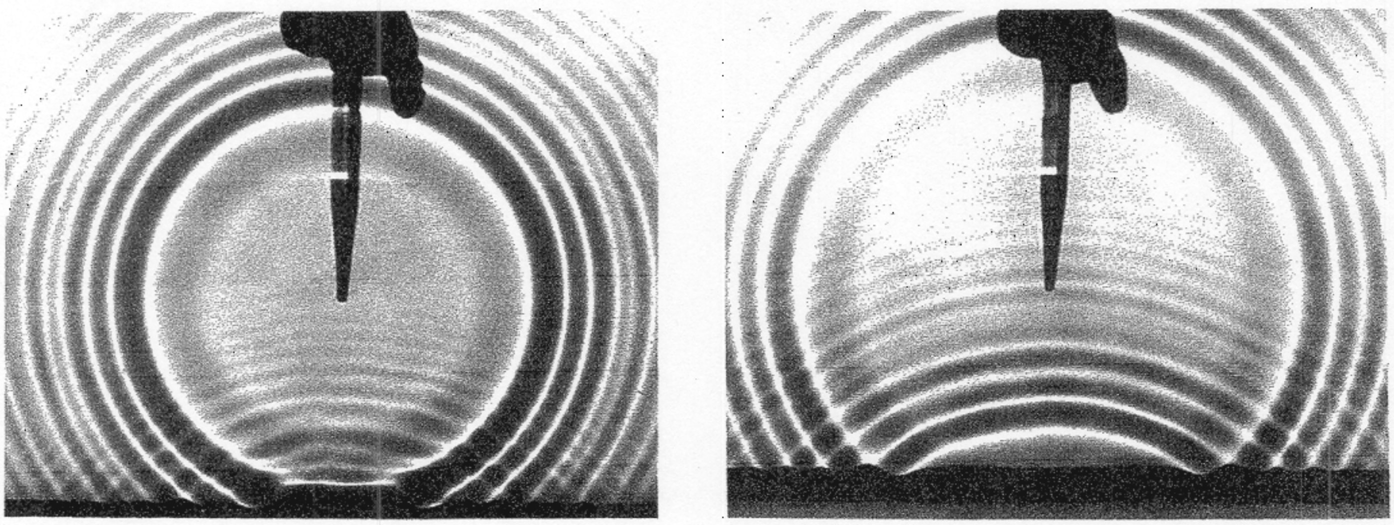


Figure 4.50 (Photos courtesy *Physics*, Boston, D. C. Heath & Co., 1960.)

at a broad source through a thick piece of glass ($\approx\frac{1}{4}$ inch) or a stack (four will do) of microscope slides *held at an angle.* There will be an obvious shift between the region of the source seen directly and the region viewed through the glass.

4.21 Suppose a lightwave that is linearly polarized in the plane of incidence impinges at 30° on a crown-glass ($n_g = 1.52$) plate in air. Compute the appropriate amplitude reflection and transmission coefficients at the interface. Compare your results with Fig. 4.22.

4.22 Show that even in the nonstatic case the tangential component of the electric field intensity **E** is continuous across an interface. [*Hint:* using Fig. 4.57 and Eq. (3.5), shrink sides *FB* and *CD*, thereby letting the area bounded go to zero.]

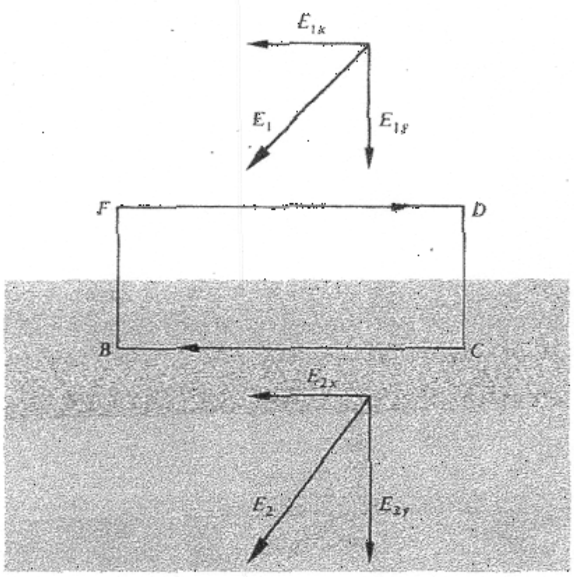


Figure 4.57

4.23 Derive Eqs. (4.42) through (4.45) for $r_\perp, r_\parallel, t_\perp$, and $t_\parallel$.

4.24 Prove that

$$t_\perp + (-r_\perp) = 1 \qquad [4.49]$$

for all θ_i, first from the boundary conditions and then from the Fresnel equations.

4.25* Verify that

$$t_\perp + (-r_\perp) = 1 \qquad [4.4$$

for $\theta_i = 30°$ at a crown glass and air interface (n_{ti} 1.52).

4.26* Calculate the critical angle beyond which ther is total internal reflection at an air–glass ($n_g = 1.5$) inte face. Compare this result with that of Problem 4.8.

4.27 Derive an expression for the speed of the evanes cent wave in the case of internal reflection. Write it i terms of c, n_i, and θ_i.

4.28 Light having a vacuum wavelength of 600 nm traveling in a glass ($n_g = 1.50$) block, is incident at 45 on a glass–air interface. It is then totally internall reflected. Determine the distance into the air at whicl the amplitude of the evanescent wave has dropped tc a value of $1/e$ of its maximum value at the interface.

