The Propagation of Light ①

- To treat in detail need Maxwell's equations
- To treat quantum aspects the electric and magnetic fields become operators and the field is in fact defined by a wave function
- To begin with can circumvent these and introduce them when needed
- A very useful principle is that of <u>Huygen's</u> To define these we introdure a "wavefront"

Let a field component be $E(z,t)$

$$\bar{E}(z,t) = \frac{E_0}{2}\left(e^{i(kz-\omega t)} + c.c.\right)\hat{x}$$

$\perp$'r to $k\hat{z}$

$$= |E_0| \cos(\underbrace{kz - \omega t + \angle E}) \hat{x}$$

This is a plane wave because kz is independent of x and y, that is $kz - \omega t + \angle E = \text{const}$ in the planes given by $z = (\text{const} - \angle E + \omega t)/k$

The $\perp$'r to this plane is the direction of propagation (in fact it is given by $\nabla(kz - \omega t + \angle E) = k\hat{z}$

The unit vector $\hat{z}$ is an example of a <u>ray</u> vector, in this case $\hat{z}$

Generalization

$$\tilde{E}(z,t) = \frac{E_0}{2} e^{iS(x,y,z) - i\omega t}$$

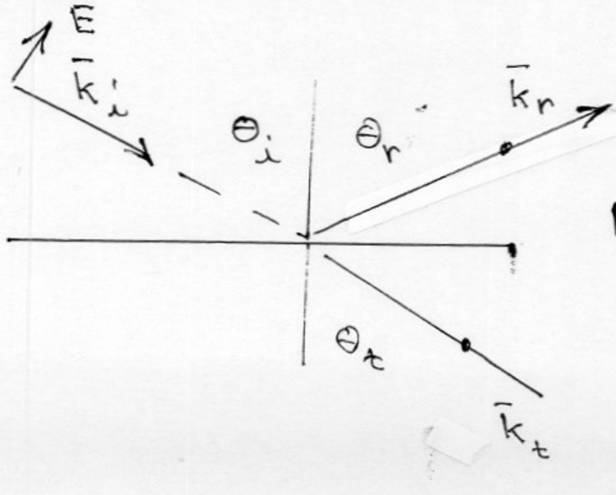
$\nabla S(x,y,z)$ is $\perp$'r to $S = \text{const}$ (wavefront)

$$\frac{\nabla S(x,y,z)}{|\nabla S(x,y,z)|} = \bar{S} \quad \text{the ray vector}$$

Huygen! - Every point on a primary wavefront serves as a source of spherical secondary wavelets such that the primary wavefront is the envelope of these wavelets at some given following time. Moreover the wavelets advance at the speed and frequency of the primary wave front. (For a spherical wave $E = E_0 \cos(kr - \omega t + \angle E)/r$).

This is in actuality the superposition principle in three dimensions. This is useful particularly for diffraction and refraction

Snell's Law

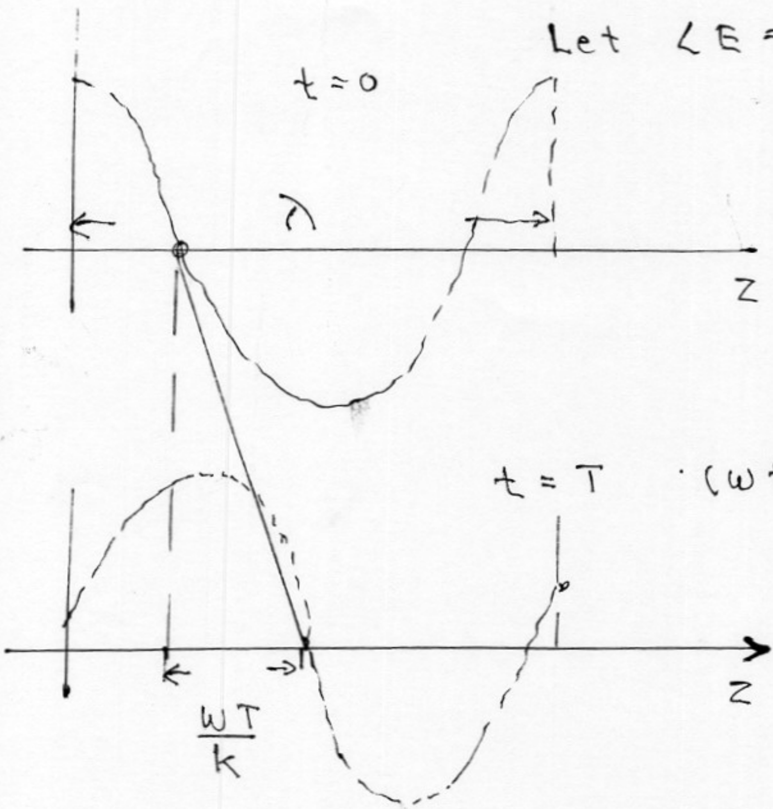


boundary

If wave speeds differ in the two media then a reflection generally occurs and $\Theta_t \neq \Theta_i$ (and in fact can be imaginary wave speed?

