EE119 Homework 2

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Due Friday, February 6, 2009

1. Consider the following x- and y- components of the electric field of a light wave that is propagating along the z-direction:

   (a) $E_x = \cos(kz - \omega t)$ $E_y = \sin(kz - \omega t)$

   (b) $E_x = \sin(kz - \omega t)$ $E_y = \cos(kz - \omega t + \frac{\pi}{2})$

   (c) $E_x = \cos(kz - \omega t)$ $E_y = \cos(kz - \omega t + \frac{\pi}{2})$

   (d) $E_x = \cos(kz - \omega t)$ $E_y = \cos(kz - \omega t + \frac{\pi}{4})$

   (e) $E_x = \cos(kz - \omega t)$ $E_y = \cos(kz - \omega t - \frac{\pi}{4})$

   (f) $E_x = \cos(kz - \omega t)$ $E_y = \cos(kz - \omega t - \frac{\pi}{3})$

   (g) $E_x = \cos(kz - \omega t)$ $E_y = \cos(kz - \omega t - \frac{\pi}{6})$

   For each combination of electric fields, plot the electric field amplitude of x and y components on the same axis as a function of displacement in the direction of propagation at time t=0. Also, plot the electric field amplitude of x and y components on the same axis as a function of time at a fixed point in space (for instance, z=0). Describe the polarization of each type of wave (linear, circular, elliptical). If the polarization is linear, specify the angle of polarization from the x-axis. If the polarization is elliptical, draw the ellipse that's formed by the resultant E-field vector in the x-y plane. Specify the direction in which the electric field is rotating and the angle α that the major axis of the ellipse forms with the x-axis (see Hecht p. 329).

2. (a) The angle of deviation for a prism is given in the notes. Plot the angle of deviation (δ) versus incident angle for n=1.5 and apex angle of 60°.

   (b) The angle of deviation should have a minimum. Derive a general expression for the incidence angle at which the deviation δ is minimized. Your answer should be given as a function of the apex angle and refractive index of the prism.

   (c) Prove that the ray for which the deviation is a minimum traverses the prism symmetrically. This means that the incident angle and the exit angle are the same. This position for the prism is also approximated in most spectral instruments because it gives the highest spectral resolution (part 2e), also the largest diameter beam to pass through a given prism and also produce the smallest amount of loss due to surface reflections.

   (d) In the case when the angle of deviation is a minimum, find an expression for a refractive index of the prism. Express the refractive index in terms of the minimum angle of deviation and the apex angle. This equation forms the basis of one of the most accurate techniques for determining the refractive index of a transparent substance.

(e) We want to design a prism spectrometer. For a better prism spectrometer, we want to have the angular dispersion of the prism as large as possible. Show that the angular dispersion of different colors $\frac{\partial \delta}{\partial \lambda}$ has a maximum value in the position of minimum deviation. (The position of minimum deviation is the position at which the deviation δ is minimized.)

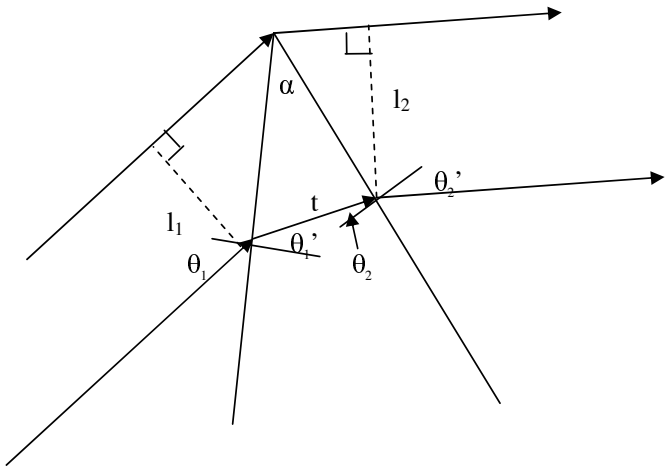

Figure 1: In problem 2, solve for the incident angle θ_1 at which the dispersion is maximized.

3. Measuring the index of refraction of a transparent material

   (a) Plot the Fresnel Coefficients of transmission and reflection for both s- and p-polarized light as a function of incident angle for four different values of the refractive index between 1 and 2. Specify what those values are in your plot. Which incident angle gives you the greatest sensitivity to changes in refractive index? Which polarization is more sensitive to changes in refractive index? If you wanted to determine the refractive index of a material and could only make one measurement, what angle and polarization would you measure at? Would you measure transmission or reflection?

