EE119 Homework 11 Solutions: Lasers and Diffraction

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1. We have discussed some of the very common lasers in class. There are many other lasers, which we have not covered in the lecture. In this problem, you should do some library (or internet) research on the following lasers. You should describe their special characteristics, how they operate, and typical operating wavelengths. Include diagrams to get more points and state some common applications to get more points.

   (a) Liquid lasers (dye lasers)

   Dye lasers use a liquid solution of a highly emissive dye molecule as the active medium. Dye lasers are usually optically pumped, and rely on vibrational relaxation (i.e. phonons) to create the top two levels of a three-level (or 4-level) system. A great many synthetic dyes have been developed for this application, and the colors of the light emitted by dye lasers can range all over the visible spectrum. A key property of these dyes is that they must have a long excited state lifetime–they molecule must remain in an excited state for long enough that it can contribute to gain, which means that non-radiative decay pathways such as vibrations must be relatively slow. The spontaneous emission lifetime of typical dye lasers is a few nanoseconds. The emission spectrum of dyes sometimes depends on the solvent the dye is in, so this adds another variable that enables tuning of the gain spectrum to a specific application.

   Before the discovery and development of nonlinear semiconducting crystals (such as Ti:Sapphire) dye lasers were a common gain medium for pulsed lasers. Getting broad bandwidth out of lasers made it possible to produce mode-locked pulses of light that were shorter than what could be controlled by electronics. Dye lasers were also frequently pumped by flash lamps, which required large voltages stored in capacitors.

   A common dye used as the gain medium is Rhodamine 6G. When dissolved in ethanol, rhodamine 6G absorbs light between 450 nm and 550 nm, with its absorption maximum at 510 nm. It's emission is maximized in the range of 550 nm in ethanol.

   (b) Excimer lasers

   Excimer lasers have gain due to a chemical reaction between two or more atoms. Excimer is short for "excited dimer." Excimer lasers are electrically pumped, and the electrical current creates a The source of gain in Excimer lasers is the dissociation of the excited dimer into two separate atoms. The dimer gives off energy by releasing light, and then the two atoms return to the ground state and are ready to be recharged again. When the excimer relaxes, the two atoms are close together because they were a dimer in the excited state; however, when they are not excited the atoms repel each other, so the lower state of the lasing transition, which is the state of the non-excited dimer, has a very short lifetime. This makes it possible to achieve very high population inversions if you pump the gas of atoms hard enough

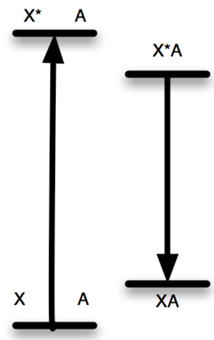

Figure 1: An excimer laser is 4-level system where each level is a different configuration of atoms/molecules.

A qualitative energy level diagram is shown in Figure 1

More generally, there is no reason why only 2 atoms can participate in this reaction; you can have an "excited complex" of atoms, and this is sometimes referred to as an "exciplex." Excimer/Exciplex lasers are used to produce ultraviolet laser light. Because UV light can effectively break chemical bonds in biological tissues, common applications of Excimer lasers are in surgery such as LASIK that involve destroying biological tissue in a controlled fashion (this is called "ablation").

(c) Plasma X-ray lasers

For a short history of x-ray lasers you can read this web page https://www.llnl.gov/str/Dunn.html.

(d) Free electron lasers

A free electron laser uses free electrons–that is, electrons that aren't bound to atoms– as the gain medium. The electrons are accelerated in an accelerated, and they emit light when their direction of motion rapidly changes. This is done by passing electrons through magnetic structures called undulators-as the electrons decelerate and change direction, they emit light. since the energy the electron lose is determined by controllable factors such as the electron speed and the strength of the magnetic fields in the undulators, the wavelength of the electron is fully tunable. However, the conventional concept of feedback doesn't really occur in these lasers because all the light is released in one shot.

(e) VCSEL (Vertical Cavity Surface Emitting Lasers)

VCSELs are a type of semiconductor laser. They are different from conventional semiconductor lasers because the reflective surface is deposited parallel to the substrate, rather than coming from cleaving a crystal at the end of the fabrication process. The mechanism for gain is the same as in semiconductor lasers, but because VSELS emit light perpendicular to the surface, they can be integrated easily with other devices, such as fiber optics. Furthermore, the fabrication of VCSELs is more reliable because the process can be tested at each step, rather than only at the end when the crystal is cleaved, as is the case with semiconductor lasers.

