EE119 Homework 3

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Due Monday, February 16, 2009

- 1. In class we have discussed that the behavior of an optical system changes when immersed in a liquid. Show that the longitudinal image magnification for an optical system not immersed in air is $(n_2/n_1)m^2$, where the object is immersed in a material of index n_1 and the image is immersed in a material of object n_2 . You can also refer to the diagram on the bottom of p. 17 of the notes for the labeling of n_1 and n_2 .
- 2. You will explore some of the differences between real and paraxial rays in this problem. For each part below, trace the specified ray and determine where it crosses the optical axis. Show all calculations and include a diagram. Report your answers to 4 decimal places.
 - (a) Find L when $\theta = 5^{\circ}$ with real ray (no paraxial approximation).
 - (b) Find L when $\theta = 0.5^{\circ}$ with real ray (no paraxial approximation).
 - (c) Repeat part (a) with the paraxial ray (paraxial approximation).
 - (d) Repeat part (b) with the paraxial ray (paraxial approximation).
 - (e) Is there a difference between your answers in (a) and (b)? Is there any difference between your answers in (c) and (d)?
 - (f) Now compare your answers in (b) and either (c) or (d). Why are they so similar?



Figure 1: diagram for problem 2.

- 3. Consider a thin, spherical plano-convex lens having a radius of curvature of 50.0mm and a refractive index of 1.5.
 - (a) Determine the focal length in air.
 - (b) Suppose this lens is placed right on the surface of a tank of water. At what depth below the surface would a collimated light beam from above come to a focus? (The refractive index of water is 1.3.)

4. A spherical wave can be mathematically described by:

$$E = \frac{E_0}{r} \operatorname{Re}\left(e^{-ik \cdot r + \phi}\right) = \frac{E_0}{r} \cos(-kr + \phi)$$

Where r is the distance of the wave front from the source in 3D space, k is the wave vector (the amplitude of k is given by $\frac{2\pi}{\lambda}$, and ϕ is a constant phase. In this problem, the optic axis is on the z-axis.

(a) Show that at a large distance from the point source, close to the optic axis, at constant z, the wave can be approximated by

$$E = \frac{E_0}{z} \cos[-\frac{k(x^2 + y^2)}{2z} + \phi]$$

In this approximation, $z \gg (x,y)$. Hint: the taylor expansion of $\sqrt{1+x}$ for $x \ll 1$ is $\sqrt{1+x} \approx 1 + \frac{x}{2}$.

(b) Show that when this wave is propagating in the z-direction, it can be approximated by a plane wave of constant amplitude. Assume that the distance it travels, Δz , is much smaller than the distance from the point source. A plane wave can be mathematically described by

$$E = E_0 \cos(kz + \phi)$$

Where E_0 is the amplitude, k is the wave vector and ϕ is a constant phase.

- (c) Show that a spherical wave emerging from a given object point will be converted into a spherical wave converging to the image point given by the usual Gaussian lens law, by using the thin-lens-phase shift.
- 5. Two thin lenses with focal lengths of + 6.0 cm and + 8.0 cm and apertures of 8.0 cm and 9.0 cm, respectively, are located 4 cm apart. An aperture stop (3 cm in diameter) is located between the two lenses, 3 cm from the first lens. An object 3 cm high is located with its center 12 cm in front of the first lens.
 - (a) Draw a diagram that shows the entire imaging system.
 - (b) Find the position and size of the entrance and exit pupils and draw them in the diagram from part (a).
 - (c) Find the position and size of the final image and draw it in the diagram from part (a).
 - (d) Draw the two marginal rays and the chief ray from the top end of the object and trace them all the way to the image.
- 6. [Adapted from Hecht 6.14] A compound lens is composed of two thin lenses separated by 10 cm. The first of these has a focal length of +20cm, and the second a focal length of 30cm. Determine the focal length of the combination and locate the corresponding principal points. Draw a diagram of the system.