UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Scences
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Problem Set No. 2

Problem Number one)Snell's Law and the Lens Formula
Hecht Problem 5.5

Problem Number two) Cartesian oval
Hecht Problem 5.1

Problem Number three) Imaging with a Thin Lens
Hecht Problem 5.12

Problem Number four) Camera Imaging (film is a C.C.D. array)
Hecht Problem 5.14

Problem Number five) A lens embedded in an index material
Hecht Problem 5.28

Problem Number six) Measurement of the focal distance of a lens
Problem 5.32

Problem Number seven) Effective focal length of lens combination
Problem 5.65

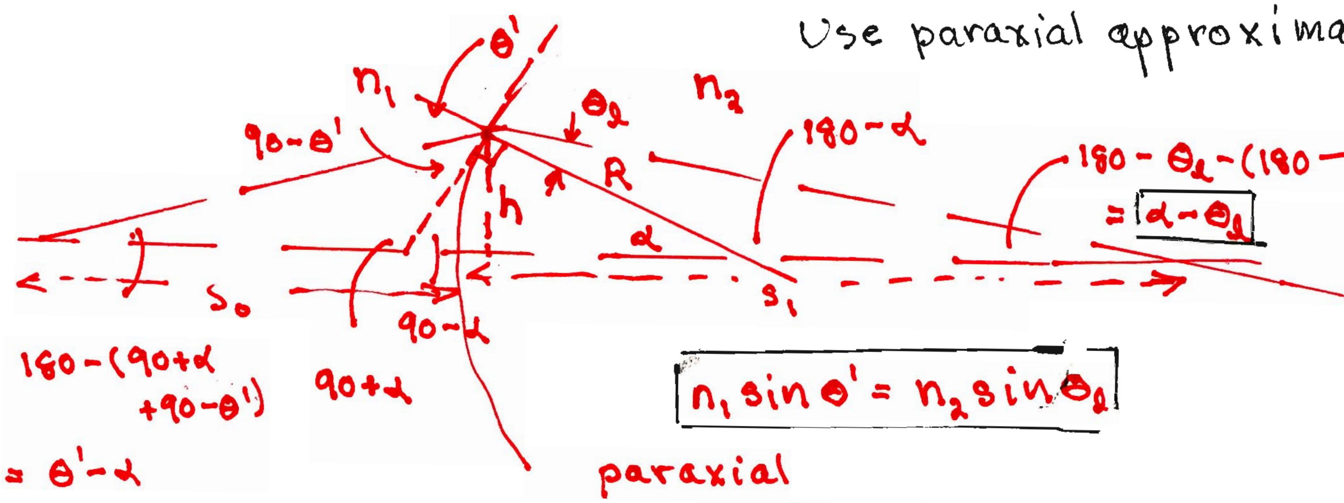
Problem Number eight)
A laser pointer has a measured minimum beam radius of .5 mm near the exit of the beam. If it is to be focussed to a radius of .01 mm what focal length lens should be used and how far from the lens is the beam focussed.

EECS 119

Prob Set No. 2. Solutions

Problem 1) Hecht 5.5 (Snells Law and the lens formula)

Use paraxial approximation



$$\boxed{n_1 \sin\theta' = n_2 \sin\theta_2}$$

paraxial

$$n_1\theta' \approx n_2\theta_2$$

consider: $\alpha - n_2\theta_2 = \alpha - n_1\theta'$

$$= \alpha - n_1(\theta' - \alpha) - n_1\alpha$$

$$n_2(\alpha - \theta_2) + (\alpha - n_2\alpha) = \alpha - n_1(\theta'-\alpha) - n_1\alpha$$

$$(\alpha - \theta_2)\, s_i \hat{=} h \qquad (\theta' - \alpha)\, s_o \hat{=} h$$

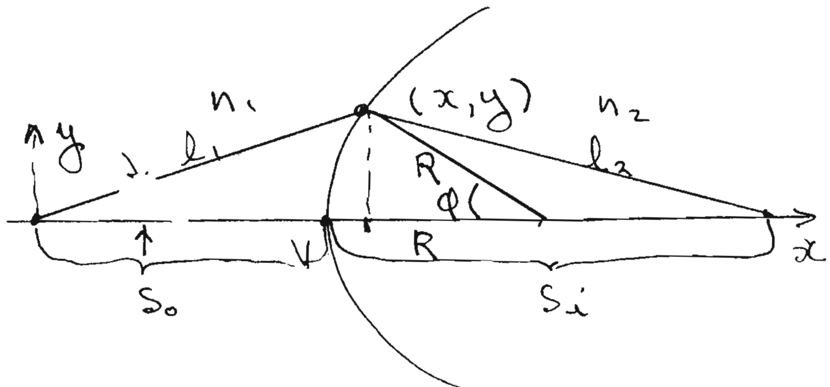
$$\therefore\ n_2 \frac{h}{s_i} \quad (1 - n_2)\alpha = \alpha - n_1 \frac{h}{s_o} - n_1\alpha$$

$$\therefore\ \frac{n_2}{s_i} + \frac{n_1}{s_o} = \left(+(1-n_1)\alpha + (-1+n_2)\alpha\right) h^{-1}$$

$$\alpha \approx \left(\frac{h}{R}\right)$$

$$\therefore\ \boxed{\frac{n_2}{s_i} + \frac{n_1}{s_o} = \frac{n_2 - n_1}{R}}$$ same as Fermat

