# EE 119 Homework 9

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1. A laser consists of two nearly perfectly reflecting mirrors, M, and a gain medium, G, of bandwidth Δf centered at $f_0$.
   (a) What are the allowed frequencies for laser operation in this optical cavity? Express you answer in terms of τ, the time takes light to make one round trip in the cavity.
   (b) If it is desired to produce a pulse of one picosecond ($10^{-12}$ sec) duration at a wavelength of 6000Å, what bandwidth Δf is required? [Hint: Use uncertainty principle Δf×Δt~1] What is the corresponding range of wavelengths? And how many laser modes would this involve? (Let L=1.5 m)

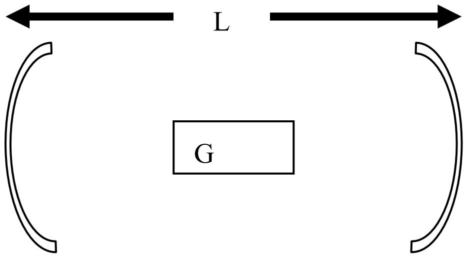



Figure 1

Solution:

(a) The only modes allowed in the cavity are standing waves for which the wavelengths λ and frequencies υ are given by

$$m\lambda = 2L,\quad f = \frac{c}{\lambda} = \frac{mc}{2L} = \frac{m}{\tau}$$, in which n is an integer.

(b) The pulse duration, Δt, is related to the bandwidth, Δf, by the uncertainty principle

$$\Delta t \cdot \Delta f \sim 1, \text{ thus } \Delta f \sim \frac{1}{\Delta t} = 10^{12}\,Hz$$

As $f = \frac{c}{\lambda}$, $\Delta f = -\frac{c}{\lambda^2}\Delta\lambda$

The corresponding range of wavelength is

$$|\Delta\lambda| = \frac{\lambda^2}{c}\Delta f \approx 12\,\text{Å}$$

For L = 1.5m, the spacing between cavity mode, $\Delta f_n = \frac{c}{2L} = 10^8\,Hz$.

Accordingly, the number of laser modes is

$$N \approx \frac{\Delta f}{\Delta f_n} = \frac{10^{12}}{10^8} = 10^4.$$

2. Identify each of the following broadening mechanisms as homogeneous or

inhomogeneous. Explain your answer.

(a) Collisions between atoms in a gas

(b) Randomly spaced impurities in a semiconductor crystal

(c) Temperature differences between different regions of the gain medium.

(d) Vibrational relaxation within an energy band of an atom or semiconductor (this is the same thing as dissipation of electronic energy into phonons within an energy band).

Solution:

(a) This is **homogeneous broadening** because all the atoms experience the same effect.

(b) This is **inhomogeneous broadening** because the amount of broadening you get from this effect varies depending on the distance from an impurity, which varies from atom to atom.

(c) This is also **inhomogeneous** because different areas of the gain medium have different amounts of such broadening.

(d) This is **homogeneous**, because each atom undergoes this process in the same way.

3. A $TEM_{00}$ (Transverse Electric Mode) He-Ne laser ($\lambda$=632.8nm) has a cavity that is 0.34 m long, a fully reflecting mirror of Radius R=10m (concave inward), and an output mirror of radius R = 10 m (also concave inward).

(a) From the symmetry of mirror geometries and the boundary condition that wavefront and mirror cavities match at the mirrors, determine the location of the beam waist in the cavity. Set z=0 at this location to be the reference plane.

(b) Determine the beam waist ($w_0$).

(c) Determine the beam spot size w(z) at the left and right cavity mirrors.

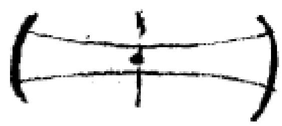
(d) Determine the half-angle beam divergence ($\theta$) for this laser.

(e) Where is the far field for this laser if you use the criterion $z_{FF} \geq 50(\pi w_0^2/\lambda)$?

(f) If the laser emits a constant beam of power 5mW, what is the average intensity ($W/m^2$) at the position where $z_{FF} = 50(\pi w_0^2/\lambda)$?

Solution:

(a) The beam waist will be at the center of the cavity by symmetry, 0.17m from each mirror.



(b) Because the radius of curvature of the beam at the end mirrors is the same as that of the mirrors, we know that R(z = 0.17) = 10m. The Rayleigh range, and from it the beam waist, is fully determined from this information.

An expression for the radius of curvature as a function of displacement from the waist is $R(z) = z + \dfrac{z_R^2}{z}$.

