EE119 Homework 6: Cameras and Telescopes

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1. (a) If a photograph of a race car is perfectly exposed, but blurred, at a shutter speed of 1/30s and and aperture stop setting of f/11, what must the aperture stop setting be if the shutter speed is raised to 1/120s in order to stop the motion? Give your answer as an F# number. (See p. 44 of the notes)

   Solution:

   Since we want the photograph to be perfectly exposed, the amount of light reaching the film should be the same. The speed is increased by a factor of 4, so the aperture stop needs to let in 4 times as much light in order for the amount of light to be the same. This means that the diameter of the aperture stop needs to increase by a factor of 2. We do some arithmetic: $(f/11) \times 2 = f/5.5$. So the new F# is 5.5.

   (b) If the camera lens is a diffraction limited system, what is the resolution limited angle at the new setting (use 500nm for the wavelength)?

   Solution:

   The numerical aperture is $1/2F = 1/11$, so we use

   $$h = \frac{0.61\lambda}{\text{NA}} = 0.61 \times 500 \times 10^{-9} \times 11 = 3.35 \times 10^{-6}$$

   You got credit if you just solved for his. However, this is the diffraction limit on the image side, so we can find the diffraction limited angle in the paraxial approximation by assuming that the image is located 1 focal length from the aperture. In that case, the diffraction angle is

   $$\alpha = \frac{h}{f}$$

2. You buy a new digital camera and want to test your skills by photographing two ninjas sparring. The cameras shutter speeds are: 1-1, 2-1, 4-1, 8-1, 16-1, 32-1, 64-1, 125-1, 250-1, 500-1, and 1000-1, and the aperture stops (F-stops) are 2, 2.8, 4, 5.6, 8, 11, and 16.

   (a) You took a picture of a ninja's fist flying through the air (with 1/16 s and f/16) and found that the image is blurred. Now you decide to use a faster shutter speed 1/32s to obtain a sharper image. What should be the corresponding aperture stop (F-stop?).

   Solution:

   The shutter speed increased by a factor of 2, so the light enters the camera for half as long. To have the same total amount of light hitting the film, the aperture stop needs to let in twice as much light. This means that the area of the aperture stop needs to be twice as big, which means that the diameter of the aperture stop needs to be $\sqrt{2}$ times as big. So we need to decrease the F# by one stop, giving a new F# of f/11.

(b) In your original picture (with 1/16 s and f/16), the ninja's fist was about to smash a board in half. You were positioned behind the fist, and you were able to have both the board and the fist just barely in focus. Will both objects still be in focus with the new settings you found in part a?

Solution:

The objects won't be in focus any more. The depth of field is proportional to F^2, and since the F-number decreased, the objects that were just in focus will no longer be in focus.

(c) If the setting found in part (a) is barely good enough to take a sharp picture of the ninja's fist (30mph), can you take sharp pictures of the fist of the world-class ninja who can move three times faster? Assume that the required shutter speed is inversely proportional to the speed of the moving object. That is, the new shutter speed will have to be three times faster. Find the settings required to take sharp pictures of the world-class ninja's fist.

Solution:

The new shutter speed will have to be $1/3 \times 32 = 1/96$s. To prevent blurring, you use the next fastest shutter speed, which is 125^-1. With this setting, the amount of light coming in decreased by 125/32=3.8$\approx$ 4. The aperture area needs to be four times bigger than the one used in part a, which was f/11. The aperture diameter needs to increase by a factor of 2, so you go down by 2 stops to f/5.6

(d) If the settings on your camera remain the same, will using larger film change your picture? If so, how?

Solution:

Larger film will make your image larger, so you will capture more of the image on your photograph.

3. Captain James Cook is sailing through the south pacific on a mission from the Royal society to observe the passage of Venus across the sun. He has a telescope with him, but he doesn't know how good it is, so he brings you along to help him figure it out. The eyepiece of the telescope is 2 cm in diameter and has a focal length of 4 cm. The clear aperture of the exit pupil is 0.35 cm. The telescope has an angular magnification of 45. Assume normal visual acuity, which means that the spacing between the detectors in the eye of the viewer is 0.3 milliradians.

(a) What is the focal length of the objective lens?

Solution:

The angular magnification of a telescope is $m = f_o/f_e$ so the objective lens focal length is $f_o = m * f_e = 4 \times 45 = 180$ cm.

(b) What is the diameter of the objective lens?

