EE119 Homework 12 Solutions: Diffraction and Interference

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1. A molecule sometimes emits light at 600 nm and sometimes emits light at 650 nm. You want to determine the relative intensity of emission at these two wavelengths, so you decide to split the light with a diffraction grating and direct the two first-order diffracted beams of different colored light onto two different photodetectors. You want to separate the centers of the photodiodes by 2 cm. The active area of the photodiode is 0.5 mm × 0.5 mm, and to maximize efficiency you want all of the light at the two wavelengths to hit the active area. Design the first-order diffraction grating you will use, and determine how far away from the diffraction grating you should place your photodiodes.

Solution:

You want the spacing between the first order diffracted beam at 650 to be 2cm away from the first order diffracted beam at 600. This means that

$$x_{650} - x_{600} = 0.02\text{m}$$

You know that the first diffracted beam occurs at $x \pm f_0 \lambda z$ where f_0 is the grating frequency and z is the distance between the photodiode and the grating. So our restriction means that

$$f_0 z (650 - 600) \times 10^{-9} \geq 2 \times 10^{-2}\text{m}$$

$$f_0 z \geq 4 \times 10^5$$

this is a unitless number that tells us the relationship between the frequency of the groove and the distance from the grating. We have another restriction, which is that the width of a peak must be less that 0.5 mm=5×10^{-3}m. This means

$$\frac{\lambda z}{w} \leq 5 \times 10^{-4}\text{m}$$

so for the larger wavelength, 650 nm,

$$\frac{z}{w} \leq \frac{5}{6.5} \times 10^3$$

This is our other constraint. We have some choice because we have 3 variables and 2 constraints, but for practical reasons we wouldn't want z to be too large. Let's pick z to be 10 cm–this is about how far we might want to put the photodetector.. Then we have the conditions

$$w \geq \frac{10 \times 6.5}{5} \times 10^{-3} = 13 \times 10^{-3}\text{m} = 1.3\text{cm}$$

So our grating must be at least 1.3 cm wide. For the grating frequency we must have

$$f_0 \geq \frac{4 \times 10^4}{1 \times 10^{-1}}\text{m} = 4 \times 10^5\text{grooves/m}$$

So the grating must have at least 400,000 grooves per meter or 400 grooves per mm. You can see from this table
http://gratings.newport.com/products/table6.asp
that this is quite attainable.

2. Interference: Sketch the interference pattern produced in the x-y plane by two plane waves, where the wavevector for wave 1 is $k_1 = (2\pi/\lambda)(x + y + z)$, and the wavevector for wave 2 is $k_2 = (2\pi/\lambda)z$. Take $\lambda = 500$nm. Quantitatively label the dimensions on your sketch.

Solution:

The electric field of each wave at point r is is $E(r) = E_0 \cos(k \cdot r)$, so the total electric field is

$$E = E_{01} \cos(\frac{2\pi}{\lambda} x r_x + y r_y + z r_z) + E_{02}\cos(\frac{2\pi}{\lambda} z r_z)$$

The intensity will be the square of the electric field:

$$I = E_{01}^2\cos^2(\frac{2\pi}{\lambda}(x r_x + y r_y + z r)) + E_{02}^2\cos^2(\frac{2\pi}{\lambda}(z r_z)) + E_{01}E_{02}\cos(\frac{2\pi}{\lambda} x r_x + y r_y + z r_z)\cos(\frac{2\pi}{\lambda} z r_z)$$

So assuming that the magnitudes of the electric fields are equal, and looking only at the interference terms, we get

$$I_{\text{interference}} = E_0^2 \cos(\frac{2\pi}{\lambda} x r_x + y r_y + z r_z)\cos(\frac{2\pi}{\lambda} z r_z)$$

Using double angle formulas:

$$I_{\text{interference}} = E_0^2(\cos(\frac{2\pi}{\lambda}(x r_x + y r_y + z r_z + z r_z)) + \cos(\frac{2\pi}{\lambda}(x r_x + y r_y + z r_z - z r_z)))$$

$$I_{\text{interference}} = E_0^2(\cos(\frac{2\pi}{\lambda}(x r_x + y r_y + 2 z r_z)) + \cos(\frac{2\pi}{\lambda}(x r_x + y r_y)))$$

