EE 119 Homework 2 Solution

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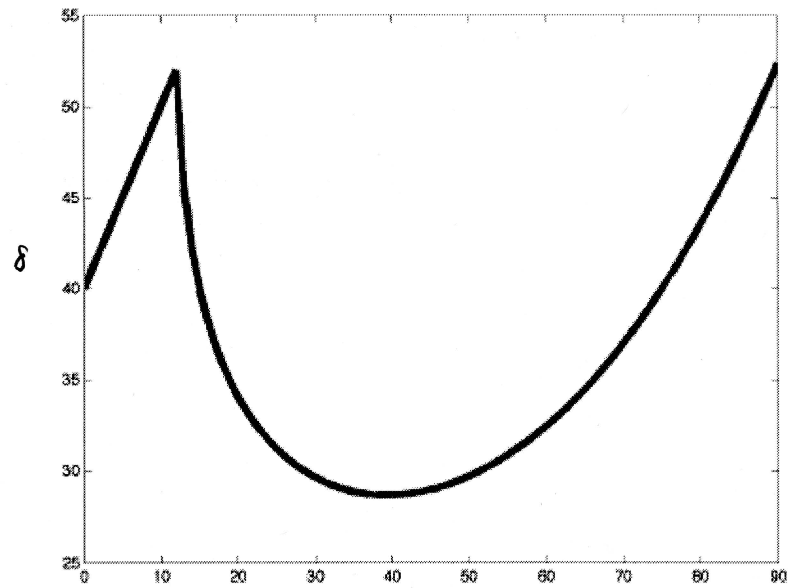
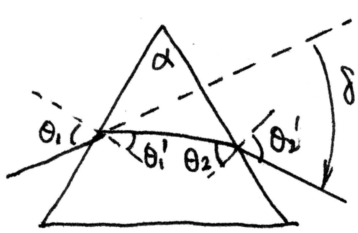
1. (a) The plot is shown as below.

Angle of deviation of a prism for $\alpha = 60°$



(b) Follow Hecht p 187 - 188 ; Start with the formulas



$$\delta = (\theta_1 - \theta_1') + (\theta_2' - \theta_2)$$

$$\alpha = \theta_1' + \theta_2 \qquad ①$$

Thus $\delta = \theta_1 + \theta_2' - \alpha$

From Snell's Law, $\sin\theta_1 = n\sin\theta_1' \qquad ②$

$$\sin\theta_2' = n\sin\theta_2 \qquad ③$$

We want to find the value of θ_1 for which δ is minimized. We can take the derivative of δ and set it to zero:

$$\frac{d\delta}{d\theta_1} = 1 + \frac{d\theta_2'}{d\theta_1} = 0$$

so this means that at the position of minimum deviation,

$$\frac{d\theta_2'}{d\theta_1} = -1$$

Taking derivative of Snell's Law (eqn. ② ③), we get

$$\cos\theta_1 d\theta_1 = n\cos\theta_1' d\theta_1' \qquad ④$$

$$\cos\theta_2' d\theta_2' = n\cos\theta_2 d\theta_2 \qquad ⑤$$

And on differentiating Eq ①, $d\theta_1' = -d\theta_2$

Dividing ④ by ⑤ on both sides, and substituting for the derivatives leads to $\dfrac{\cos\theta_1}{\cos\theta_2'} = \dfrac{\cos\theta_1'}{\cos\theta_2}$

$$\Rightarrow \frac{\cos^2\theta_1}{\cos^2\theta_2'} = \frac{\cos^2\theta_1'}{\cos^2\theta_2}$$

$$\Rightarrow \frac{1-\sin^2\theta_1}{1-\sin^2\theta_2'} = \frac{1-\frac{\sin^2\theta_1}{n^2}}{1-\frac{\sin^2\theta_2'}{n^2}} = \frac{n^2-\sin^2\theta_1}{n^2-\sin^2\theta_2'}$$

The value of θ_1 that satisfies the above equation is the one for which $d\delta/d\theta_1 = 0$. Since $n \neq 1$, it follows that

$$\theta_1 = \theta_2'$$

and therefore, $\theta_1' = \theta_2$.

This shows that the ray for which the deviation is a minimum traverses the prism symmetrically.

