EE119 Homework 7: Microscopes, Projectors and Photomultiplier Tubes

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Due Monday, March 30, 2009, or by the end of spring break

1. Design a simple (single lens) microscope to resolve an object size of 3 microns such that the image can be viewed with a fully relaxed eye. Assume normal visual acuity of the user.

Solution:

Since the eye can normally resolve 75 microns, the magnification should be 75/3=25. In order for the eye to be relaxed, the image should be at about 25 cm. The magnification of a lens in this configuration is 25/f, so we need a focal length of 1 cm.

2. Compound Microscope

A microscope consists of an objective and an eyepiece. The total magnification is 20. The eyepiece has a focal length of 5 mm, and the tube length of this microscope is the standardized 160 mm. The working distance of a microscope is the distance between the sample and the objective lens, or s_1 in the lecture notes. What is the working distance of the microscope?

Solution:

The total magnification is the product of the magnifications of the eyepiece and the objective. At normal viewing distance, where the object appears to be 25 cm from the eye, the magnification of the eyepiece is 25/f=25 cm/5 mm=250/5=50. Since the overall magnification is 20, the magnification of the objective is 50/20=0.4.

The magnification of the objective is given by x'/f_o , where f_o is the focal length of the objective. Since x' is 160 mm, we know that $f_o=160 \text{mm}/0.4=40$ cm. In order for the objective lens to form a real image at a distance $f_o + x'$ away from the lens, the object must be located a distance s_1 from the lens. We can find s_1 using the lens law:

$$\frac{1}{56} = -\frac{1}{s_1} + \frac{1}{40}$$
$$\frac{1}{s_1} = \frac{1}{40} - \frac{1}{56} = \frac{16}{2240} = \frac{1}{140}$$

so $s_1 = 140$ cm. This is a lot bigger than for a normal microscope, where the working distance would be about a centimeter.

3. Microscope Design

Design a compound microscope with the following characteristics:

• It can resolve 0.5 micron feature size (or lower)



Figure 1: distances in a microscope

- The working distance should be greater than 5 mm
- Standardized Tube Length (160 mm)
- The microscope is not diffraction-limited

Provide in your solution your reasoning, a ray trace of your system, the focal lengths of the optics and Numerical Aperture (NA) of the objective. Show that your design meets specifications. Assume normal visual acuity.

Solution:

Since normal visual acuity can resolve 75 microns at a comfortable viewing distance, we need to a magnification of at least 75/0.5=150. The magnification is the product of the transverse magnification of the objective with the angular magnification of the eyepiece. From the notes, we have

$$M_{\rm TOT} = \frac{x'}{f_o} \frac{25}{f_e} = \frac{16}{f_o} \frac{25}{f_e} = \frac{400}{f_o f_e}$$

So the product of the objective and eyepiece focal lengths needs to be less than 400/150=8/3 cm.

If the working distance has to be greater than 5 mm, then this places a constraint on the objective focal length:

$$\frac{1}{16\text{cm} + f_o} = \frac{1}{f_o} - \frac{1}{0.5\text{cm}}$$

$$\frac{1}{0.5\text{cm}} = \frac{1}{f_o} - \frac{1}{16\text{cm} + f_o} = \frac{16}{f_o(f_o + 16)}$$

Rearranging terms gives

$$f_o(f_o + 16) = 8$$

$$f_o^2 + 16f_o - 8 = 0$$
 (1)

This is a quadratic formula, which has solution

$$f_o = \frac{-16 \pm \sqrt{256 + 32}}{2} = \frac{-16 \pm \sqrt{288}}{2} \approx \frac{-16 \pm 17}{2}$$

The positive solution to this quadratic equation is $f_o = 0.5$. Notice that if the working distance were larger, then f_o could be larger too. But let's use $f_o = 0.5here$. This means that the eyepiece focal length should be $8/(3\times0.5)=16/3=5.333$ cm.

Now we need to find some diameters for these lenses. For spherical lenses, you can't have the diameter of a lens be greater than twice its radius of curvature, and remember that the focal length for a glass (index 1.5) lens is approximately half the radius of curvature. So the largest diameter we could have for an objective lens is 2 cm, but 1 cm or even less is more realistic, especially when you consider the effects of spherical aberration. For a 1 cm entrance pupil and 5 mm working distance, the numerical aperture is

$$NA = \sin(\tan^{-1}(\frac{0.5}{0.5})) = \sin(\tan^{-1}(1)) = \sin(45^{\circ}) \approx 0.7$$

The diffraction limited resolution at 500 nm is

$$h_{\min} = \frac{0.61\lambda}{NA} = \frac{0.61 \times 5nm}{0.7} \approx 430nm = 0.43microns$$

So the diffraction limit is slightly smaller the the feature size we need to resolve, but not by much. If we were using a longer wavelength, greater than 580 nm, we would have a problem. The diameter of the eyepiece should be big enough to capture the marginal rays going through the objective. Since the magnification of the objective is 16/0.5=32

- 4. On his voyage last week through the South Pacific, Captain James Cook encountered some pirates and barely survived the attack. He decides to scare off the pirates while he sails around Tahiti by projecting a big skull and crossbones on the sail of his ship. Fortunately, he has brought you along, and you have a transparency of a skull and crossbones, as well as a lightbulb and some lenses. The focal length of your lenses is 30 cm and 8 cm. The lamp has a coil filament, so you will have to design a projector to illuminate the slides uniformly. Assume that you need a magnification of M = -100 in order to project the skull and crossbones onto the sail.
 - (a) Draw a picture of the illumination system with the two lenses. The picture does not need to be to scale, but rays should be included to indicate where the filament of the lamp imaged. Label the lamp, the projector lens, the condenser, the slide and the screen in your diagram. Which lens will you use as the projector and which will you use as the condenser? Put the slide in contact with the condenser lens.

