EE 119 Homework 7 Solution

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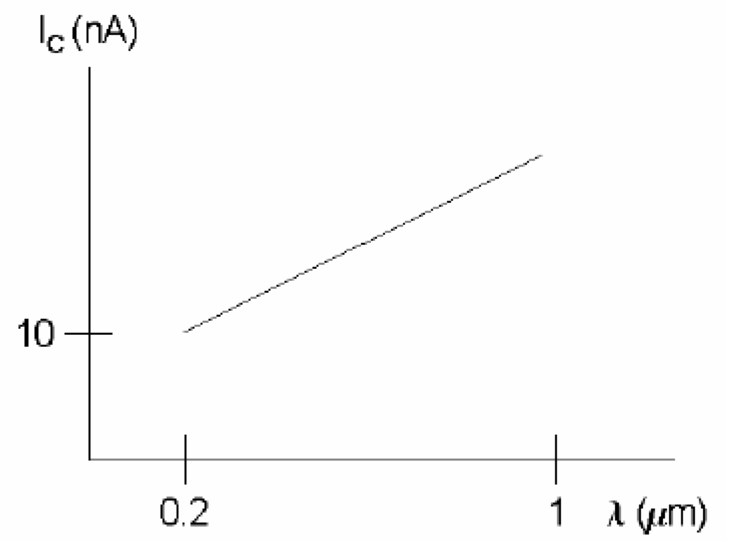
1. Photomultiplier tube
   a) Assume constant quantum efficiency and constant power. Sketch the variation of cathode current as a function of wavelength between 200nm and 1μm. (Assume a current of 10nA at 200nm).

   Solution:

$$I_{cathode} \propto n_p \cdot QE = \frac{P}{hv} \cdot QE = \left[\frac{P}{h \cdot c} \cdot QE \right] \lambda = C \cdot \lambda$$

in which C is a constant. Thus, the sketch is as the following.



   b) If the photocathode material has a work function of 1.8eV, what is the maximum wavelength it can detect?

   Solution:

The maximum wavelength it can detect is $\lambda_{\max} = \dfrac{1.24 eV \cdot um}{1.8 eV} = 0.689 um$

Or $\phi = 1.8 eV = hv_{\min} = \dfrac{hc}{\lambda_{\max}}$,

$$\lambda_{\max} = (4.17 \times 10^{-15} eV \cdot s) \times (3 \times 10^8 \frac{m}{s}) \times (\frac{1}{1.8 eV}) = 695 nm$$

2. Dark Current
   a) If the cathode dark current of a PMT is found to be $2x10^4$e/sec at room temperature (300K), what is the equivalent shot noise to this dark current? Assume the bandwidth is 1Hz.

   Solution:

Since the dark current is N = 2x10^4 electrons per second, the shot noise associated with it will be $\sqrt{N}$ = 141 electrons per second.

b) We can actually distinguish a light signal that only exceeds the shot noise level of the dark current. Based on this criterion, what is the minimum power that can be detected? Assume the quantum efficiency is 50% at λ = 630nm.
Solution: The minimum power that can be detected which just exceeds the shot noise level of the dark current is

$P_{\min} = \frac{1}{QE} h\nu \cdot n_{se} \cdot \Delta \nu$, where n_{se} is the number of electrons due to the shot noise. Therefore $P_{\min} = \frac{1}{0.5} h\frac{c}{\lambda} \cdot 141 \cdot 1 = 8.9 \times 10^{-17} W$

c) If the PMT is cooled down by 50°C (250K), what are the cathode dark current and the minimum detectable power? Assume the work function is 1.4eV.
Solution:
When the PMT is cooled down from 300K to 250K, the dark current is reduced by a factor of

$$\sigma_T = \frac{\exp(-\frac{\phi}{k \times 250})}{\exp(-\frac{\phi}{k \times 300})} = \exp(-1.4 \times (\frac{1}{0.026 \times \frac{250}{300}} - \frac{1}{0.026})) = 2.104 \times 10^{-5}.$$

So the cathode dark current is reduced to

$I'_{cathod} = 2 \times 10^4 \times 2.104 \times 10^{-5} = 0.4208$ electrons per second

Then the associated shot noise is reduced by a factor of

$\sqrt{\sigma_T} = 4.587 \times 10^{-3}$

So the minimum detectable power is reduced by the same factor to

$P'_{\min} = 8.9 \times 10^{-17} \times 4.587 \times 10^{-3} W = 4.082 \times 10^{-19} W$

3. a) A certain PMT has 12 stages. In the first five stages, each primary electron can stimulate four secondary electrons. In the next seven stages, each primary electron can stimulate five secondary electrons. What is the gain of this PMT?
Solution:

The gain is $G = \delta_1^{N1} \cdot \delta_2^{N2} = 4^5 \times 5^7 = 8 \times 10^7$.

b) The pulse width is 8nsec. With the load resistance of 50 Ohms, what is the approximate peak voltage observed for the single photon? Assume a quantum efficiency of 1.