As $\theta_1' + \theta_2 = \alpha$, we get that at the position of least deviation $\theta_1' = \alpha/2$.

$$\Rightarrow \theta_1 = \sin^{-1}\left(n\sin\frac{\alpha}{2}\right)$$

(c) In the case when $\delta = \delta_m$, $\delta_m = \theta_1 + \theta_2' - \alpha$

$$= 2\theta_{1m} - \alpha$$

$$\Rightarrow \theta_{1m} = \frac{\delta_m + \alpha}{2}$$

From (b) $\theta_{1m}' = \frac{\alpha}{2}$.

Due to Snell's Law, $n = \dfrac{\sin\theta_{1m}}{\sin\theta_{1m}'} = \dfrac{\sin\left(\frac{\delta_m+\alpha}{2}\right)}{\sin(\alpha/2)}$

(c)

Now that the prism is put inside a liquid with refractive index n_1, we have to modify the expression of δ given in the lecture note.

$$\delta = \theta_1 - \theta_1' + \theta_2' - \theta_2 = \theta_1 + \theta_2' - \alpha$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_1',$$

$$n_2 \sin\theta_2 = n_1 \sin\theta_2', \qquad \alpha = \theta_1' + \theta_2$$

$$\Rightarrow \delta = \theta_1 - \alpha + \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right)$$

$$= \theta_1 - \alpha + \sin^{-1}\left[\frac{n_2}{n_1}\sin(\alpha - \theta_1')\right]$$

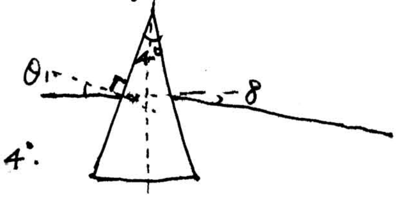
$$= \theta_1 - \alpha + \sin^{-1}\left[\frac{n_2}{n_1}(\sin\alpha\cos\theta_1' - \cos\alpha\sin\theta_1')\right]$$

$$= \theta_1 - \alpha + \sin^{-1}\left[\frac{n_2}{n_1}\left(\sin\alpha\cdot\left(1 - \frac{n_1^2}{n_2^2}\sin^2\theta_1\right)^{1/2} - \cos\alpha\cdot\frac{n_1}{n_2}\sin\theta_1\right)\right]$$

$$= \theta_1 - \alpha + \sin^{-1}\left[\sin\alpha\left(\frac{n_2^2}{n_1^2} - \sin^2\theta_1\right)^{1/2} - \cos\alpha\cdot\sin\theta_1\right]$$

$$\boxed{\delta = \theta_1 - \alpha + \sin^{-1}\left[\sin\alpha\left(\frac{n_2^2}{n_1^2} - \sin^2\theta_1\right)^{1/2} - \cos\alpha\cdot\sin\theta_1\right]}$$

The incident angle θ_1 is found to be $2°$. Substituting all values into the modified expression for δ, we have



$$\delta = 2° - 4° + \sin^{-1}\left[\sin 4°\cdot\left(\frac{1.5^2}{1.33^2} - \sin^2 2°\right)^{1/2} - \cos 4°\cdot\sin 2°\right]$$

$$= 0.62°$$

In order for the reflected beam to be parallel to the incoming laser beam, the mirror has to be rotated by $0.62°/2 = \underline{0.31°}$

2.

The ordinary component of the incoming light is the component perpendicular to x-axis (optical axis), and the extraordinary component is the component along the x-axis.

(a) The o-component and the e-component are originally in phase. $\Delta\varphi_0 = 0$. After passing through the quartz plate, their phase

$$\Delta\varphi = k n_o \Delta z - k n_e \Delta z + \Delta\varphi_0$$

$$= \frac{2\pi}{\lambda}(n_o - n_e)\Delta z + 0$$

$$= \frac{2\pi}{589.3\times10^{-9}\,m}\times(1.54424 - 1.55335)\times 1.618\times10^{-5}\,m$$

$$= -\frac{\pi}{2}$$

So the o-component (y-component) lags e-component (x-component) by $\frac{\pi}{2}$ after the quartz plate. Also as the two components have equal amplitudes, the existing light is <u>right-circularly polarized</u>

(b) From (a), we know that after passing through the quartz plane, the phase delay difference between the o-component and the e-component is $k n_o \Delta z - k n_e \Delta z = -\frac{\pi}{2}$.

Here the original phase difference between these two components is $\Delta\phi_0 = \pi$.