Solution:

See Figure 2

(b) Find the distance between the slide and the projector lens.

Solution:

We want the magnification of -100, which will be the ratio d_2/d_1 . So $d_2 = -100d_1$. We can solve for d_1 because we know the magnification

$$m = \frac{f_p}{d_1 + f_p} = \frac{30}{d_1 + 30} = -100$$

$$3 = -10d_1 - 300$$
$$d_1 = \frac{-303}{10} = 30.3$$

so the distance between the slide and the projector is 30.3 cm.



Figure 2: lenses in a projector

(c) Find the distance between the projector lens and the screen.

Solution:

Since the magnification is 100, the distance between the projector lens and the screen is 30.3 meters.

(d) The light from the filament should be focused on the projector lens–in other words, the image of the filament should be on the projector lens. Find the distance between the filament and the condenser.

Solution:

The distance from the projector and the condenser lens is 30.3 cm. The focal length of the condenser is 8 cm, so the object position of the filament is

$$\frac{1}{30.3} = \frac{1}{8} - \frac{1}{s_1}$$
$$\frac{1}{s_1} = \frac{1}{8} - \frac{1}{30.3} = \frac{1}{10.87}$$

so the distance from the projector and condenser is 10.87 cm.

Also: See section 8.12 in Hecht for a description of liquid crystal displays, if you're interested

- 5. For a photo-multiplier tube, assume constant quantum efficiency and constant light power.
 - (a) Plot the variation of cathode current as a function of wavelength between 200 nm and 1000 nm. Assume a current of 10nA at 200nm.
 - (b) If the photocathode material has a work-function of 1.7 eV, what is the maximum wavelength it can detect?

Solution:

We use 1240=1.7 eV·nm, so the minimum detectable wavelength (maximum detectable energy) is 730 nm.

6. A certain PMT has 12 stages. In the first five stages, each primary electron can stimulate four secondary electrons. In the next seven stages, each primary electron can stimulate five secondary electrons.

(a) What is the gain of this PMT?

Solution:

 $gain=4^5 \times 5^7 = 8 \times 10^7$

(b) The pulse width is 9 nsec, and the rise time is 3 nsec. The load resistance is 50Ω . What is the approximate peak voltage observed for the single photon?

Solution:

A single photon will generate 10^7 electrons in 9 nsec. the peak current will then be

$$\frac{1 \times 10^7 \text{electrons} \times 1.6 \times 10^{-19} C/\text{electron}}{9 \times 10^{-9} \text{seconds}} = 0.18 \times 10^{-3} \text{Amps} = 0.18 mA.$$

and the voltage of this current over a 50 Ω resistor is 8.9 mV.

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(c) This problem is optional. Choose a coupling capacitor that is suitable for this photon counting system (ie. Choose a capacitor so that the pulse width is not broadened). Assume that both the PMT load resistor and the input impedance of the amplifier are 50Ω .

Solution:

You want to choose a coupling capacitor so that the time constant is smaller than the rise time by a lot. The rise time of the pulse is 3 ns, so you want to find C such that

$$50 \times C \ll 3 \times 10^{-9}$$

which gives

$$C \ll 6 \times 10^{-11} = 60 pF$$

- 7. A PMT has a cathode dark current of 2 $\times 10^4$ electrons per second at 300K (room temperature).
 - (a) What is the equivalent shot noise to this dark current? Assume the bandwidth is 1Hz.

Solution:

If we detect N electrons, the shot noise associated with these electrons will be \sqrt{N} , so the shot noise of this dark current is 141 electrons per second.

(b) We can distinguish a light signal if it just barely exceeds the shot noise level of the dark current. Based on this criterion, what is the minimum power that can be detected? Assume the quantum efficiency is 50 % at $\lambda = 630$ nm.

Solution:

The energy in this photon ins 1240/630 nm=1.97 eV. The power that would be required to generate the shot noise is $n_s \times \text{energy per photon}/QE = 141 \times 2 \times 1.97 \text{eV}=555 \text{ eV/second}=8.89 \times 10^{-17} \text{Watts}$. This is the minimum detectable power.

(c) If the PMT is cooled down by 80 °C (down to to 220 K), what are the cathode dark current and the minimum detectable power? Assume the work-function is 1.4 eV. Solution:

The dark current is proportional to e^{-E/k_BT} . Here E=1.4 eV and at 300 Kelvin, $k_BT = 1/40$. At 220 K, $k_BT = 1/40 \times 220/300 = 5.5/300 \approx 1/55$ eV. So the dark curren goes down by a factor of $e^{-55 \times 1.4}/e^{-40 \times 1.4} = e^{-15 \times 1.4} = 7.5 \times 10^{-10}$. So the

dark current goes down to 3.2×10^{-6} electrons/second; this is really really low. The shot noise is the square root of this, which is about 1.8×10^{-3} electrons/second, and the minimum detectable power becomes $8.89 \times 10^{-17} \times 1.8 \times 10^{-3}/141 = 1.1279 \times 10^{-21}$ W