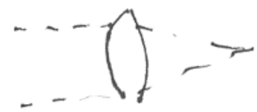
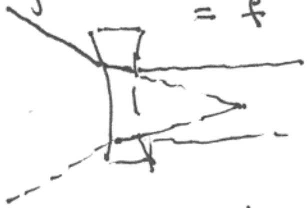
Focal Points and Planes

Opposite



$s_o = f \quad s_i = \infty$ | $s_o = \infty \quad s_i = f$

object focal length $F_o = f$ | image focal length $= f = F_i$



$s_o = \infty$

$s_i = f < 0$ image focal length

$R_1 < 0,\ R_2 > 0 \quad n > 1$

$f < 0$

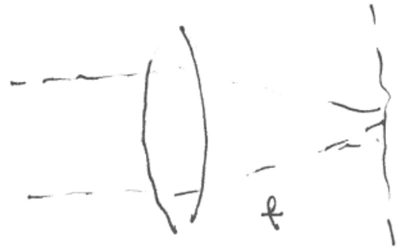
Once again $\dfrac{1}{s_o} + \dfrac{1}{s_i} = \dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$s_o = f$ (negative number)

$s_i = \infty$

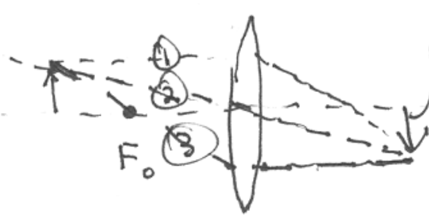
(Note: if $n < 1$ this becomes effectively a concave lens [what other consequences])

Focal Plane



Finite imagery - ray tracing - <u>use three rays to get image</u>

real (rays forming image exist)



inverted

optical axis

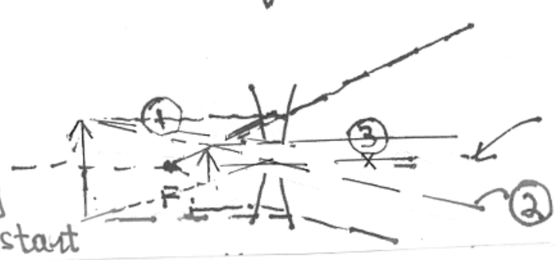
<u>Real</u>

$$\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$$

$r_1 = 0 \quad r_2 = r_2$

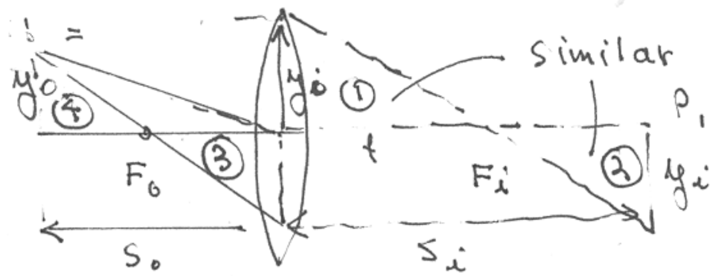
$r_2' = r_1'$

virtual (rays forming image non-existant at image)



optical axis

Fig. 5.22 page 163



1 & 2: $\dfrac{y_o}{|y_i|} = \dfrac{f}{\underbrace{s_i - f}_{x_i}}$; 3 & 4: $\dfrac{|y_i|}{y_o} = \dfrac{f}{\underbrace{s_o - f}_{x_o}}$

3 + 4 & 2 + 1: $\dfrac{y_o}{|y_i|} = \dfrac{s_o}{s_i}$

$$\frac{y_o}{|y_i|} \rightarrow \frac{s_o}{s_i} = \frac{f}{s_i - f} = \frac{f}{x_i} = \frac{x_o}{f} \longrightarrow x_o x_i = f^2$$

Transverse Magnification

$$M_T = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$ (Hecht uses a − sign for an inverted image)