$$\cos(kz - \omega t + \angle E) = \text{const}$$

Let $\angle E = 0$



$k\lambda = 2\pi$

$(\omega t < \frac{\pi}{2})$

phase speed $= \frac{\omega T}{k} \cdot \frac{1}{T} = \frac{\omega}{k} = v_{ph}$

$\omega = 2\pi f = 2\pi \frac{1}{T}$; $\frac{2\pi f}{k} = v_{ph} = f\lambda$
$= c$ in vacuum

If no in a vacuum it is reduced by a factor n the <u>index of refraction</u>.

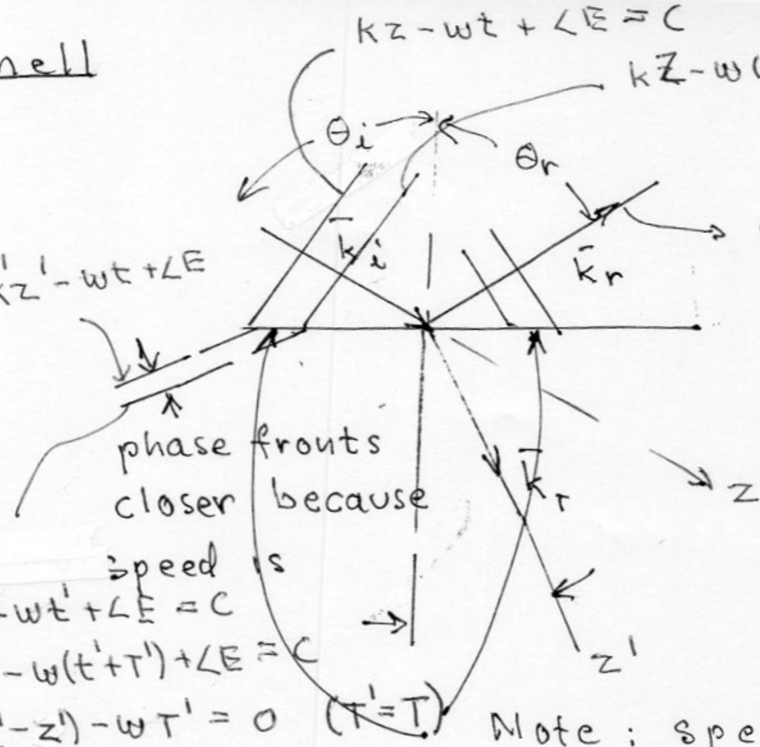
$$c \rightarrow c/n$$

The field is of the form

$$\bar{E} = \frac{E_0}{2} \cos\left(-\omega t + \omega \frac{n}{c} z + \angle E\right)$$

Source <u>of n</u> : atomic or molecular vibrations, electronic oscillations, acoustical waves, etc.

Snell



$k'z' - \omega t' + \angle E = C$

$k'z' - \omega(t'+T') + \angle E = C$

$k'(z'-z') - \omega T' = 0 \quad (T' = T)$

$\Delta z' = \omega T/k'$

Note: speed along the boundary is the same for all three waves.

generalize direction of propagation

$$\cos(\omega t - kz + \angle E) = \cos(-k\hat{z}\cdot(z\hat{z}) + \omega t + \angle E)$$

$$= \cos(-\underbrace{\bar{k}\cdot\bar{r}} + \omega t + \angle E)$$

true for any $\bar{k}$-vector

For $x = y = 0$, this is

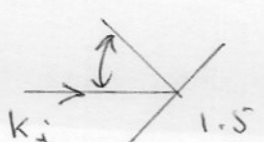
$$\cos(\omega t - k_z z + \angle E) \quad \text{with } k_z = k\sin\theta$$

Thus for phase speeds equal along the boundary

$$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$$

$$n_1 \sin\theta_i = n_1 \sin\theta_r = n_2 \sin\theta_t$$

Critical angle $\quad \theta_t = 90^0 \quad \sin\theta_i = \frac{n_2}{n_1} \quad$ n_2 must be $< n_1$



$\theta_i = \sin^{-1}\frac{1}{1.5} =$