   (b) Plot the angle of deviation of a prism with apex angle of 50°as a function of incident angle for the same four values of refractive index you used in part (a). At what incident angle is the sensitivity to changes in refractive index maximized? (this is very similar to what you did in problem 2a)

4. Photoelectric effect

   (a) What wavelength of light must you shine on a material with a 2 eV work function to get it to emit electrons?

   (b) Marty McFly is teleported back time and space to Nevada in 1875 where silver mining was well under way. He has with him a tunable light source, so he decides to strike it big by finding pieces of platinum that look like silver, buying them for the cost of silver, and selling them later at the higher cost of platinum. He remember from physics class that the work function of silver is about 4.3 eV and the work function of platinum is about 5.6 eV. He plans to illuminate metals with light that will cause photoemission in silver, but not in platinum. Then if he doesn't see photoemission

in a metal, he plans to raise the energy of his light source enough to check if it is platinum. What range must his tunable light source be able to produce in order for his plan to work?

(c) A year goes by, and Marty amasses loads of platinum. Doc pays him a visit from the future and, impressed with his business, offers to give him a light source is five times more intense, in exchange for half of his amassed platinum. Should Marty accept the offer? Will a more intense light source help him distinguish platinum and silver?

5. In this problem, consider the direction of propagation of light to be the z-axis.

(a) A birefringent plate with $n_e = 1.4$ and $n_o = 1.6$ that is 5 mm thick is placed perpendicular to the z-axis as shown in figure 2. The extraordinary axis, $n_e = 1.4$, is aligned along the y-axis. 500 nm light that is linearly polarized at 20°from the y-axis passes through the plate. What are the phase delays of the x- and y-components of the electric fields? Remember, the phase delay (see p. 24 of the class notes) is $\phi = kn\Delta z$ where k is the wave number, n is the refractive index of the medium, and z is the distance that the light has traveled through the material. Up to a factor of 2π, the phase delay is equal to the difference in the phase of the electric field vector between the start and the end of the material. Is the emerging light linearly polarized or elliptically polarized? If it is linearly polarized, specify the angle from the y-axis; if it is elliptically polarized, specify the direction of rotation and the angle of the semi-major ellipse from from the x-axis (see Hecht p. 329).

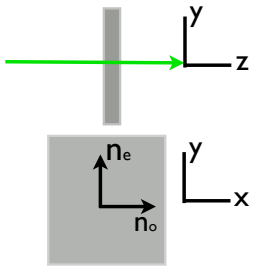

Figure 2: diagram for problem 5a.

(b) Now you cut the birefringent plate in problem 5a differently, so that n_o is parallel to the surface of the plate, and n_e is perpendicular to the surface of the plate, as shown in figure 3. Only one of either s- or p-polarized light ever interacts with n_e. Which polarization is the one that interacts with n_e?

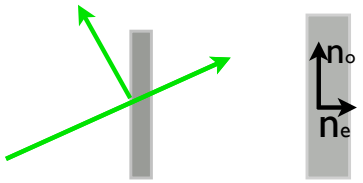

Figure 3: diagram for problem 5b

(c) [Hecht problem 8.34] A ray of light is incident on a calcite plate at 50°. the plate is cut so that the optic axis is parallel to the front face and perpendicular to the plane-of-incidence. Find the angular separation between the two emerging rays. The table in section 8.4.3 (p. 243) of Hecht gives a value of $n_o = 1.6584$ and $n_e = 1.4868$ for calcite.

6. Lex Luthor obtains a new form of kryptonite that will shrink Superman down to be 10 cm tall. He decides to carry around a lens with him that will restore his appearance to his former 2 meter height.

   (a) What transverse magnification must this lens have for superman to look the height that he wants?

   (b) Superman wants to be able to stand up to 10 meters behind his lens. What focal length does the lens need to have in order to achieve the magnification required in part 6a?

   (c) If Superman wants to appear right-side up when he's walking on the street, on what side of the focal point does he need to be relative to the lens? Draw a diagram. Will people looking through the lens see a real image or a virtual one?