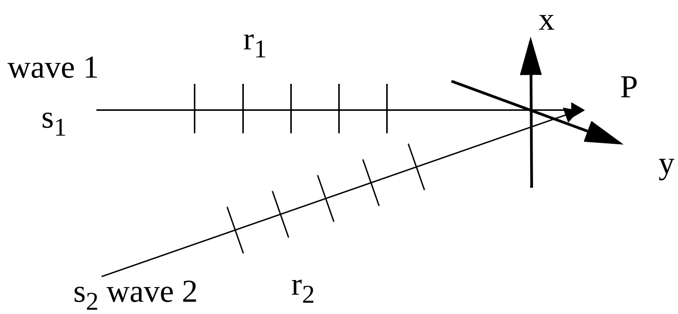
Chapter 7
INTERFERENCE

[Reading assignment: Hecht 9.1, 9.3 (to p. 396 only), 9.4, 9.7.2, 9.8.2, 9.8.3]

Interference occurs when light from different sources or different paths are superimposed. As an electromagnetic wave, when two waves superimpose, it is the electric field amplitudes that add.



Let the two sources radiate plane waves so that

$$E_1 = A_1 \cos\left(\omega_1 t - \frac{2\pi r_1}{\lambda_1} + \phi_1\right) = A_1 \cos(\omega_1 t + \alpha_1)$$

$$E_2 = A_2 \cos\left(\omega_2 t - \frac{2\pi r_2}{\lambda_2} + \phi_2\right) = A_2 \cos(\omega_2 t + \alpha_2)$$

At P, we add the fields

The intensity is generally what is detected

where the average is a time average over detector response time,

$$= \langle A_1^2 \cos^2(\omega_1 t + \alpha_1) + A_2^2 \cos^2(\omega_2 t + \alpha_2)$$
$$+ A_1 A_2 \cos[(\omega_1 + \omega_2)t + (\alpha_1 + \alpha_2)] + A_1 A_2 \cos[(\omega_1 - \omega_2)t + (\alpha_1 - \alpha_2)]\rangle$$

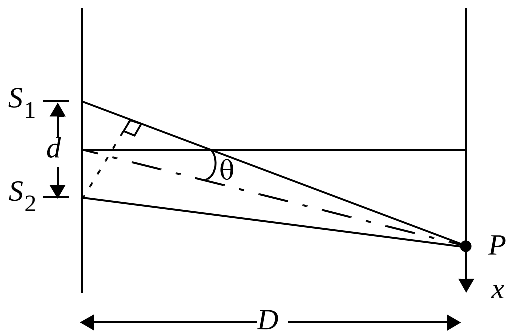
When $\omega_1 \neq \omega_2$, the $\cos(\omega_1 \pm \omega_2)t$ terms average out

If , then we get an interference:

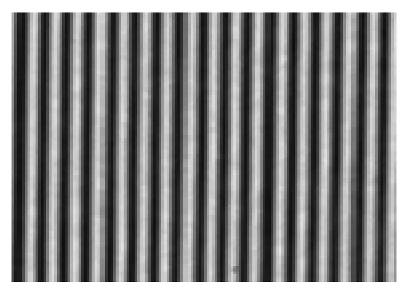
$$\begin{aligned} I(P) &= I_1 + I_2 + \langle A_1 A_2 \cos[2\omega t + (\alpha_1 + \alpha_2)] + A_1 A_2 \cos(\alpha_1 - \alpha_2)\rangle \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\alpha_1 - \alpha_2) \\ &= I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\left[\left(\frac{2\pi}{\lambda}\right)((r_1 - r_2) + (\phi_1 - \phi_2))\right]}_{\text{interference term}} \end{aligned}$$

The intensity observed shows maxima + minima as the *path length difference* $(r_1 - r_2)$ varies [assumes phase factors do not vary (ϕ_1, ϕ_2)] This gives rise to constructive and destructive interference. The phase difference is .

Young's two-slit interference experiment



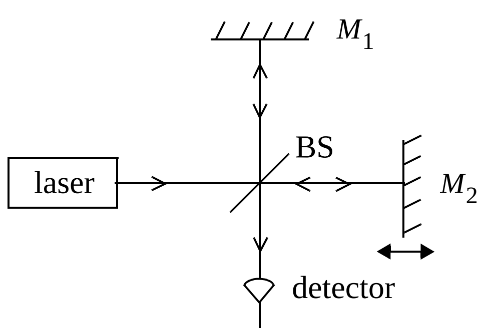
Phase difference



$$I = 2I_1 + 2I_1 \cos\frac{2\pi dx}{\lambda D}$$

"fringe pattern"

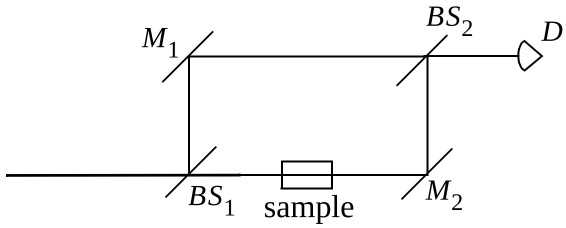
Michelson interferometer



As M_2 moves, the detected intensity changes. The light double passes the M_2 arm, so when M_2 moves by $\lambda/2$, the detected intensity changes sinusoidally from I_{max} to I_{min} to I_{max} again (1 full cycle).

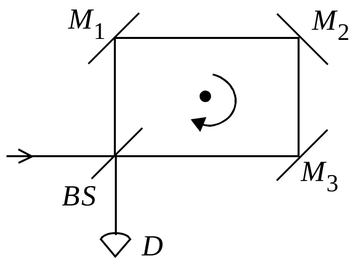
Used for distance measuring. As M_2 moves, we count cycles using the detector. With good S/N ratio and a stable laser, movement as small as $\lambda/1000$ (~ 5A°!) can be measured. Large movement can be measured with this accuracy. Such interferometers are very useful for very precise servo control of positioning systems.