2. Give short (a few sentences) answers to the following questions:

(a) Why is there no lasing from the Helium atoms in a HeNe laser?

Solution:

The excited state of Helium has the outer electron in an s orbital, so the state has a total angular momentum of zero. The ground state also has zero angular momentum. For a radiative transition, the angular momentum of the atom must change by 1 unit of $\hbar$, so there are no radiative transitions from the relevant excited states of Helium.

Note: sometimes people will talk about "forbidden transitions" where you do observe light emitted from a state that violates this rule. This has to do with coupling of the involved states with other states, leading to "perturbations" in properties such as the energy and angular momentum of the state.

(b) If you could achieve lasing from all the transitions in Neon simultaneously in a single cavity, which transition would yield the longest collimation length? Give the wavelength of the transition, the energy levels involved, and explain your answer.

Solution:

The Rayleigh range, which is a measure of collimation length, is inversely proportional to wavelength. Consequently, the shortest -wavelength (highest energy) transition will give the longest collimation length. Of the states of Neon shown on p. 79, the shortest wavelength is 5944 Angstroms, between the states $2p^5 3p$ and $2p^5 3s$.

3. Find an expression for the intensity distribution in the Fraunhofer diffraction pattern of the aperture shown in Figure 2. Assume unit-amplitude, normally incident plane-wave illumination. The aperture is curcular and has a circular central obstruction. (Hint: Use the result given below for the simple circular aperture and then use superposition.) Outer radius is R_1 and inner radius is R_2.

$$E(r) = e^{jkr} e^{jk\frac{r^2}{2z}} \frac{A}{jz\lambda} \left[2\frac{J_1(kwr/z)}{kwr/z} \right]$$

Where J_1 is the Bessel function of order 1, k is a wavevector and w is the size of the circular pupil (same notations as those used in class). A table of Bessel functions is on p.470 of Hecht (section 10.2.5)

Solution:

Electric field of the Fraunhofer diffraction pattern from a circular aperture:

$$E(r) = e^{jkr} e^{jkr^2/2z} \frac{2A}{jz\lambda} \left[\frac{J_1(kR_1 r/z)}{kR_1 r/z} - \frac{J_1(kR_2 r/z)}{kR_1 r/z} \right]$$

Intensity distribution:

$$I(r) = \frac{4A^2}{z^2\lambda^2} \left[\frac{J_1(kR_1 r/z)}{kR_1 r/z} - \frac{J_1(kR_2 r/z)}{kR_1 r/z} \right]^2$$

4. (Hecht 10.7) A single slit in an opaque screen 0.10 mm wide is illuminated (in air) by plane waves from a krypton ion laser ($\lambda_0 = 461.9$ nm). If the observing screen is 1.0 m away, determine whether or not the resulting diffraction pattern will be of the far-field variety and then compute the angular width of the central maximum.

Solution:

The diffraction pattern will be of the far field variety if the Fraunhofer condition is satisfied. The Fraunhofer condition is

$$z \gg \frac{k(\xi^2 + \eta^2)}{2} = \frac{\pi(\xi^2 + \eta^2)}{\lambda}$$

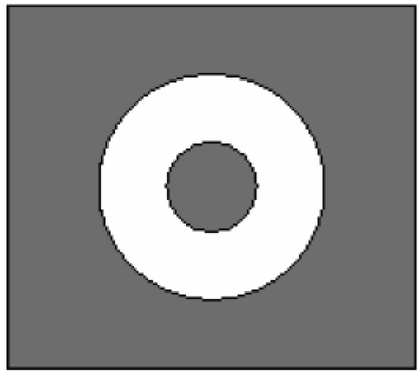

Figure 2: Circular aperture

In this case the distance z is 1 meter, and the dimension of the diffracting screen is 1×10^{-4} meter. The right hand side of the expression is

$$\frac{\pi \times 1 \times 10^{-8}}{461.9 \times 10^{-9}} = 0.068\text{meters}$$

This is about 20 times smaller than z, so the diffraction is of the far field variety, so we can use the Fraunhofer diffraction pattern.

The diffraction half-angle of the central maximum for a slit of width 2w is equal to $\lambda/2w$. Since our slit width is .10 mm, is equal to

$$\theta = \frac{\lambda}{1 \times 10^{-4}} = 0.0046$$

radians, so the angular width of the first maximum is twice that, 0.0046 radians.

