Lecture 10

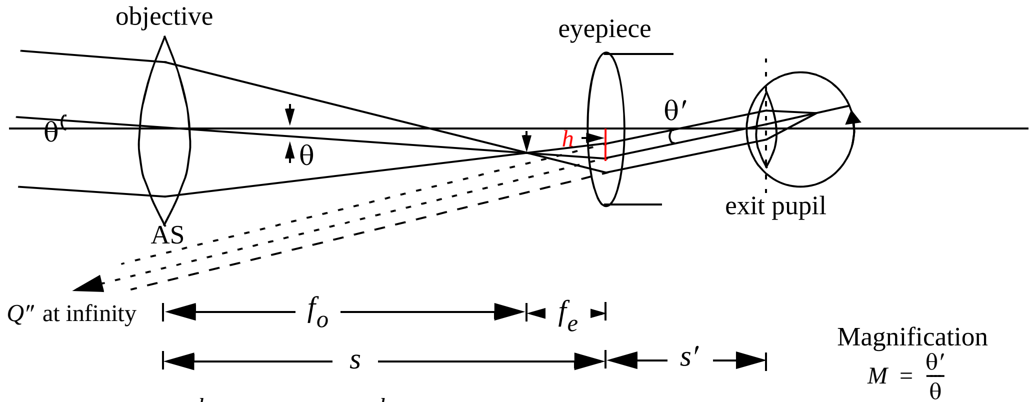
Telescope

[Reading assignment: Hecht 5.7.4, 5.7.7]

A telescope enlarges the apparent size of a distant object so that the image subtends a larger angle (from the eye) than does the object.

The telescope is an *afocal system*, which means that both the object and image are at infinity.

<u>Astronomical telescope</u>



$\tan\theta = \frac{-h}{s}$ $\qquad \tan\theta' = -\frac{h}{s'}$

Using the lens law for the eyepiece:

$$\boxed{\begin{aligned} &\frac{1}{s'} - \frac{1}{s} = \frac{1}{f_e} \qquad s = -(f_o + f_e) \\ &\frac{1}{s'} + \frac{1}{f_o + f_e} = \frac{1}{f_e} \\ &\frac{1}{s'} = \frac{1}{f_e} - \frac{1}{f_o + f_e} = \frac{f_0}{f_e(f_o + f_e)} \end{aligned}}$$

So $\tan\theta = \frac{+h}{+(f_o + f_e)} \qquad \tan\theta' = -\frac{hf_o}{f_e(f_o + f_e)}$.

For small angles, $\tan\theta \approx \theta \qquad \tan\theta' \approx \theta'$, then $\boxed{M = \frac{\theta'}{\theta} = -\frac{f_o}{f_e}}$.

The *exit pupil* is the image of the AS.

Define CA_o = entrance pupil clear aperture
CA_e = exit pupil clear aperture

From the diagram, it is clear that

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$$\boxed{\frac{CA_o}{CA_e} = \frac{s}{s'} = \frac{\theta'}{\theta} = M}$$

The eye is placed at the exit pupil, so a CA_e much larger than 3 mm is not very useful. However, making it somewhat larger makes it easier to align the eye to the eyepiece. Binoculars may have $CA_e \sim 5\text{ mm}$.

Resolution

The resolution of the eye is 1 arc min = 60 arc sec. So in a telescope, the eye can resolve objects separated by an angle α if

$$M > \frac{1}{\alpha} \qquad (\alpha \text{ in min.})$$

Now, the diffraction limit of the telescope can be written as $\alpha_T = 5.5/CA_o$, with α_T in sec. and CA_o in inches (for 550nm wavelength).

At the diffraction limit, the finest detail in the image has an angular separation of $M\alpha_T$. If this angle is at least 60 sec, the eye can resolve the detail. So, with

$$\boxed{60 = 5.5\frac{M}{CA_o} \rightarrow M_{\max} = 11CA_o}$$

At this magnification, the diffraction limit and the resolution of the eye are equal. Magnification much larger than this means that the diffraction blur spot is larger than the smallest feature that the eye can resolve. The eye sees a rather blurry image.

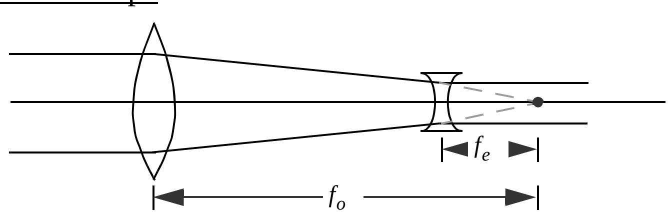
Example: $2\frac{1}{2}''$ refractor telescope $f_o = 700\text{ mm}$

$$M_{\max} \cong 28$$

$f_e = 25\text{mm objective} \rightarrow M = 28$

$f_e = 9\text{mm objective} \rightarrow M = 78$ – no increase in resolution
– hard to align the eye

Galilean Telescope



$$\boxed{M = -\frac{f_o}{f_e}}$$

f_e is negative, so $M > 0$. **Non-inverting**.