So after the quartz plate, the phase difference between the e-component and o-component is

$$\Delta\phi = -\frac{\pi}{2} + \Delta\phi_0 = \frac{\pi}{2},$$

which means the y-component leads the x-component by $\frac{\pi}{2}$.
We also know that $|E_y| = |E_x|$, therefore the existing beam is left-circularly polarized.

(c) the original phase difference between o- and e-component is $\Delta\phi_0 = 0$, But $|E_y| = |E|\cos 30°$, $|E_x| = |E|\sin 30°$

$$|E_y| \neq |E_x|$$

$$\Delta\phi = -\frac{\pi}{2} + \Delta\phi_0 = -\frac{\pi}{2}$$

Therefore the existing beam is right-elliptically polarized.

(d) The incident beam has only e-component, so it's itself an e-ray. Hence after existing the quartz plate, it's still linearly polarized along the same direction.

Notice that the quartz plate in this problem is a quarter-wave plate at this wave length ($\lambda = 589.3\,nm$).

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3. (a) In order for Conan to restore his appearance to his original height from his picture in the transparency, he needs the imaging system with the ability of magnifying transversely

$$M = \frac{1.6m}{8cm} = 20$$

Since both the object (Conan's picture) and the image projected are real, they should be on different side of the thin lens, and thus the image should be inverted from the object. Therefore, Conan should put the transparency with his head down so that his image projected on the wall is standing with head up.

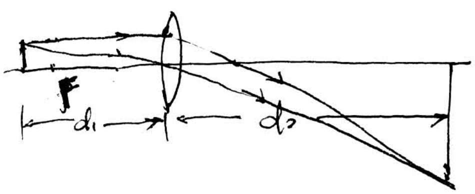
Considering the image is inverted from the object, we should add a minus sign to the transverse magnification,

$$\underline{M = -20}$$

(b) $M = \frac{d_2}{d_1} = -20$

As $d_2 = -10\text{m}$, following the sign convention, $d_1 = 0.5\text{m}$.

$$\frac{1}{d_2} = \frac{1}{f} + \frac{1}{d_1}$$



$$\Rightarrow \quad f = \left(\frac{1}{d_1} - \frac{1}{d_2}\right)^{-1}$$

$$= \left(\frac{1}{0.5} - \frac{1}{-10}\right)^{-1}$$

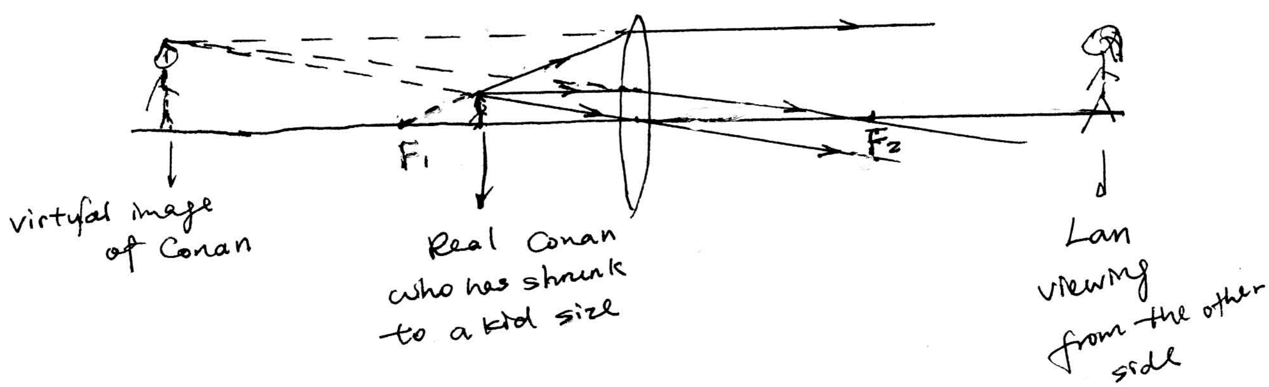
$$= \underline{0.476\text{m}}$$

The focal length of the lens is therefore 0.476m.