In the x-y plane, we can ignore the dependence on z (and pick a constant value for z, say z=0), and the variation of intensity wil be $\cos(\frac{2\pi}{\lambda}(x r_x + y r_y)))$ You can see this variation in electric field in figure figure 1 And you can see this variation in intensity in figure figure 2 I generated the figure with the following matlab code:

```
lambda=500
x=linspace(0, 2*lambda, 100)
y=linspace(0, 2*lambda, 100)

kx=2*pi/lambda*ones(size(x));
ky=2*pi/lambda*ones(size(y));
figure(1)
set(gcf,'DefaultAxesFontSize',18, 'DefaultAxesFontWeight', 'bold')% to make axes visible
E=(cos(kx'*x+y'*ky));;
surf(x, y, E)
```

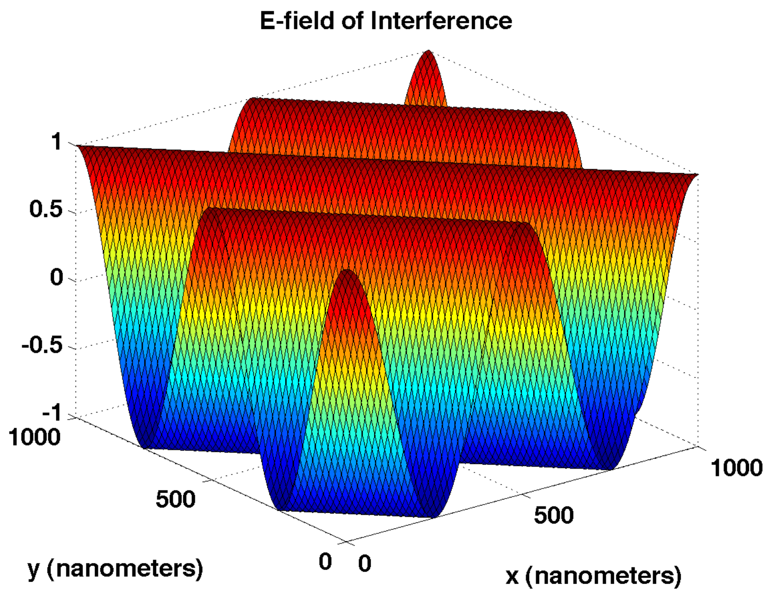

Figure 1: Interference pattern in x-y plane

```
xlabel('x (nanometers)')
ylabel('y (nanometers)')
title('E-field of Interference')
figure(2)
set(gcf,'DefaultAxesFontSize',18, 'DefaultAxesFontWeight', 'bold')% to make axes visible
E=(cos(kx'*x+y'*ky)).^2;
surf(x, y, E)
xlabel('x (nanometers)')
ylabel('y (nanometers)')
title('Intensity of Interference')

print -f1 -dpng interferenceE
print -f2 -dpng interferenceI
```

3. Youngs double-slit experiment is performed with orange light from a krypton arc (λ= 6058 angstroms). If the fringes are measured with a micrometer eyepiece at a distance 100cm from the double slit, it is found that 25 of them occupy a distance of 12.87 mm between centers. Find the distance between the centers of the two slits.

   Solution:

   The spacing between adjacent fringes is 12.87/25=0.5148 mm. This spacing, which is the distance between two maxima or two minima on the screen, is the spacing Δx when equal to $d\Delta x/\lambda D = n\pi$ (on p. 93 of notes). Since D=100 cm and λ =6065$\times 10^{-10}$m, the distance between the centers of the slits, d, is

$$d = \frac{1 \times 6065 \times 10^{-10}}{5.148 \times 10^{-4}} = 0.00118 = 1.18\text{mm}$$