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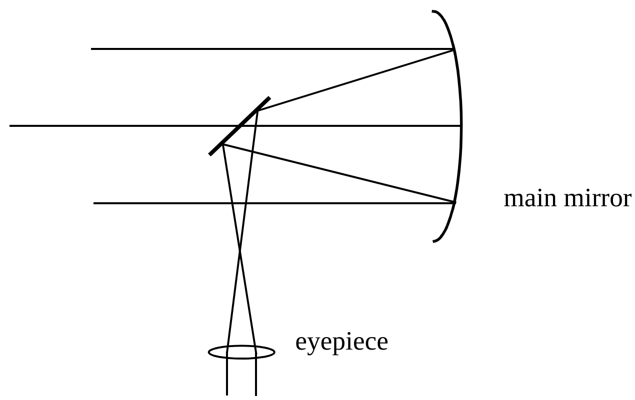
This telescope would seem to be a good candidate for binoculars. Inexpensive "field glasses" or "opera glasses" are indeed made according to this design, but it turns out to have a very limited field of view

Reflecting Telescope



All modern astronomical telescopes have this basic configuration because it is much more practical to fabricate large mirrors than lenses. The size of the large main mirror (the entrance pupil) sets the diffraction limit. Also, a larger entrance pupil gathers more light, so that faint objects can be detected. Ground-based telescopes are limited by atmospheric turbulence, which introduces unavoidable aberrations. One solution is to go into space, above the atmosphere.

The configuration shown above, with a parabolic mirror is called a Newtonian reflector. It has fairly good performance and is inexpensive, but does suffer from coma aberration for off-axis objects.

"Catadioptric" designs use a combination of mirrors and lenses to fold the optics and form an image. There are two popular designs: the Schmidt-Cassegrain and the Maksutov-Cassegrain. In the Schmidt-Cassegrain the light enters through a thin aspheric "Schmidt" correcting lens, then strikes the spherical primary mirror and is reflected back up the tube and intercepted by a small secondary mirror which reflects the light out an opening in the rear of the instrument where the image is formed at the eyepiece. The corrector lens reduces the off-axis aberrations, giving good images over a wider field than the Newtonian. An additional advantage is that the lens seals the telescope tube, which protects the primary mirror from contamination, as well as stiffening the structure.

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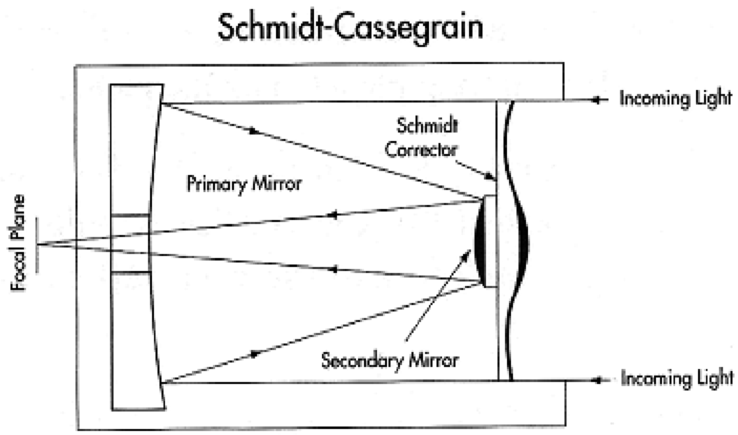
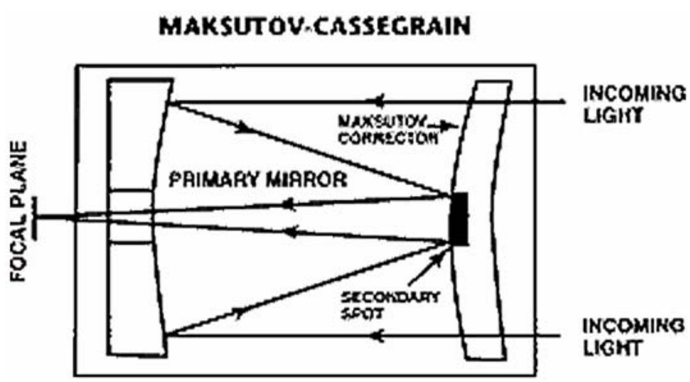