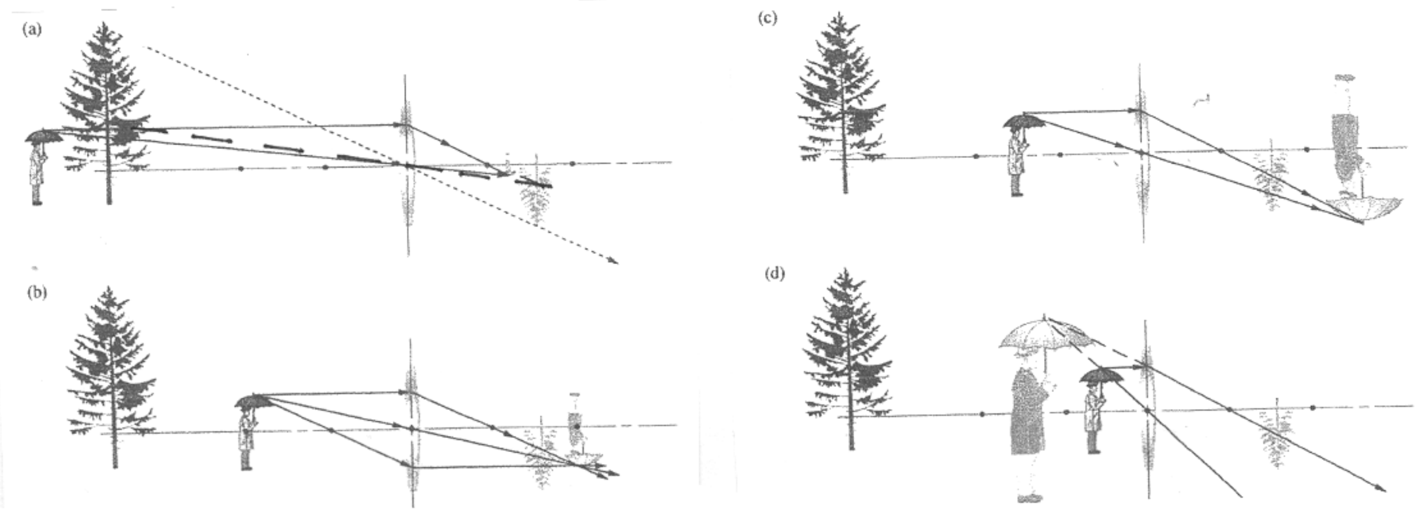
$$= -\frac{f}{x_o} > 1 \quad \text{if} \quad x_o < f$$

Longitudinal magnification

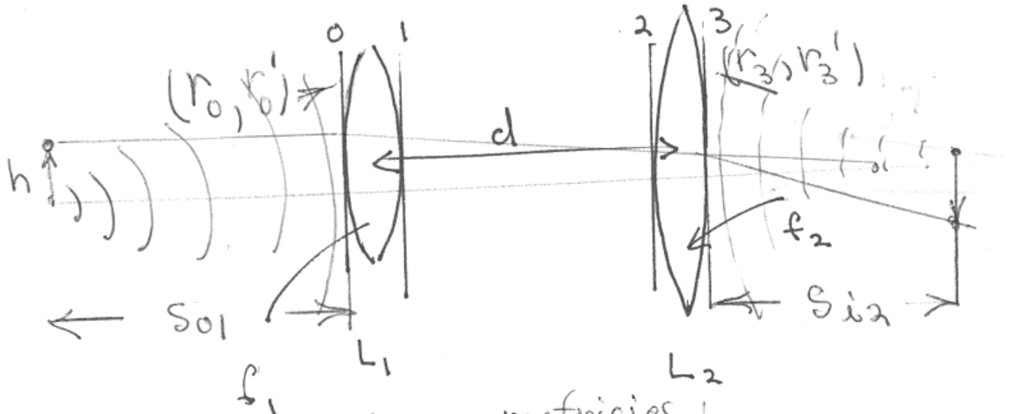
$$M_L = \frac{dx_i}{dx_o}$$

$$= \frac{d}{dx_o}\left(f^2/x_o\right) = -\frac{f^2}{x_o^2} = -M_T^2$$

Figure 5.24 The image-forming behavior of a thin positive lens.



