Lecture 7

Wave Front Aberration

In a wave-optics picture, the thin lens is represented by phase delay.

$$\phi(x, y) = -k\frac{x^2 + y^2}{2} = -k\Delta(x, y)$$

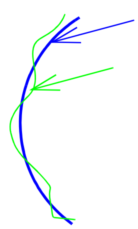
Which gives Gaussian imaging. Aberrations modify ϕ. A spherical lens only gives this ϕ in the paraxial approximation.

- For a complex optical system, we can collect the effects of all the lenses and represent them as a phase delay in the exit pupil. Usually, we subtract the quadratic phase to find the aberration. The residual is called the wave front error; or wfe

$$\Delta(x, y) = -\frac{x^2 + y^2}{2f} + W(x,y)$$

Aberration wfe

$\Delta(x, y)$ usually depends on the field coordinate. In other words, the aberrations can vary depending on where you are in the field of view.



Expressed in this way, the primary aberrations are written as:

Spherical aberration: $\boxed{W(x,y)=A_s\rho^4}$

Coma: $\boxed{A_c\rho^3 h'\cos\theta}$

Astigmatism: $\boxed{A_a\rho^2 h'^2\cos^2\theta}$

ρ : is normalized radial coordinate in the pupil

h' : image height

Field Curvature: $A_d\rho^2 h'^2$

Distortion: $A_t h'^3\rho\cos\theta$

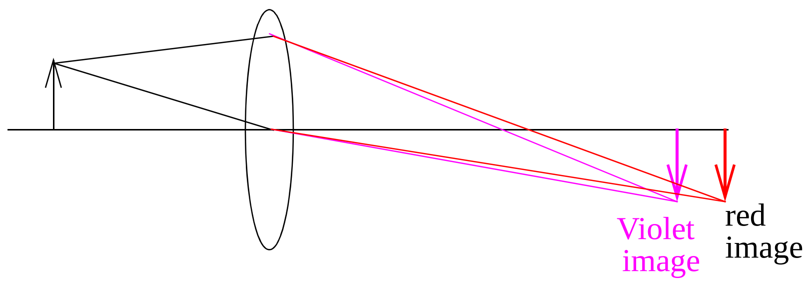
Monochromatic Aberrations: All of the preceding discussion refers to aberrations that do not depend on wavelength.

Chromatic Aberrations: Dependance of wavefront on wavelength.

Consider the simple thin lens equation:

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

The index n is generally λ dependent, $n(\lambda)$, so f is λ dependent.



Change in image distance: longitudinal chromatic aberration

Change in magnification: lateral color. Lateral color is usually more noticeable

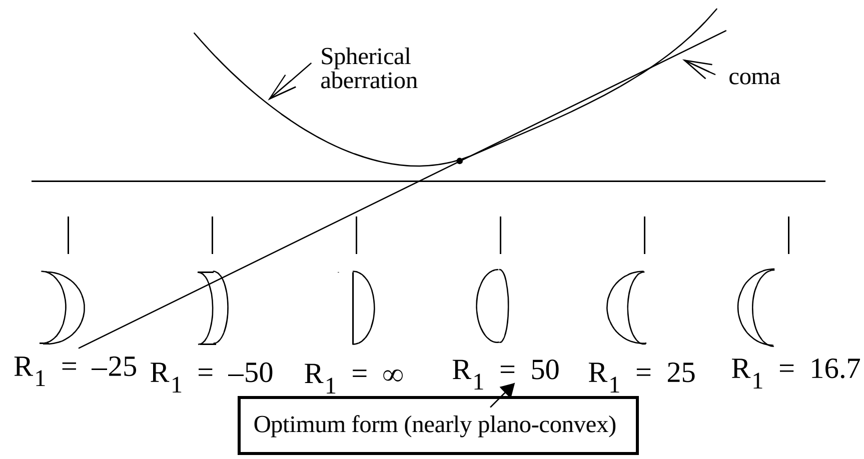
Achromat: lens designed to cancel chromatic aberration.

Lens Design:

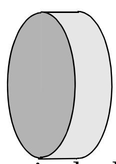
- The general problem of lens design involves cancelling aberrations
- Aberration depends on the lens index, as well as the surface radii.
- Complex lens systems can minimize aberrations

Simple singlet case: For a given desired focal length, there is freedom to choose one of the radii for a singlet The spherical aberration and coma depend on the particular choice, so these aberrations can be minimized by the design form. This is illustrated in the following diagram:

design: 100mm focal length $\pm 17^\circ$ field.