4.29 Figure 4.58 shows a laserbeam incident on a we piece of filter paper atop a sheet of glass whose index of refraction is to be measured—the photograph shows the resulting light pattern. Explain what is happening and derive an expression for n_i in terms of R and d.

4.30 Consider the common mirage associated with an inhomogeneous distribution of air situated above a warm roadway. Envision the bending of the rays as if it were instead a problem in total internal reflection. If an observer, at whose head $n_a = 1.00029$, sees an apparent wet spot at $\theta_i \geq 88.7°$ down the road, find the index of the air immediately above the road.

4.31* Use the Fresnel equations to prove that light incident at $\theta_p = \frac{1}{2}\pi - \theta_t$ results in a reflected beam that is indeed polarized.

4.32 Show that $\tan\theta_p = n_t/n_i$ and calculate the polarization angle for external incidence on a plate of crown glass ($n_g = 1.52$) in air.

4.33* Beginning with Eq. (4.38), show that for

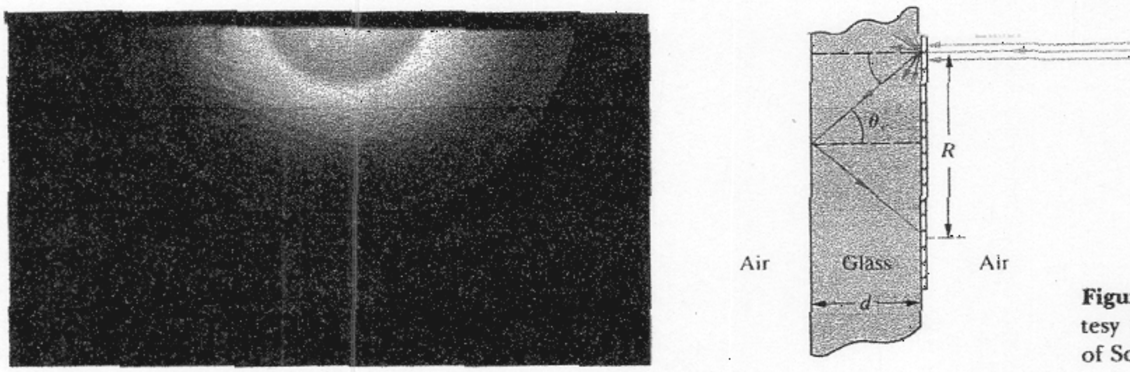


Figure 4.58 (Photo and diagram courtesy S. Reich, The Weizmann Institute of Science, Israel.)

two dielectric media, in general $\tan\theta_p = [\epsilon_t(\epsilon_t\mu_i - \epsilon_i\mu_t)/\epsilon_i(\epsilon_t\mu_t - \epsilon_i\mu_i)]^{1/2}$.

4.34 Show that the polarization angles for internal and external reflection at a given interface are complementary, that is, $\theta_p + \theta'_p = 90°$ (see Problem 4.32).

4.35 It is often useful to work with the *azimuthal angle* γ, which is defined as the angle between the plane of vibration and the plane of incidence. Thus for linearly polarized light,

$$\tan\gamma_i = [E_{0i}]_\perp/[E_{0i}]_\parallel \qquad (4.94)$$

$$\tan\gamma_t = [E_{0t}]_\perp/[E_{0t}]_\parallel \qquad (4.95)$$

and

$$\tan\gamma_r = [E_{0r}]_\perp/[E_{0r}]_\parallel. \qquad (4.96)$$

Figure 4.59 is a plot of γ_r versus θ_i for internal and external reflection at an air–glass interface ($n_{ga} = 1.51$), where $\gamma_i = 45°$. Verify a few of the points on the curves and in addition show that

$$\tan\gamma_r = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_t)}\tan\gamma_i. \qquad (4.97)$$

4.36* Making use of the definitions of the azimuthal angles in Problem 4.35, show that

$$R = R_\parallel \cos^2\gamma_i + R_\perp \sin^2\gamma_i \qquad (4.98)$$

and

$$T = T_\parallel \cos^2\gamma_i + T_\perp \sin^2\gamma_i. \qquad (4.99)$$

4.37 Make a sketch of $R_\perp$ and $R_\parallel$ for $n_i = 1.5$ and $n_t = 1$ (i.e., internal reflection).

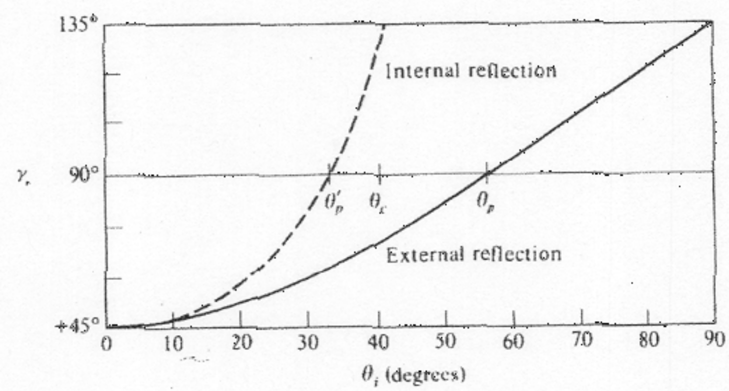


Figure 4.59