Prob 2) Problem. 5.1 of Hecht



<u>Fermat again</u> - ~~using~~ the cosine law

$$(1) - \quad (x^2+y^2)^{1/2} n_1 + n_2((S_o+S_i - x)^2+y^2)^{1/2} = \text{Optical path length} = \text{OPL} = f(\varphi) = \text{const}$$

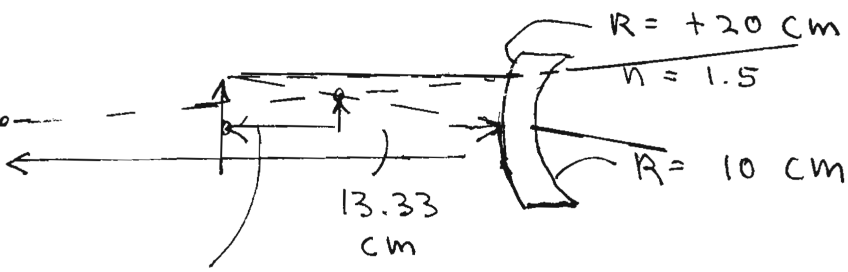
maximize w.r.t. S_i

This is a function φ in general

Define the surface such that $f(\varphi) = \text{const}$

then $\frac{df}{d\varphi} = 0$ and Fermats principle is satisfied

Prob. 3) Hecht 5.12



<u>thin lens</u>

$$\frac{1}{f} = (.5)\left(\frac{1}{20} - \frac{1}{10}\right)$$

$$= -.5 \frac{1\cancel{0}}{20\cancel{0}} = \frac{-.5}{20}$$

$$f = -40$$

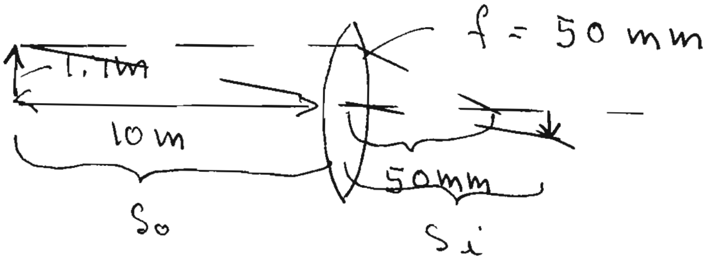
$\therefore \quad \frac{1}{S_o} + \frac{1}{S_i} = -(40)^{-1}$

$$\frac{1}{20} + \frac{1}{S_i} = -(40)^{-1}$$

$$.05 + .025 = -1/S_i$$

$$S_i = -1/.075 = -13.33$$

Problem No. 4 Prob. 5.14 Hecht.

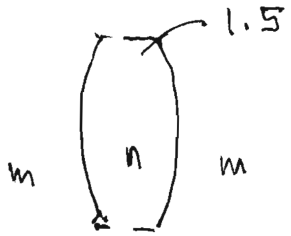


$$\frac{1}{10} + \frac{1}{S_i} = \frac{1}{50 \times 10^{-3}}$$

$$.1 + \frac{1}{S_i} = 20$$

$$S_i = \frac{1}{19.9} = 50.25$$

Prob. No. 5 Prob. 5.28 Hecht



$f =$ >0 for (>0 for (

$$\frac{n_m}{S_{o1}} + \frac{n_m}{S_{i2}} = (n_l - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

In air $(n_l - 1)\left(\frac{1}{R_1} + \frac{1}{|R_2|}\right)$ (double convex)

$$= 10 \text{ cm} = (1.5 - 1.)\left(\frac{1}{R_1} + \frac{1}{|R_2|}\right)$$

in water $\frac{(1.56 - 1.33)}{1.33}\left(\frac{1}{R_1} + \frac{1}{|R_2|}\right)$

$$= \frac{(1.56 - 1.33)}{1.33} \frac{10}{(.5)} = 3.458$$

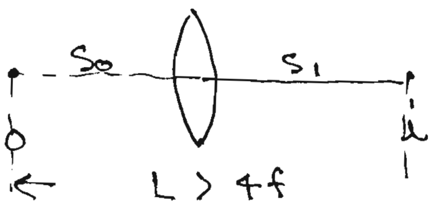
$$\frac{1}{S_{o1}} + \frac{1}{S_{i2}} = \frac{1}{3.458}$$ (viewed in water)