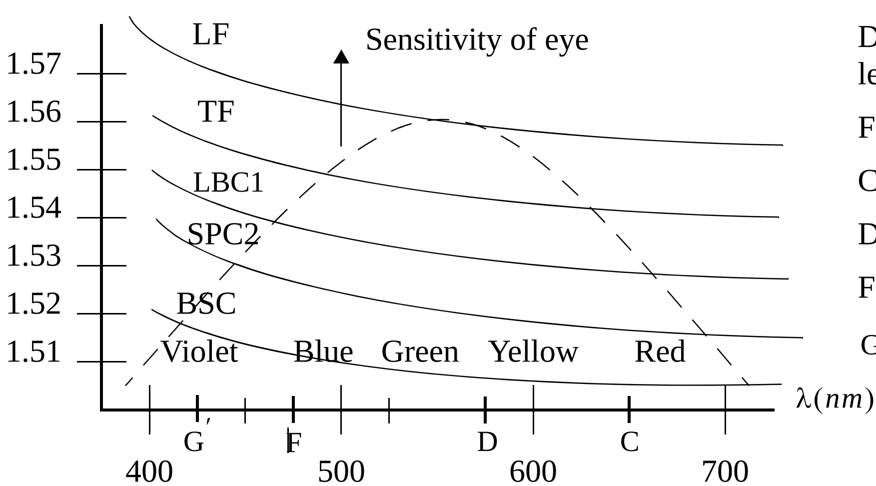
<u>Achromatic doublet.</u> Two elements made from different glass materials



achromatic doublet

positive element:	undercorrected spherical undercorrected chromatic
negative element:	both overcorrected

We generally choose design an achromat to minimize chromatic aberration across the visible part of the spectrum.

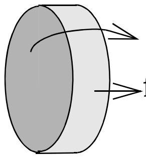


Different glasses for use in lenses.

Fraunhofer designations.

C H	656.3 nm
D Na	589.2
F H	486.1
G′ H	434.0

Design of Cemented Doublet Achromat



The ‘D’ wavelength, near the center of visual brightness curve is chosen as the nominal wavelength for specifying focal length. We then choose 2 indices on either side, for achromatization, for example, ‘C’, and ‘F’.

For 2 thin lenses in contact

$$\frac{1}{f_D} = \frac{1}{f_{D'}} + \frac{1}{f_{D''}}$$

prime: crown glass

double prime: flint glass

define lens power $\boxed{P = \frac{1}{f}}$ with f in meters,

P units are <u>diopters</u>

$$\boxed{P_D = P_{D'} + P_{D''}}$$

$$= (n_D' - 1)\left(\frac{1}{r_1'} - \frac{1}{r_2'}\right) + (n_D'' - 1)\left(\frac{1}{r_1''} - \frac{1}{r_2''}\right)$$

Define $K' = \left(\frac{1}{r_1'} - \frac{1}{r_2'}\right)$ $K'' = \left(\frac{1}{r_1''} - \frac{1}{r_2''}\right)$

$$P_D = (n_D' - 1)K' + (n_D'' - 1)K''$$

$$P_F = (n_F' - 1)K' + (n_F'' - 1)K''$$

$$P_C = (n_C' - 1)K' + (n_C'' - 1)K''$$

Achromatic design means we make $\boxed{P_C = P_F}$

$$(n_F' - 1)K' + (n_F'' - 1)K'' = (n_C' - 1)K' + (n_C'' - 1)K''$$

Simplifies to: $\boxed{\frac{K'}{K''} = -\frac{n_F'' - n_C''}{n_F' - n_C'}}$

For normal dispersion K' has the opposite sign from K''. One lens must be positive one lens must be negative.

For the center of the spectrum (D-line)

$$P_D' = (n_D' - 1)K'$$

$$P_D'' = (n_D'' - 1)K'' \text{, so}$$

$$\frac{K'}{K''} = \frac{(n_D'' - 1)P_D'}{(n_D' - 1)P_D''}$$

Combining results, we find:

$$\frac{P_D''}{P_D'} = -\frac{(n_D'' - 1)\ (n_F' - n_C')}{(n_D' - 1)\ (n_F'' - n_C'')} \equiv -\frac{\nu''}{\nu'} \tag{1.1}$$

$\boxed{\nu \equiv \frac{n_D - 1}{n_F - n_C}}$ is a property of a given glass called the “dispersion constant”

ν is called the “dispersive power” or V-number. Glass manufacturers spec these numbers for use by designers. Now, from Eq. (1.1),

$$\boxed{\frac{P_D''}{\nu''} + \frac{P_D'}{\nu'} = 0} \tag{1.2}$$

and

$$P_D' = P_D\frac{\nu'}{\nu' - \nu''} \qquad\qquad P_D'' = -P_D\frac{\nu''}{\nu' - \nu''} \tag{1.3}$$

Eqs. (1.2) and (1.3) are the design equations.