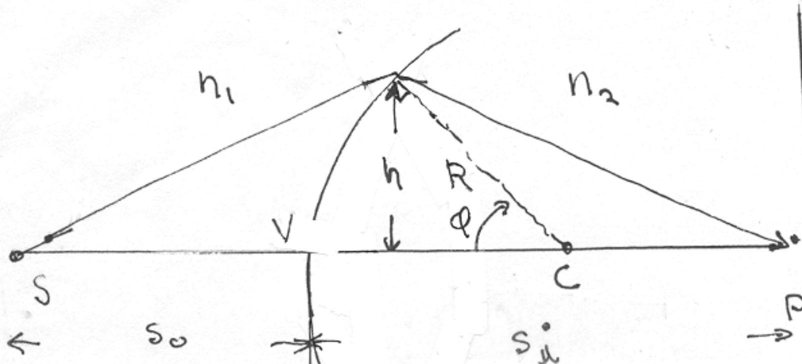
Ray propagation, the thick and thin lens, ABCD matricies

Consider a spherical wave emminating from S, a point source

$$\text{Field} = \mathrm{Re}\left(\frac{A}{r} e^{i\omega t - i\bar{k}\cdot\bar{r}} \right)$$

ray vector $\frac{k\,\hat{r}}{k} = \hat{r}$

We have shown $\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R}$



Signs For S_o, S_i (Hecht: 154)
S_o is positive (S_o to the left of V)
S_i is positive (S_i to the right of V)
$R > 0$ (C to right of V) (see page 154 of Hecht)

Lens formula for rays

$$\frac{n_1}{S_o} + \frac{n_2}{S_i} = \frac{n_2 - n_1}{R}$$

A, B, C, D matrix for rays

$\left(\frac{h}{S_o}\right) = r_o' =$ slope of o-ray

$-\left(\frac{h}{S_i}\right) = r_i' =$ slope of i-ray

$h = r_o =$ location from axis of o ray

$h = r_i =$ location i ray

$$n_1 r_o' - n_2 r_i' = \frac{n_2 - n_1}{R} r_o$$

$$r_i = r_o$$

$$r_i' = -\frac{n_2 - n_1}{n_2 R} r_o + \frac{n_1}{n_2} r_o'$$

Agrees with Yariv page 41 (since R > 0 here)

$$\begin{pmatrix} r_i \\ r_i' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & n_1/n_2 \end{pmatrix} \begin{pmatrix} r_o \\ r_o' \end{pmatrix} = \begin{pmatrix} A, & B \\ C, & D \end{pmatrix} \begin{pmatrix} r_o \\ r_o' \end{pmatrix}$$

In terms of curvature
$R(z) > 0$ if the center is at $z' < z$

$$\frac{n_1}{R_1} - \frac{n_2}{R_2} = \frac{n_2 - n_1}{R\,n_2} \quad \text{or}$$

In terms of usual notation

$$\frac{n_1}{q_1} - \frac{n_2}{q_2} = \frac{n_2 - n_1}{R\,n_2}; \quad q_1 = R_1, \; q_2 = R_2$$

(q will be the "Gaussian beam parameter but here it is just curvature R)

$$\frac{1}{q_2} = \frac{1}{q_1}\frac{n_1}{n_2} - \frac{n_2 - n_1}{n_2 R}$$

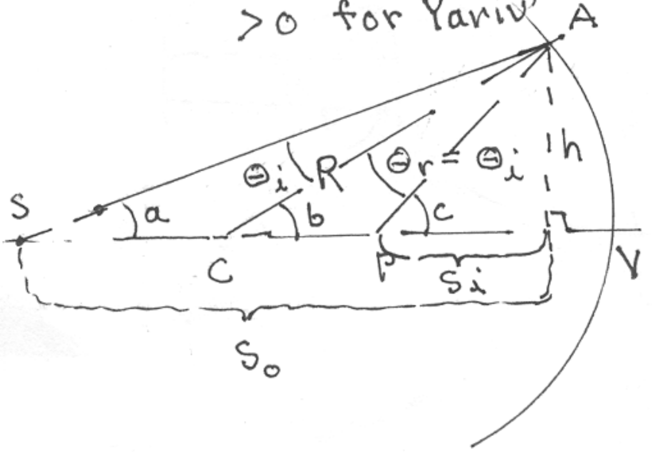
$$q_2 = \frac{q_1}{-\frac{n_2 - n_1}{n_1 R} q_1 + \frac{n_1}{n_2}}$$