4. Refer to the notes on p. 93 to identify the components of the Michelson Interferometer.

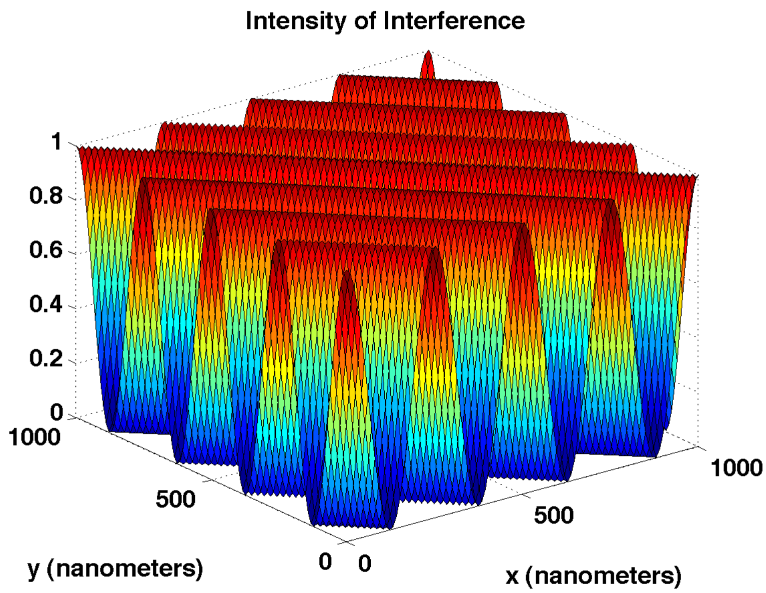

Figure 2: Interference pattern in x-y plane

(a) How far must the movable mirror of a Michelson interferometer be displaced for 2500 fringes of the red cadmium line (6438 angstroms) to cross the center of the field of view?

Solution:

$$\Delta d = \Delta m \frac{\lambda}{2} = 2500 \frac{6.438 \times 10^{-4}\text{mm}}{2} = 0.8\text{mm}$$

(b) If the mirror of a Michelson interferometer is moved 1.0 mm, how many fringes of the blue cadmium line (4799.92 angstroms) will be counted crossing the field of view?

Solution:

$$\Delta m = 2\Delta d \frac{1}{\lambda} = \frac{2 \times 1\text{mm}}{4.79994 \times 10^{-4}\text{mm}} = 4167\text{fringes}$$

5. Design an anti-reflective coating for light of wavelength 950nm to place on top of GaAs (n=3.6) Explain your design process. Specifically, what are the criteria that must be met to ensure zero reflected intensity?

Solution:

When designing an AR coating, you choose two things: the refractive index and the thickness of the coating. You choose the refractive index so that the intensity of the two reflected beams are the same:

$$n = \sqrt{n_1 n_2} = \sqrt{1 \times 3.6} = 1.9$$

You choose the thickness so that the phase shift between the two reflected beams is a quarter wavelength:

$$nd = \frac{\lambda}{4}$$

$$d = \frac{950}{4 \times 1.9} = 125\text{nm}$$

6. In an experiment involving Newton's rings between a curved and flat surface in air, the diameters of the fifth and fifteenth bright rings formed by sodium yellow light (589 nm) are measured to be 2.303 and 4.134 mm, respectively. Calculate the radius of curvature of the convex glass surface. Notes:

   (a) The wavelength of the yellow sodium line is 589 nm.

   (b) You can assume that the interference occurs in air, so the refractive index of the "thin film" material is 1.

   (c) There is a bright ring when m=0 in expression 9.42 of Hecht.

   Solution:

   Page 407 of Hecht (section 9.4) tells us that the radius of the mth bright ring is

$$x_m = \left[(m + \frac{1}{2})\lambda_f R \right]^{1/2}$$

   here we use 4 and 14 for m because we have a bright right (the "zeroth" one) when m=0;

$$2.303 \times 10^{-3} = [4.5\lambda_f R]^{1/2}$$

   so this tells us that

$$\lambda_f R = \frac{2.303^2 \times 10^{-6}}{4.5} = 1.1786 \times 10^{-6}$$

   Since the film is made in air, the refractive index is 1, so $\lambda = 589$ nm. Therefore,

$$R = \frac{1.1786 \times 10^{-6}}{589 \times 10^{-9}} = 1.98\text{meters}$$