Plugging in numbers and solving for the Rayleigh range $z_R$:

$$10 = 0.17 + \frac{z_R^2}{0.17}$$

$$z_R = \sqrt{1.7 - 0.0289} = 1.3\,\text{m}.$$

and since the waist size is related to the Rayeleigh range through

$$z_R = \frac{\pi w_o^2}{\lambda}$$

we find that $w_0 = \sqrt{\dfrac{1.3 \times 632.8 \times 10^{-9}}{\pi}} = 5.1 \times 10^{-4} = 0.5mm$

(c) The spot size at distance z from the waist is given by

$$w(z) = w_0 \sqrt{1 + (\frac{z}{z_R})^2}\ .$$

So at z = 0.17,

$$w(z) = 0.5\sqrt{1 + (\frac{0.17}{1.3})^2} = 0.5mm$$

(d) In the paraxial regime, $\theta_d = \dfrac{\lambda}{\pi w_0} = 4 \times 10^{-6}$.

(e) The far field region occurs when $z_{FF} \geq 50(\pi w_0^2/\lambda) = 50 \times 1.29 = 64.6$ m.

(f) At z = 94.6m, the waist will be

$$w(z) = 0.5\sqrt{1 + (\frac{64.6}{1.3})^2} = 24.6mm = 0.0246m\ .$$

The area of the beam is then $\pi w(z)^2 = \pi \times 0.0246^2 = 0.0019m^3$.

The average intensity is then $I = \dfrac{P}{A} = \dfrac{5 \times 10^{-3}}{2 \times 10^{-3}} = 2.5\,\text{W/m}^2$.

4. The laser resonator shown in Figure 2 with z = 0 located at the flat mirror and its output impinges on a lens of focal length 10cm. Assume the beam waist size, $w_0$=0.5mm; laser wavelength, $\lambda$ = 632.8nm; and distance of the lens to laser output mirror, d=50 cm.
   (a) What is the far-field beam divergence of the laser in mrad if the lens is not present?
   (b) What are the spot size and wavefront radius of curvature of the output laser beam on the lens?
   (c) What is the wavefront radius of curvature after passing through the lens?

(d) What is the spot size at the focal point after the lens if the clear aperture of the lens is 1.5cm in radius?
(e) What is the beam radius if the laser beam is propagated 1m further after the focal point? And what is the far-field beam divergence after the beam passes through the focus?

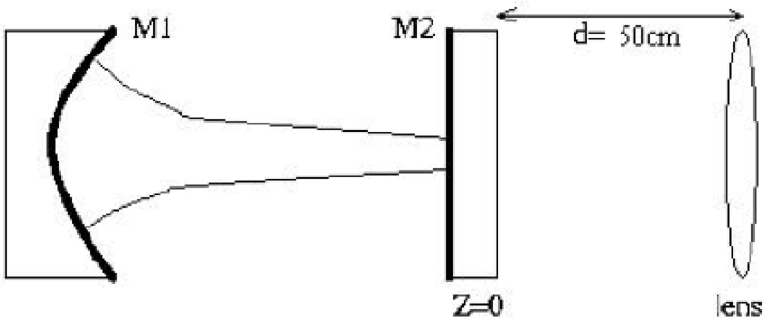



Figure 2

Solution:

(a) The far field divergence is $\theta_d = \frac{\lambda}{\pi w_0} = 4 \times 10^{-4} rad$ .

(b) The Rayleigh range of the laser beam is

$$z_R = \frac{\pi \omega_0^2}{\lambda} = \frac{\pi 25 \times 10^{-8}}{632.8 \times 10^{-9}} = 1.24m$$

The radius of curvature is

$$R(z) = z + \frac{z_R^2}{z} = 0.5 + 3.08 = 3.58m$$

And the spot size is

$$w(z = 0.5m) = w_0 \sqrt{1 + [z / z_R]^2} = 5 \times 10^{-4} \times (1 + (0.5/1.24)^2) = 5.4 \times 10^{-4} m$$

(c) The "source" of the beam is to the left, so its radius of curvature is negative.

$$\frac{1}{R_0} = \frac{1}{f} + \frac{1}{R(z)} = \frac{1}{0.1} - \frac{1}{3.58} = \frac{1}{0.103}$$

The radius of curvature of the beam after passing through the lens is 10.03cm.

(d) The clear aperture of the lens is larger than the beam (1.5>0.54). The spot size at the focal length is

$$w_0 = \frac{R_0 \lambda}{\pi w_{lens}} = \frac{0.103 \times 632.8 \times 10^{-9}}{\pi \times 0.5 \times 10^{-4}} = 3.63 \times 10^{-5} m$$

The spot size at the focal point is 0.0363 mm in radius.