S_{o1} ↑ 100 cm

$S_{i2} \approx 3.458$ cm

Problem No. six

Prob. 5-32



$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

$$S_o + S_i = \cancel{d}\, L$$

$$\frac{1}{S_o} + \frac{1}{(L - S_o)} = \frac{1}{f}$$

$$\cancel{S}L - \cancel{S_o} + \cancel{S_o} = S_o\left(\frac{L}{f} - \frac{S_o}{f}\right)$$

quadratic Equation

$$S_o^2 - S_o(L) + Lf$$

$$S_o = \frac{L}{2} \pm \sqrt{\frac{(L)^2}{4} - Lf}$$

Difference between these is

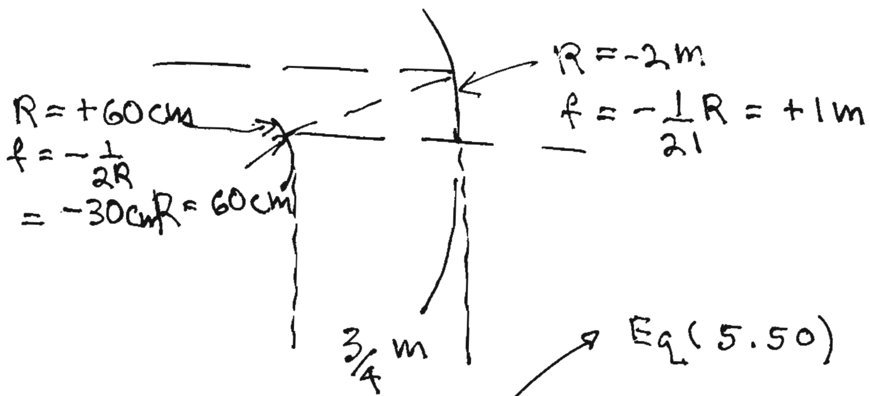
$$2\sqrt{\left(\frac{L}{2}\right)^2 - Lf} = d$$

$$\frac{L^2}{4} - Lf = \frac{d^2}{4}$$

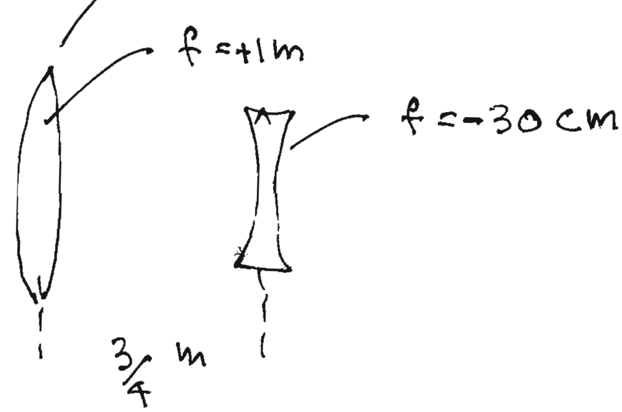
$$f = \frac{-d^2 + L^2}{4L}$$

Problem No, seven

Prob. 5.65



Same as



A two lens system (Eq. 5.33)

From notes

$$S_{i2} = \left(\frac{f_2 d - f_2 S_{o1} f_1/(S_{o1}-f_1)}{d - f_2 - S_{o1} f_1/(S_{o1}-f_1)} \right);$$

$$R_{i2} \Rightarrow \frac{(f_1 - d) f_2}{-(f_2 - d + f_1)} \text{ for } R_{o1} \to \infty$$

$$= \frac{(1 - .75)(-.3)}{-(-.3 - .75 + f_1)} = \frac{.25(-.3)}{.05}$$

$$= -1.5 \text{ m}$$

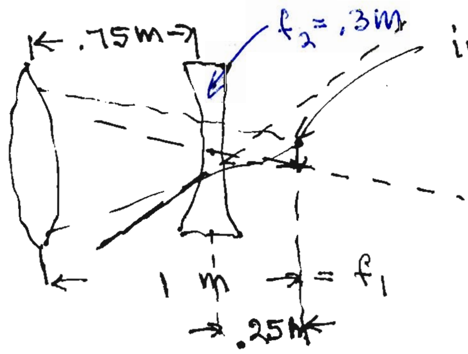
$$= -S_{i2}$$

$$= \frac{-.3 \times .75 - (-.3) S_{o1} 1/(S_{o1} - 1)}{.75 + .3 - S_{o1} 1/(S_{o1} - 1)}$$