Solution:

The clear aperture of the exit pupil is 3.5 mm. The distance between the objective lens and the eyepiece is $-d_1 = f_0 + f_e$=184 cm. The magnification at such a distance is m=$f_e/(f_e + d_1)$=$f_e/(f_e - f_o + f_e) = -f_e/f_0$=-1/45. So the exit pupil is 45 times bigger than the objective lens, meaning that the objective lens has a diameter of 3.5$\times$45=157.5 mm

(c) What is the object field angle? The object field angle is the maximum angle that defines the field of view of the telescope due to the diameter of the lenses (and not to diffraction)

Solution:

The object field angle is the angle subtended by the chief ray that barely touches the outside of the eyepiece lens. So it is given by

$$\tan(\theta) = \frac{d_e}{2} \frac{1}{f_e + f_o} = \frac{d_e}{2(f_e + f_o)} = \frac{1}{184} = 0.0054$$

$$\theta_o = 0.3114^\circ$$

(d) What is the image field angle?

Solution:

The object and image field angles are related to each other by the angular magnification, so the image field angle is $M \times \theta_o = 14^\circ$

(e) What is the limit of the telescope's resolution due to diffraction?

Solution:

The diffraction limited angle is determined by diffraction at the entrance pupil, which is the objective lens. It has a diameter of 15.75 cm=0.1575 m

$$\theta \approx \sin(\theta) = \frac{1.22\lambda}{D} = \frac{1.22 \times 5 \times 10^{-7}}{0.1575} = 3.873 \times 10^{-6}$$

The image angle that this corresponds to is $45 \times \theta = 1.7429 \times 10^{-4} = 0.17$ mrad This is half of the resolution of the eye due to spacing of cones, so the limit of resolution on the telescope is your eye, not diffraction.

(f) Captain Cook sees a ship in the distance, and he wants you to tell him if the ship is a pirate ship or a friendly merchant ship. The black flag of the ship has white letters, so the only way to distinguish it from a pirate ship with a skull and crossbones is to read the text on the flag. You can read the text on the flag if you can separate features that are 10 cm apart. Using this telescope you've just analyzed, how close do you have to be to the ship before you can tell Captain Cook if you're dealing with merchants or pirates?

Solution:

Since the limit on resolution of the microscope is the eye, you can resolve the an angle of 0.3 mrad/M=$3 \times 10^{-4}/45 = 6.7 \times 10^{-6}$mrad. Since this angle is small, we can use the paraxial approximation, so $6.7 \times 10^{-6} = 0.1/D$ where D is the minimum distance in meters between the Eminent British Captain and the mysterious black-flagged ship of potential doom. $D = 0.1/6.7 \times 10^{-6} = 1.5 \times 10^4$ meters= 15 km.

4. Captain Cook decides that he wants you to modify his telescope to make it more useful for observing the night sky. He wants you to increase the clear aperture of the exit pupil to 8 mm to make it practical to view at night. Design an astronomical telescope which can resolve an object on the moon which is 1/50 the size of the moon (the moon is 3476 km in diameter and orbits at roughly 384,300 km from the earth). Captain Cook doesn't like wasting money, so he doesn't want you to make any of the lenses larger or more powerful than they need to be. Assume normal visual acuity. Your solution should include

- the overall magnification of the telescope
- a diagram and ray trace of the system;
- focal lengths and diameters of the lenses;
- sizes of the aperture stop, entrance pupil, and exit pupil;
- The limits on the resolution of the telescope due to diffraction and your eye's resolution

Since this is a design problem, there is no one right answer, so it is very important for you to clearly explain each step and make it easy for whoever is reading your problem to understand your reasoning.

Solution:

The telescope must be able to resolve a feature that is 3476/50=69.5 km wide and 384,300 km away, which corresponds to an angle of $\theta \approx \tan(\theta) = 69.5/384300 = 1.8 \times 10^{-4}$. The aperture diameter that corresponds to this limit on diffraction is $D = 1.22\lambda/1.4 \times 10^{-4} = 3.4 \times 10^{-3}$. The magnification that would be required for the cones of the eye to resolve this is $3 \times 10^{-}4/1.8 \times 10^{-4} = 1.6$. This is not very strong–you don't have to spend too much of Captain Cook's money. So you can pick an objective lens of focal length of, say, 5 cm, and an eyepiece of focal length of 3 cm. Or 50 and 30 cm, if you prefer.

The exit pupil must be at least 8 mm large. The magnification of the objective lens is $f_e/f_o = 1/1.6$ so the diameter of the objective lens must be at least $8 \times 1.6 = 12.8mm$.

So the cheapest thing you could get away with is using a 5 cm objective lens with diameter 1.2 cm, and a 3 cm eyepiece lens. But you probably should do better than that.