(f) The rayleigh range is $z_R = \dfrac{\pi\omega_0^2}{\lambda} = 0.065m$

So the beam spot size at 1 meter from the focal point is

$$w(z=1m) = w_0\sqrt{1+\left[z/z_R\right]^2} = 15.4\times 3.64\times 10^{-5} = 5.6\times 10^{-4}m$$

The beam radius is 1.7 mm at 1 meter past the focal length. And (you didn't have to solve for this) the radius of curvature is

$$R = 1 + 0.065^2/1 = 1.00m$$

The far-field beam divergence with the lens is

$$\theta_d = \frac{\lambda}{\pi w_0} = 0.0055rad$$

5. Compare the irradiance at the retina that results when looking:
   (a) Directly at the sun. The sun subtends an angle of 0.5 degree. At the earth's surface, the sun's irradiance is $1kW/m^2$. Assume that the pupil of the bright-adapted eye is 2mm in diameter and focal length is 22.5mm.
   (b) Into a 1-mW He-Ne laser. Assume the beam waist of the laser is 1mm, and the laser is located 1m from the eye.
   (c) Which one will damage your eye? Eye-damaging intensities are in the range of $10\ \mu W/cm^2$.

Solution:

(a) The radius of the pupil is 1 mm, so the area is $\pi\times 10^{-6}$ square meters. The power entering the eye is

$$P = 1\times 10^3 \times \pi \times 10^{-6} = \pi \times 10^{-3} = 3.14mW$$

The sun subtends an angle of 0.5 degrees at the pupil, and the light travels 22.5 mm to the retina, so the radius of the image of the sun ends up being 22.5 mm $\times$ tan(0.5) = 0.1964mm. The area of this image is $0.12\ mm^2$. The intensity at the eye is then

$$I_{sun} = \frac{3.14\times 10^{-3}}{1.2\times 10^{-7}} = 2.4\times 10^4 W/m^2 = 2.4W/cm^2 .$$

(b) The Rayleigh range of the laser is

$$z_R = \frac{\pi\omega_0^2}{\lambda} = \frac{3.14\times 1\times 10^{-6}}{632.8\times 10^{-9}} = 4.96m$$

The beam radius of curvature at 1m from the waist (at the eye) is

$$w(z=1m) = w_0\sqrt{1+\left[z/z_R\right]^2} = 1\times 10^{-3}\sqrt{1+(1/4.96)^2} = 1.02mm$$

The area of the beam is $3.77\times 10^{-6}\ m^2$. The radius of curvature of the beam is

$$R(z=1m) = z + \frac{z_R^2}{z} = 1 + 4.96^2 = 25.6m$$

After passing through the eye with focal length 22.5 mm, the radius of curvature of the beam will be

$$\frac{1}{R_{eye}} = \frac{1}{f} + \frac{1}{R(z)} = \frac{1}{0.0225m}$$

(Notice that the curvature of the beam is so small (radius so big) that you can ignore it). Now we have a Gaussian beam traveling into the eye, with radius of curvature 0.0225 meters at z=0.0225. We can solve for the beam waist:

$$w_0 \approx \frac{R_{eye}\lambda}{\pi w_{lens}} = \frac{0.0225 \times 632 \times 10^{-9}}{\pi \times 0.00102} = 4.44 \times 10^{-6}$$

So the area of the beam at the retina is $6.2 \times 10^{-11} m^2$. The intensity, then, is

$$I_{sun} = \frac{1 \times 10^{-3}}{6.2 \times 10^{-11}} = 1.61 \times 10^7 W/m^2 = 1.61 \times 10^3 W/cm^2$$

(c) Both will damage your eye. But the LASER will cause MORE DAMAGE!!!

6. [Hecht 13.26] A He-Ne c-w laser has a Doppler-broadened transition bandwidth of about 1.4 GHz at 632.8 nm. Assuming n = 1.0.

(a) Determine the maximum cavity length for single-axial mode operation.

(b) What is the transition rate for the neon atoms in the laser if the power output is 1.0 mW and the energy drop is 1.96 eV?

Solution:

(a) The condition for single-axial mode operation is

$$\Delta v = 1.4 \times 10^9 = c/2L \quad \text{for n=1}$$

Hence, $L = c/2\Delta v = 11cm$.

(b) The transition rate is

$$N/t = P/\Delta E = (1.0 \times 10^{-3} J \cdot s^{-1})/(1.96 \times 1.602 \times 10^{-19} J) = 3.2 \times 10^{15} s^{-1}$$

Hence, we need $3.2 \times 10^{15}$ transitions per second.