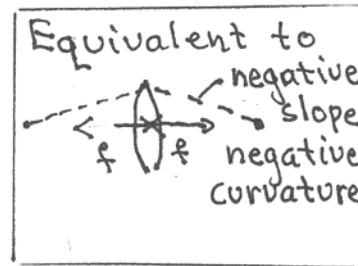
Observe $q = \dfrac{A q_1 + B}{C q_1 + D}$

2: The reflecting lens

Note $R<0$ for Hecht (C to left of V) but >0 for Yariv

page 182





$$b = a + \theta_i$$

$$c = b + \theta_i$$

$$a + c = 2b$$

$$a = \frac{h}{S_o} \;;\; b = \frac{h}{R} \;;\; c = \frac{h}{s_i}$$

Thus $\frac{1}{S_o} + \frac{1}{s_i} = \frac{2}{R} = \frac{1}{f}$ ← + for both Hecht & Yariv

↑ Yariv

$S_o = R_o \quad s_i = -R_i$

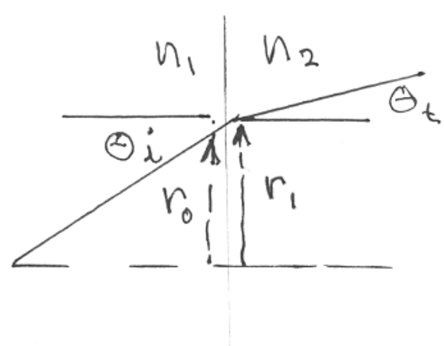
$$\therefore R_i = \frac{R_o}{-\frac{1}{f}R_o + 1} \;;\; \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

(page 41, Yariv)

Also for ray and ray slopes

$$\begin{pmatrix} r_i \\ r_i' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_o \\ r_o' \end{pmatrix}$$

Planar boundary



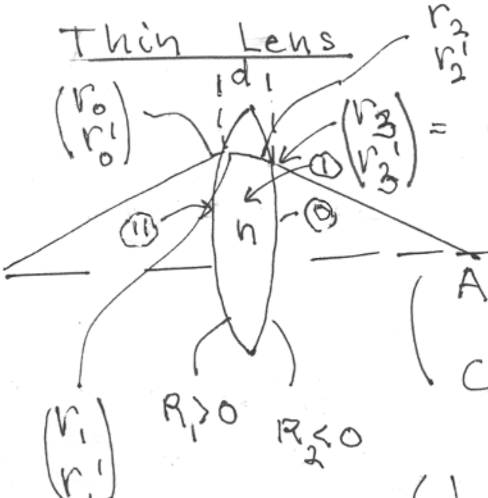
$$r_1 = r_o$$

$$n_2 \theta_t = n_1 \theta_i$$

$$r_1' = \frac{n_1}{n_2}\theta_i = \frac{n_1}{n_2} r_o'$$

$$\begin{pmatrix} A, & B \\ C, & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$$ (Yariv page 41)

Thin Lens



$$\begin{pmatrix} r_3 \\ r_3' \end{pmatrix} = \begin{pmatrix} A^0, & B^0 \\ C^0, & D^0 \end{pmatrix} \begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A^0, & B^0 \\ C^0, & D^0 \end{pmatrix} \begin{pmatrix} A', & B' \\ C', & D' \end{pmatrix} \begin{pmatrix} A'', & B'' \\ C'', & D'' \end{pmatrix} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix}$$

$$\begin{pmatrix} A & , B \\ C & , D \end{pmatrix} \underset{\substack{n_1 = n \\ n_2 = 1}}{=} \begin{pmatrix} 1 & 0 \\ -\frac{(1-n)}{R_2}, & n \end{pmatrix} \begin{pmatrix} 1, & d \approx 0 \\ 0, & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ -\frac{(n-1)}{nR_1}, & \frac{1}{n} \end{pmatrix}$$

$n_2 = n, n_1 = 1$

Assuming $d = 0$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{(1-n)}{R_2}, & n \end{pmatrix} \begin{pmatrix} 1 & , 0 \\ -\frac{n-1}{nR_1}, & \frac{1}{n} \end{pmatrix} = \begin{pmatrix} 1 & , 0 \\ -(n-1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right), & 1 \end{pmatrix}$$

But
$$\begin{pmatrix} 1 & 0 \\ -(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

From the A, B, C, D Law.