$S_{o1} \to \infty$

$$S_{i2} = -.3(.75 - 1)/(.75 + .3 - 1) = +.3 \times \frac{.25}{.05} = +1.5\text{m}$$

quick check



↑ $S_{o1} = \infty$

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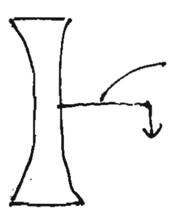


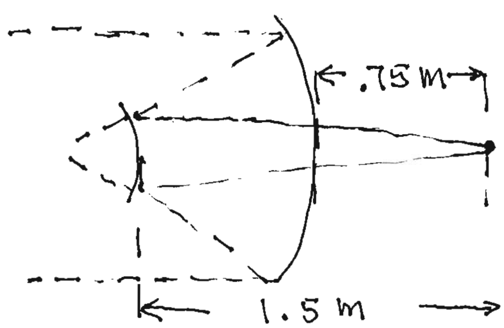
$S_{o1} < 0$ $f < 0$

$$\therefore \frac{1}{S_{o2}} - \frac{1}{.25} = -\frac{1}{.3}$$

$$\frac{1}{S_{o2}} = \frac{.05}{(.25) \times .3}$$

$$S_{o2} = \frac{.25 \times .3}{.05} = \frac{.075}{.05}$$

$$= \underline{1.5 \text{ m}}$$



The effective focal length. This is rather <u>vague</u>

One interpretation $S_o = \infty$ $S_i = 1.5$

Thus $\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f_{eff}} = \frac{1}{1.5}$ But the ABCD matrix is <u>not</u> of the form of a lens unless $d = 0$

Problem No. 8 (ref Yariv)

At the input plane 1 $\omega = \omega_{01}$, $R_1 = \infty$ so that

$$\frac{1}{q_1} = \frac{1}{R_1} - i\frac{\lambda}{\pi\omega_{01}^2 n} = -i\frac{\lambda}{\pi\omega_{01}^2 n} = -i\frac{10^{-4}}{\pi(.05)^2} = -i(1.27\times10^{-2}\text{ cm}^{-1}) = -i/z_{01}$$

$$z_{01} = 78.54\text{ cm}$$

using (2.6-8) leads to

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} = -\frac{1}{f} - i\frac{\lambda}{\pi\omega_{01}^2 n}$$

$$q_2 = \frac{1}{-1/f - i(\lambda/\pi\omega_{01}^2 n)} = \frac{-a + ib}{a^2 + b^2}$$

$$a \equiv \frac{1}{f} \qquad b \equiv \frac{\lambda}{\pi\omega_{01}^2 n}$$

At plane 3 we obtain, using (2.5-4),

$$q_3 = q_2 + l = \frac{-a}{a^2+b^2} + \frac{ib}{a^2+b^2} + l$$

$$\frac{1}{q_3} = \frac{1}{R_3} - i\frac{\lambda}{\pi\omega_3^2 n}$$

$$= \frac{[-a/(a^2+b^2) + l] - ib/(a^2+b^2)}{[-a/(a^2+b^2) + l]^2 + b^2/(a^2+b^2)^2}$$

Since plane 3 is, according to the statement of the problem, to correspond to the output beam waist, $R_3 = \infty$. Using this fact in the last equation leads to

$$l = \frac{a}{a^2+b^2} = \frac{f}{1 + (f/\pi\omega_{01}^2 n/\lambda)^2} = \frac{f}{1 + (f/z_{01})^2} \tag{2.6-11}$$

as the location of the new waist, and to

$$\frac{\omega_3}{\omega_{01}} = \frac{f\lambda/\pi\omega_{01}^2 n}{\sqrt{1 + (f\lambda/\pi\omega_{01}^2 n)^2}} = \frac{f/z_{01}}{\sqrt{1 + (f/z_{01})^2}} \tag{2.6-12}$$

for the output beam waist. The confocal beam parameter

$$z_{01} \equiv \frac{\pi\omega_{01}^2 n}{\lambda}$$

is, according to (2.5-11), the distance from the waist in which the input beam spot size increases by $\sqrt{2}$ and is a convenient measure of the convergence of the input beam. The smaller z_{01}, the "stronger" the convergence.

$$\frac{w_3}{w_{01}} = \frac{.01}{.5} = \frac{f/z_{01}}{(1+(f/z_{01})^2)^{1/2}} \Rightarrow \left(1 + \left(\frac{f}{z_{01}}\right)^2\right)\left(\frac{w_3}{w_{01}}\right)^2 = \left(\frac{f}{z_{01}}\right)^2$$

$$\therefore \left(\frac{f}{z_{01}}\right)^2 = \frac{(w_3/w_{01})^2}{1 - (w_3/w_{01})^2} \cong (.02)^2 \; ; \; f = .02 z_{01} = 1.57\text{ cm}$$

$\therefore l \cong 1.57$ cm since f/z_{01} is small