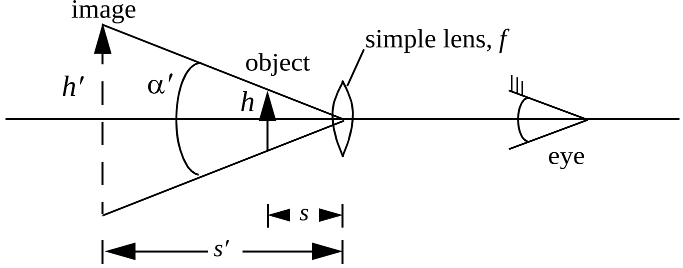
The Maksutov design uses a thick meniscus correcting lens with a strong curvature and a secondary mirror that is usually an aluminized spot on the corrector. The Maksutov secondary mirror is typically smaller than the Schmidt's giving it slightly better resolution, especially for observing extended objects, such as planets, galaxies, and nebulae.



Microscope

[Reading assignment: Hecht 5.7.3, 5.7.5]

Simple microscope (magnifier)



– object located inside lens focal length f

– virtual image is formed at s'

Simple application of the lens law gives:

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$$\boxed{h' = \frac{h(f-s')}{f}}$$

If the eye is located at the lens, the angle subtended by the image is

$$\alpha' = h'/s' = \frac{h(f-s')}{fs'}$$

If the eye views the same object at standard viewing distance (25 cm), then the angle would be

$$\boxed{\alpha = \frac{-h}{25}}$$

The magnifier enlarges the object by the ratio

$$M = \frac{\alpha'}{\alpha} = \frac{h(f-s')}{fs'} \cdot \frac{-25}{h} = \frac{25}{f} - \frac{25}{s'} \qquad (f, s' \text{ in cm})$$

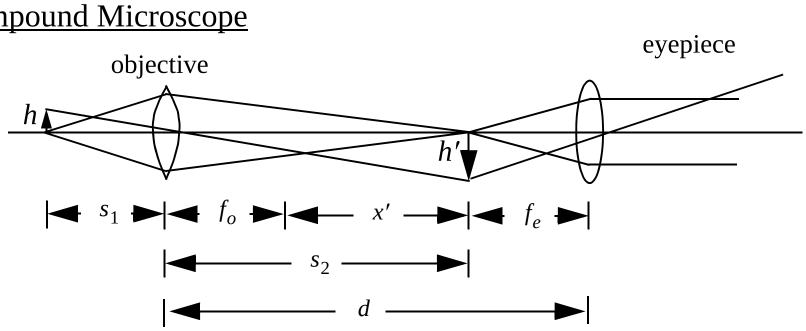
One may adjust the lens to put the image appearing at ∞, which means that it is viewed with a fully relaxed eye, then

$$\boxed{M = \frac{25}{f}}$$

With the image appearing at 25 cm (standard viewing distance), then

$$\boxed{M = \frac{25}{f} + 1}$$

<u>Compound Microscope</u>



– The objective lens produces a real (inverted), magnified image of the object.

– The eyepiece re-images to a comfortable viewing distance and provides additional magnification.

The total magnification is the product of the linear objective magnification times the eyepiece angular magnification.

$$\boxed{\begin{aligned} M_o &= \frac{h'}{h} = \frac{s_2}{s_1} = \frac{-x'}{f_o} \\ M_e &= \frac{25}{f_e} \\ M_{\mathrm{TOT}} &= M_o \cdot M_e = \frac{-x'}{f_o} \cdot \frac{25}{f_e} \end{aligned}}$$

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In laboratory microscopes, x' is called the "tube length" and is standardized to 160 mm. So, the objective magnification is given by $M_o = \frac{16}{f_o}$. Thus, a 20× objective lens has a focal length of 0.8 cm.

Resolution. The aperture stop is usually set by the size of the objective (NA). Recall that the diffraction limited linear resolution is

$$\boxed{Z = \frac{0.61\lambda}{\mathrm{NA}}}$$. This is the smallest object that can be resolved.

The eye can resolve an object size of ~0.08 mm at the distance of 25 cm, so the equivalent object size in the microscope is

$$R = \frac{0.08\ \mathrm{mm}}{M}$$

The magnification at which these two resolutions are equal is

$$\frac{0.08\ \mathrm{mm}}{M} = \frac{0.61\lambda}{\mathrm{NA}}$$
$$M = \frac{0.08}{0.61\lambda}\mathrm{NA} = \frac{0.13}{\lambda}\mathrm{NA} \qquad \text{with } \lambda \text{ in mm}$$

Take $\lambda = 0.55\mu\mathrm{m} \rightarrow M_{\mathrm{max}} \cong 240\mathrm{NA}$.

Increasing the magnification beyond this does not allow observation of smaller objects due to diffraction.

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