Mach-Zender



This interferometer can be used for measuring material properties. If the index of refraction of the sample varies, then the phase difference varies and the intensity at D varies. As an example, one can determine the temperature dependence of the index of refraction n for air or other gases.

Sagnac interferometer (modified Mach-Zender)

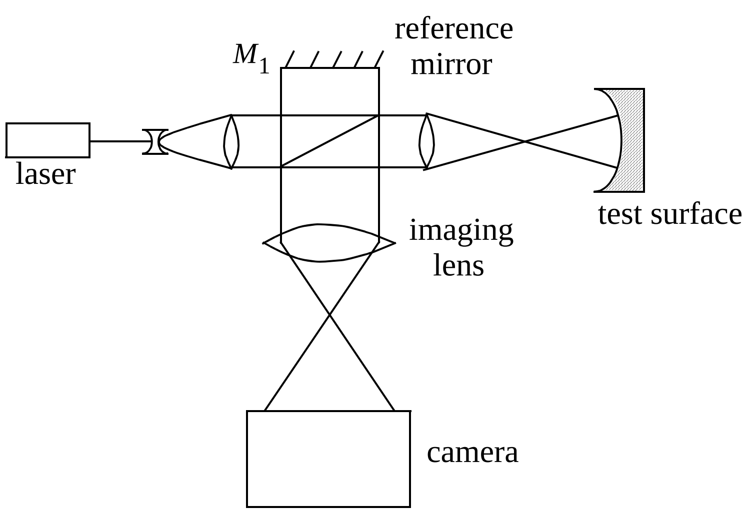


If the interferometer is rotating clockwise, the clockwise light has a longer time-of-flight than the opposite direction.

# of fringes shift $N = \dfrac{4A\Omega}{c\lambda}$ A: area
Ω: rot vel

By using a spool of fiber instead of discrete mirrors, a very stable arrangement can be made and sensitivity is increased by n, the number of turns of fiber on the spool. This is called the “fiber-ring gyro,” very popular in inertial navigation.

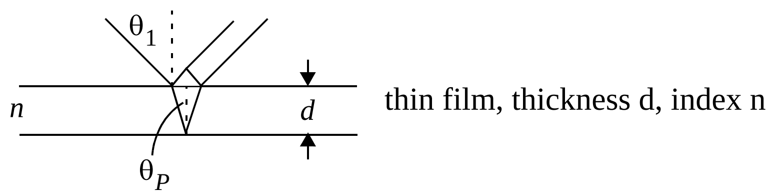
Twyman-Green interferometer



If the test surface is perfect, then the path length is identical across the beam, and the intensity is uniform on the camera. M_1 can be translated, and then like the Michelson interferometer, the intensity varies $\max \to \min$ uniformly over the camera. If the mirror has aberration, then a fringe pattern appears on the camera that gives a signature of the aberration.

A very accurate measurement of the aberration is made by varying M_1 . At each point in the image there is an oscillation in intensity as M_1 moves. With aberration, there are variations in the phase from point to point. This can be determined very accurately. This technique is called *Phase-Shifting Interferometry* (PSI).

Thin film interference

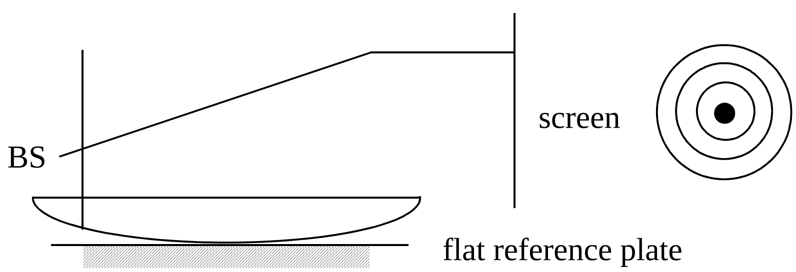


The phase difference between the reflected rays can be shown to be

For , we get a bright fringe; for , we get a dark fringe.

Variations in d, λ, n , or θ give rise to fringes.

Newton rings

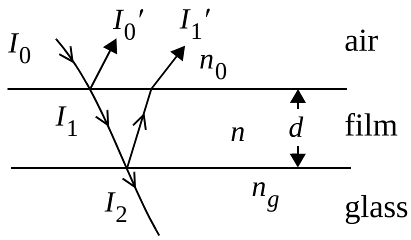


If the test surface is spherical, concentric ring fringes are observed. The reference surface must be well-known.

– Useful for testing flats. Quick test on spheres.

– Reference surface could also be spherical.

Anti-reflection (AR) coating



The Fresnel reflection coefficient at the top surface is

$$R_o = \left(\frac{n - n_0}{n + n_0}\right)^2 \qquad I_o' = R_o I_o$$

where the typical value for R_o is ~ 4%.

At the bottom surface:

$$R_g = \left(\frac{n_g - n}{n_g + n}\right)^2 \qquad I_1' \cong I_o R_g$$

I_1' and I_o' interfere *destructively* if

or $nd = (2m + 1)\dfrac{\lambda}{4} = \dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}, \ldots$

"quarter wave"

The *net* reflected intensity is zero if I_1' and I_o' are equal, but out of phase.

So, $$\frac{n - n_o}{n + n_o} = \frac{n_g - n}{n_g + n}$$

$$(n - n_o)(n_g + n) = (n_g - n)(n + n_o)$$

$$n^2 - n_o n_g - n_o n + n n_g = n_g n - n^2 - n n_o + n_o n_g$$

$$2n^2 = 2n_o n_g$$