UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Scences
EEECS119, Spring 2008

Spring 2008

Problem Set No. 4

Problem Number one) Reflectance at near normal incidence
Problem 4.47

Problem Number two) The Oscillator model for dielectrics (including metals)
a) problem 4.4
b) From x_o deduce an expression for the polarization per unit volume if the density of contributing microscopic oscillators is N.
c) From b) deduce an expression for the dielectric coefficient.
d) For a metal for which the resonant frequency is zero (free electrons), why is the reflectivity high up to the frequency $\omega_p = \sqrt{Ne^2/(m\epsilon_o}$?
e) Estimate ω_p for aluminium?
f) What is the Hagen-Rubens region? Determine an approximate expression for the reflectivity for this region. (It is slightly less than 100 %)

Problem Number three) Basic resolution and uncertainty
a) If an aperture of radius a is illuminated by a plane wave, argue from the uncertainty relation that the diffraction angle of the transmitted beam is approximately λ/a
b) For a telescope why is the entrance pupil the circular boundary of the objective lens? (additional ref Born and Wolf page 463)
c) If the radius of the objective is a, what is the approximate angular resolution?

Problem Number four) Prism
a) Show by direct differentiation of Eq (5.53) with respect to θ_{i1} that the minimum deviation of the transmitted beam occurs when the ray traverses the prism parallel to the base.

Problem Number five) Basic Reflection of of a Diffraction Grating
A boundary between two dielectrics having indicies of refraction n_1 and n_2 has a periodic square wave structure etched into it.
a) generalize Snell's equation's for reflection and transmission angles to include the influence

of the grating structure.
b) Show that the wavelength "resolution" is given by

$$\delta\lambda = \lambda/N$$

where N is the number of grating periods illuminated.
c) Show that the dispersion is $\frac{d\theta}{d\lambda} = \frac{tan\theta}{\lambda}$

Problem Number six) Grin Lens
a) From the wave equation show that the ABCD law for Gaussian beam propagation in graded index media is indeed correct.
b)A cylindrical grin lens has $\epsilon_2/\epsilon_o = 100\ cm^{-2}$. The thickness is .1 cm. It is used to focus a Gaussian beam of radius 25 microns.
How far away from the output surface is the focal region and what is the focussed beam radius?

Problem Number seven (Optional))
a) In an optical system, the difference between a field E(t) over a time S, say $E(t)-E(t-S)$, is found to be f(t), periodic with period T and possessing a zero average. What (stable) solutions for E(t) are possible with the same period (T)? An example is S, a rational number times the round-trip time in a laser cavity (2 times the length / (speed of light)), and T, the period of pulses emitted by the cavity. Generally S and T are taken equal and f(t) is degenerate (zero). The pulses g(t) are exactly periodic and presumably stable. It is interesting to pose the more general question (non-zero f(t) and S not necessarily equal to T). It relates to frequency and time standards. This more general problem is famous (relating to the so-called KAM theorem) and has attracted the attention of many well-known mathematicians.