$$q_1 = \frac{q_0}{-1/f\, q_0 + 1}$$

$$\frac{1}{q_1} = \frac{1}{q_0}\left(-\frac{1}{f} q_0 + 1\right)$$

$$-\frac{1}{q_1} + \frac{1}{q_0} = \frac{1}{f} \qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\uparrow$$

$$\frac{1}{(S_1)} + \frac{1}{(S_0)} = \frac{1}{f} \qquad \lim_{S_0 \to \infty} S_i = f_i \qquad \lim_{S_i \to \infty} S_0 = f_0$$

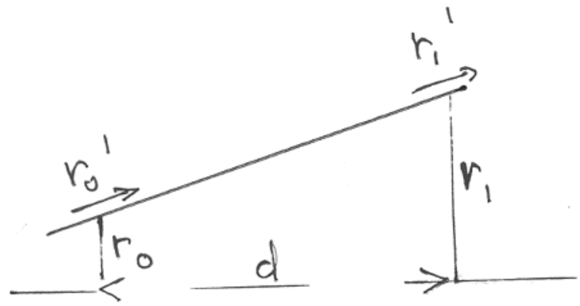
Sign Conventions for curved surfaces in Hecht (154)

S_o, f_o	+ left of V	- diverging beam
x_o	+ left of V	
S_i, f_i	+ right of V	- converging beam
x_i	+ right of V	
R	+ if C is right of V	- convex

For beam propagation + curvature is diverging - curvature is converging

Free space

ray



$$r_1 = r_0 + r_0' d$$

$$r_1' = r_0'$$

$$\begin{pmatrix} r_1 \\ r_1' \end{pmatrix} = \begin{pmatrix} 1 , & d \\ 0 , & 1 \end{pmatrix} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix}$$

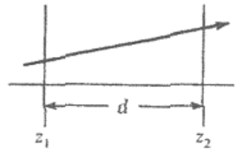
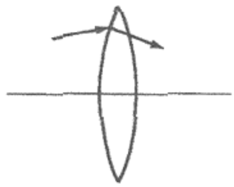
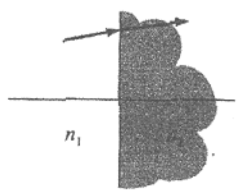
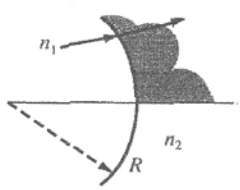
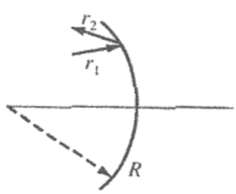
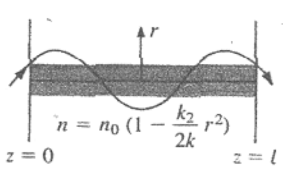
$$= \begin{pmatrix} A , & B \\ C , & D \end{pmatrix} \begin{pmatrix} r_0 \\ r_0' \end{pmatrix}$$

Gaussian parameter

$$q_1 = \frac{A q_0 + B}{C q_0 + D} = q_0 + d$$

Yariv,

Table 2-1 Ray Matrices for Some Common Optical Elements and Media

(1) Straight Section: Length d		$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
(2) Thin Lens: Focal length f ($f > 0$, converging; $f < 0$, diverging)		$\begin{bmatrix} 1 & 0 \\ \dfrac{-1}{f} & 1 \end{bmatrix}$
(3) Dielectric Interface: Refractive indices n_1, n_2		$\begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_2} \end{bmatrix}$
(4) Spherical Dielectric Interface: Radius R		$\begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_1}{n_2 R} & \dfrac{n_1}{n_2} \end{bmatrix}$
(5) Spherical Mirror: Radius of curvature R		$\begin{bmatrix} 1 & 0 \\ \dfrac{-2}{R} & 1 \end{bmatrix}$
(6) A medium with a quadratic index profile		$\begin{bmatrix} \cos\left(\sqrt{\dfrac{k_2}{k}}\, l\right) & \sqrt{\dfrac{k}{k_2}} \sin\left(\sqrt{\dfrac{k_2}{k}}\, l\right) \\ -\sqrt{\dfrac{k_2}{k}} \sin\left(\sqrt{\dfrac{k_2}{k}}\, l\right) & \cos\left(\sqrt{\dfrac{k_2}{k}}\, l\right) \end{bmatrix}$

See Chapt 6 of Hecht