Example of a Thin Lens Combination (page 107)



matricies

$$\begin{pmatrix} A, & B \\ C, & D \end{pmatrix} = \begin{pmatrix} 1, & 0 \\ -\frac{1}{f_2}, & 1 \end{pmatrix} \begin{pmatrix} 1, & d \\ 0, & 1 \end{pmatrix} \begin{pmatrix} 1, & 0 \\ -\frac{1}{f_1}, & 1 \end{pmatrix} \qquad \begin{pmatrix} r_3' \\ r_3 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_0' \\ r_0 \end{pmatrix}$$

$$= \begin{pmatrix} 1, & d \\ -\frac{1}{f_2}, & -\frac{d}{f_2}+1 \end{pmatrix} \begin{pmatrix} 1, & 0 \\ -\frac{1}{f_1}, & 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{d}{f_1}, & d \\ -\frac{1}{f_2}-\frac{1}{f_1}\left(1-\frac{d}{f_2}\right), & \left(1-\frac{d}{f_2}\right) \end{pmatrix}$$

det = 1

<u>Various cases</u> to be considered

From the A B C D matrix, we can write

$$R_{i2} = \frac{\left(1-\frac{d}{f_1}\right)R_{01} + d}{\left(-\frac{1}{f_2}-\frac{1}{f_1}\left(1-\frac{d}{f_2}\right)\right)R_{01} + \left(1-\frac{d}{f_2}\right)}$$

$$= \frac{\left(f_1 d + (f_1 - d)R_{01}\right) f_2}{\left(-f_1 - (f_2 - d)\right)R_{01} + f_1 f_2 - d f_1}$$

(Note that this is the negative of Eq (5.33)

(sign convention differences $R_{i2} = -S_{i2}$ and

$R_{01} = +S_{01}$

S_{o1} and S_{i2} are the object and image distances of the compound lens, respectively

Example – page 169 $\quad f_1 = 30\,\text{cm}$ $\quad d = 20\,\text{cm}$
$f_2 = 50\,\text{cm}$

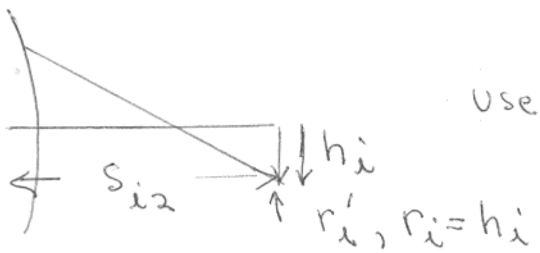
$S_{i2} = 26.2\,\text{cm}$
The image is real because $S_{i2} > 0$

Magnification

If $r_o' = 0 \quad r_o = h_o$

$r_3 = \left(1 - \frac{d}{f_1}\right) r_o \quad$ Lens ; $r_3' = \left(-\frac{1}{f_2} - \frac{1}{f_1}\left(1 - \frac{d}{f_2}\right)\right) h_o$

free propagation



use $\begin{pmatrix} r_i \\ r_i' \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r_3 \\ r_3' \end{pmatrix} \quad d = S_{i2}$

$$\therefore r_i = r_3 + S_{i2} r_3'$$

$$= \left(1 - \frac{d}{f_1}\right) h_o + S_{i2}\left(-\frac{1}{f_2} - \frac{1}{f_1}\left(1 - \frac{d}{f_2}\right)\right) h_o$$

$$M_T = \frac{r_i}{h} = 1 - d/f_1 + S_{i2}\left(-1/f_2 - 1/f_1\left(1 - d/f_2\right)\right)$$

$$= 1 - 20/30 + 26.2\left(-1/50 - 1/30\left(1 - 20/50\right)\right)$$

$$= .714$$ (as in the book page 168)

Note: To get the same analytical result solve for $1/f_2$ in the expression for S_{i2} (5.33) and eliminate it in the above

Note that as $d \to 0$ the ABCD reduce to an effective single lens with $1/f = 1/f_1 + 1/f_2$

(Eq. 5.37 of text)