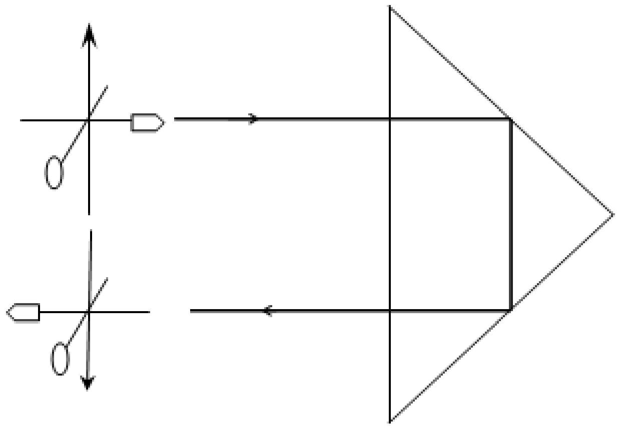
As $f > 0$, the lens is a converging lens.

(c)

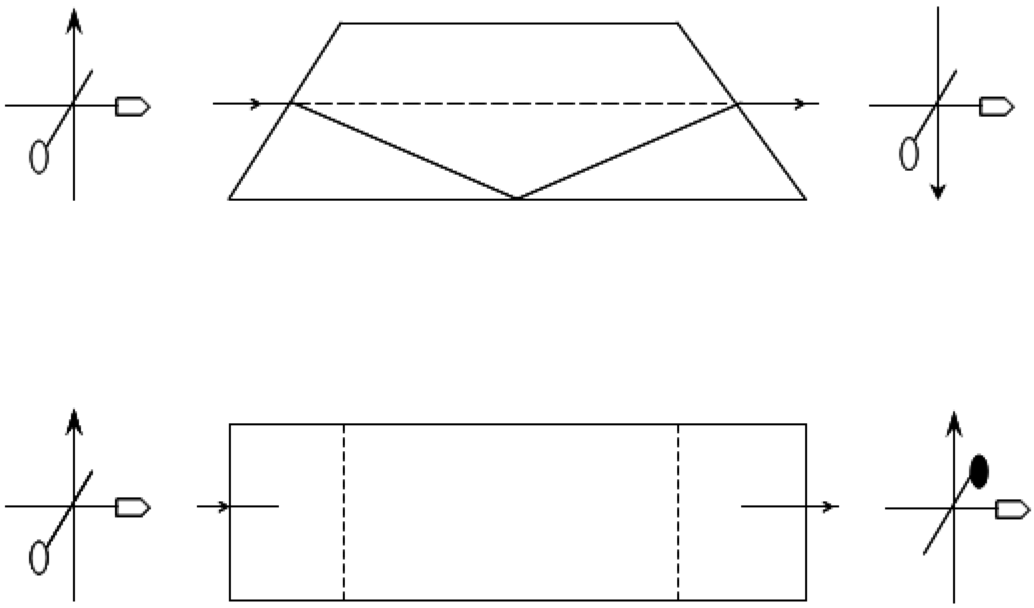
Since Lan sees Conan's image through the Lens, the image is virtual image, and it is on the same side of the lens as the real Conan is. The diagram is as below.



4. (a) **Porro prism**



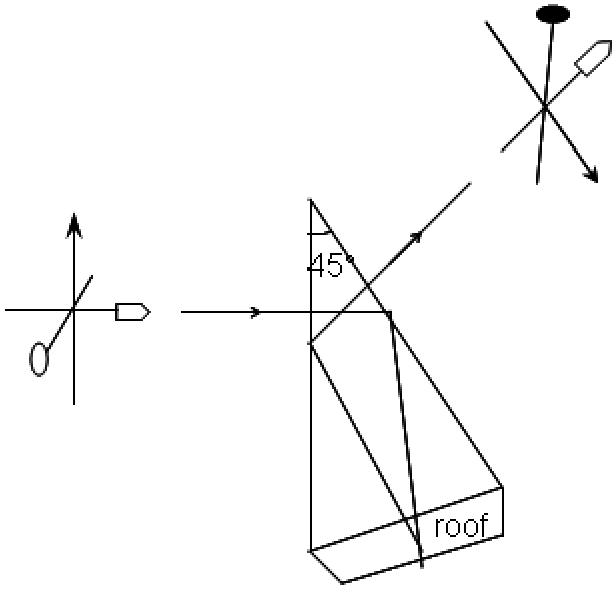
(b) **Dove prism**



We see that the orientation of the lower image, compared to that of the upper image, is rotated around by 180°the longitudinal axis of the prism, while the lower prism is rotated only by 90° around its axis in the same direction.

Again, To avoid confusion, the void circle ○ is used for lollypops pointing outside the paper plane, and the solid ● for lollypops pointing inside.

(c) **Schmidt Prism**



Yes, it can be used as an erecting prism!

(d) **Panoramic Sight**