Solution:

First find the approximate peak current-

$$i = \frac{e \cdot \# ofelectrons}{time} = \frac{1.6\times10^{-19}\times(8\times10^{7})}{8\times10^{-9}} = 1.6mA.$$

Then the approximate peak voltage observed for the single photon is $V = i \cdot R = 1.6mA \times 50\Omega = 80mV$.

4. A PMT has 10 dynodes and δ = 4. It is used to detect a faint laser beam that is operating at 632.8 nm. The quantum efficiency of the cathode at this wavelength is 25%. The laser power incident on the PMT is 1nW.

a) What is the number of incident photons per second?

Solution: The number of incident photons per second, or the photon flux,

is $$\varphi = \frac{P}{h\nu} = \frac{1\times10^{-9}W}{6.62\times10^{-34}J\times\dfrac{3\times10^{8}m/s}{632.8\times10^{-9}m}} = 3.19\times10^{9}\ photons/\sec$$

b) How many photoelectrons are generated at the cathode per second?

Solution:

The number of photoelectrons that are generated at the cathode per second

is $n_{cathode} = QE\cdot\varphi = 0.25\times3.19\times10^{9} = 7.97\times10^{8}\,electrons/\sec$.

c) What is the anode current?

Solution:

The anode current is the cathode current multiplied by the gain of the PMT,

$$I_{anode} = G\cdot I_{cathode} = \delta^{N}\cdot e\cdot n_{cathode} = 4^{10}\times1.6\times10^{-19}\times7.97\times10^{8} = 133.6\mu A.$$

d) Given an integration time of 1 second (that is, a bandwidth of 1Hz), what is the shot noise? What is the SNR?

Solution:

The shot noise is $\sqrt{n_{cathode}} = 2.82\times10^{4}$ electrons/sec, and

$$SNR = \sqrt{n_{cathode}} = 2.82\times10^{4}$$

5. Semiconductor Photodiodes: Band Gaps

The band gaps ($E_g = E_{\text{conduction band edge}} - E_{\text{valence band edge}}$) of some common photodiode materials are given in the table below. Find the equivalent wavelength (λ) in Å (to the nearest whole Å) and wavenumber (1/λ) in cm^{-1}. What spectral ranges (infrared, visible, UV…etc.) of light would these photodiodes detect?

Material	E_g (eV)
CdS	2.42
CdSe	1.73
PbS	0.37
PbTe	0.29
PbSe	0.27

Solution:

Material	E_g (eV)	λ(Å)	1/λ (cm^{-1})	spectral range
CdS	2.42	5134	19478	< visible (~green)
CdSe	1.73	7181	13926	< Near IR
PbS	0.37	33578	2978	< Far IR
PbTe	0.29	42841	2334	< Far IR
PbSe	0.27	46014	2174	< Far IR

Note

(1) The equivalent wavelength corresponding to bandgap energy is found through

$$\lambda(\mu m) = \frac{1.24(eV \cdot \mu m)}{1.8eV}$$

(2) The spectrum range that these photodiodes can detect is the range of wavelengths shorter than the λ in the third column of the table.

6. Monochromatic light is incident on the photodiodes shown in Fig. 6-1. The metal electrode is 20-nm thick and has an absorption length at this wavelength of 100 nm. The semiconductor has an absorption length at this wavelength of 600 nm. The depletion region is 300 nm with no applied bias.