5. (Hecht 10.11) Two long slits 0.10 mm wide, separated by 0.20 mm, in an opaque screen are illuminated by light with a wavelength of 500 nm. If the plane of observation is 2.5 m away, will the pattern correspond to Fraunhofer or Fresnel diffraction? How many Youngs fringes will be seen within the central bright band?

   Solution:

   The Fraunhofer condition is

$$z \gg \frac{kd^2}{2}$$

   Where z=2.5 m and d=0.3 mm. The right hand side of the condition is

$$\frac{\pi \times 9 \times 10^{-8}}{5 \times 10^{-7}} = 0.5655$$

   This is smaller than 2.5, so the Fraunhofer condition is satisfied, and we have Fraunhofer diffraction. The diffracted E-field will be the sum of the E-fields diffracted from each of the two slits. The diffraction pattern from each slit will be

$$E(x) = \frac{\sin(ax/\lambda z)}{ax/\lambda z}$$

   Where a is the length of the slit if the slit is centered at the origint, x is the position on the screen, and z is the distance from the slit to the screen. However, the slits are not centered at the origin, so the (1-dimensional) rect function that we use for the aperture has to be shifted. We'll say that one of the slits is centered at the origin, and the other is centered at 0.3 mm (if the slits are separated by 0.2 mm and each is 0.10 mm wide, then their centers are separated by 0.3 mm).

The Fourier transform of a function translated by a distance d is the fourier transform of the original function multiplied by a phase factor of $e^{2\pi jds}$ where s is the conjugate variable. In our case, the conjugate variable is $x/\lambda z$ so the Fourier transform of the second slit is equal to the Fourier transform of the first slit, multiplied by this phase:

$$E_{\text{slit } 2}(x) = e^{2\pi jdx/\lambda z}\frac{\sin(ax/\lambda z)}{ax/\lambda z}$$

So we see that we have single slit interference at a frequency $0.1/\lambda z$ modulated by double slit interference at a frequency of $0.3/\lambda z$. This means that the double slit interference will have a minimum every $\lambda z/0.3$ meters, including at zero. The single slit interference patter will have a minimum every $\lambda z/0.1$ meters, except at 0, where the sinc function equals 1. However, because the spacing between the slits is three times the width of the slit, there will be a missing order from the double-slit pattern at the each minimum of the single slit interference (see p. 460 of Hecht) so there will be 2 double-slit (Young's) fringes for each single-slit fringe. Because this missing order is in fact present at x=0, there will be a double-slit maximum at x=0. Consequenly, there will be 5 fringes in the first single-slit maximum.

6. Do problems 10.17-10.21 in Hecht. These problems require you to identify the type of aperture that gives rise to the diffraction pattern shown in the pictures. The answers to these questions are in the back of the book, which means that you should get all of them right. Make sure you understand why the answer is correct. Your grade on this problem will depend on the accuracy and completeness of your explanations.

   Solution:

   (a) Hecht 10.17. The aperture is three parallel short slits. The slits are short because we see diffraction in both dimension, so the longer dimension is not infinitely long. We know that there are three slits because there are three diffraction maxima that are modulated by the one-slit diffration pattern from the width of the slits themselves.

   (b) Hecht 10.18 The aperture is two parallel short slits. This situation is similar to problem 5, where you have 5 Young's fringes in the first single-slit maximum. The first minimum of the diffraction pattern is almost missing, suggesting that the spacing between the centers of the two slits is close to three times the width of the slits.

   (c) Hecht 10.19. the aperture is an equiliateral; We (sort of) see that the irradiance has six directions in which a sinc function decays. We know that it's an equilateral triangle and not a hexagon because the center lobe has three points that stick out, rather than six.

   (d) Hecht 10.20 The image is the electric field distribution of a cross-shaped aperture. We know that it's a cross and not a single slit because the intensity decays equally in two directions. We also know that there is 4-fold symmetry, so the aperture can't be a circle or some complex polygon. We know it's a cross and not a square because the intensity is stronger along the slit directions than it would be for a square.

(e) Hecht 10.21 The image is the electric field distribution of a rectangular aperture. We know the image is an electric field and not intensity because it goes below zero. The E-field seems to have 4-fold symmetry (although it is not perfectly symmetric, so the aperture is a rectangle but not necessarily a square. Since the decay is uniform on all four directions, it is a rectangle and not a cross.