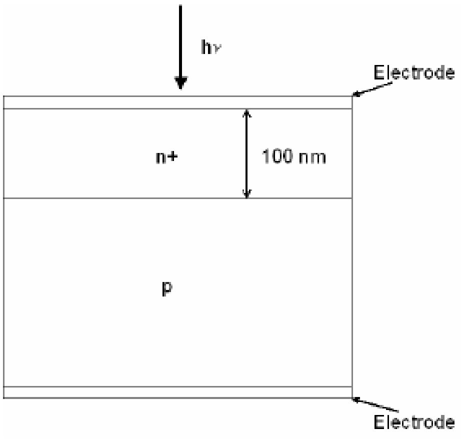


Fig. 6-1

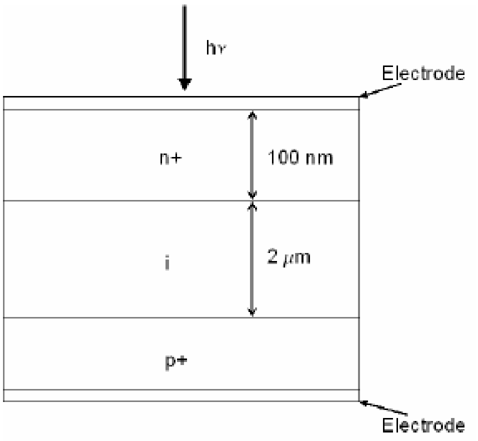


Fig. 6-2

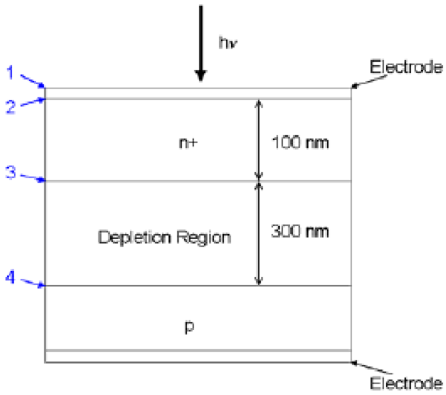
(a) Find the conversion efficiency of this photodiode neglecting any loss due to other effect such as recombination. The conversion efficiency is defined as

$$\eta \equiv \frac{\text{The number of electron} - \text{hole pairs collected in circuit}}{\text{The number of photons incident on the diode surface}}$$

Hint: light intensity decays with distance x in an absorptive medium as $I(x) = I_o \exp(-x/L_\alpha)$ in which L_α is the absorption length; only electron-hole pairs generated in the depletion region are considered collected in circuit.

Solution:

(a)



Since light intensity decays with distance x in an absorptive medium as

$$I(x) = I_o \exp(-x/L_\alpha)$$

Assume I_o is the incident light intensity at 1.

Then at 2, $I_2 = I_o \exp(-20/100) = 0.81 \cdot I_o$

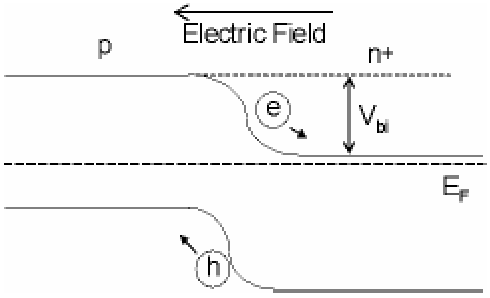
And at 3, $I_3 = 0.81 \cdot I_o \exp(-100/600) = 0.69 \cdot I_o$

And at 4, $I_4 = 0.69 \cdot I_o \exp(-300/600) = 0.42 \cdot I_o$

The amount of light absorbed in the depletion region is $I_3 - I_4 = 0.27 \cdot I_o$, therefore the conversion efficiency is 27%.

(b) Draw the energy-band diagram and label the p-side, n-side, depletion region, built-in potential and Fermi-level.

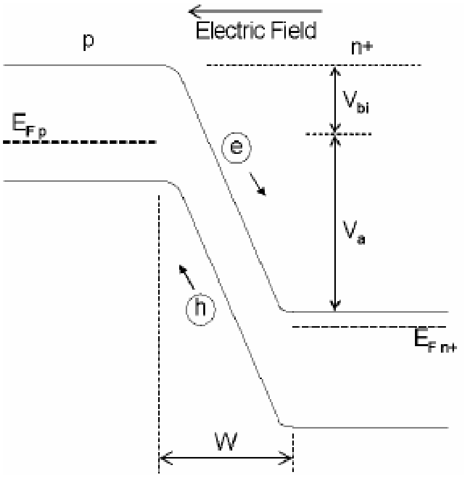
Solution:



(c) Now, suppose we apply a reverse bias to the diode. How does this change your band diagram? How would it change your answer to part (a)?

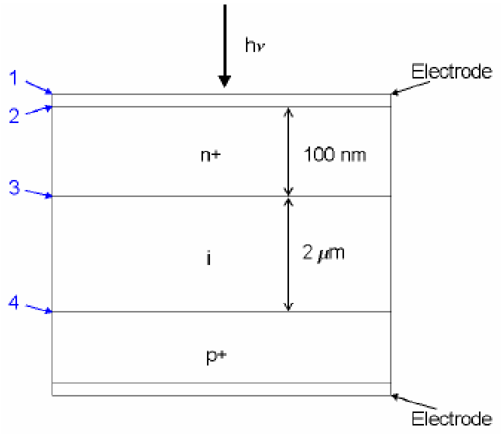
Solution:

With a reverse bias applied to the diode, the depletion region (W) becomes wider, and more electron-hole pairs are generated there and collected in the circuit. Hence, the efficiency improves.



(d) Now suppose we add a block of intrinsic silicon (2-mm long) between the p and n regions to the photodiode from the problem 1, as in Fig. 6-2. Redo the part (a) for this case. Which configuration has higher conversion efficiency?

Solution:



With the intrinsic Si added, at 4, $I_4 = 0.69 \cdot I_o \exp(-2000/600) = 0.02 \cdot I_o$.

$0.67 \cdot I_o$ is absorbed in the depletion region. The conversion efficiency is increased to 67%.