In the case of a discrete-time signal $x[n]$, we define its time-shifted version as follows:

$$y[n] = x[n - m]$$

where the shift m must be an integer; it can be positive or negative.

► **Drill Problem 1.11** The discrete-time signal $x[n]$ is defined by

$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases}$$

Find the time-shifted signal $y[n] = x[n + 3]$.

Answer: $$y[n] = \begin{cases} 1, & n = -1, -2 \\ -1, & n = -4, -5 \\ 0, & n = -3, n < -5, \text{ and } n > -1 \end{cases}$$ ◄

■ PRECEDENCE RULE FOR TIME SHIFTING AND TIME SCALING

Let $y(t)$ denote a continuous-time signal that is derived from another continuous-time signal $x(t)$ through a combination of time shifting and time scaling, as described here:

$$y(t) = x(at - b) \tag{1.25}$$

This relation between $y(t)$ and $x(t)$ satisfies the following conditions:

$$y(0) = x(-b) \tag{1.26}$$

and

$$y\left(\frac{b}{a}\right) = x(0) \tag{1.27}$$

which provide useful checks on $y(t)$ in terms of corresponding values of $x(t)$.

To correctly obtain $y(t)$ from $x(t)$, the time-shifting and time-scaling operations must be performed in the correct order. The proper order is based on the fact that the scaling operation always replaces t by at, while the time-shifting operation always replaces t by $t - b$. Hence the time-shifting operation is performed first on $x(t)$, resulting in an intermediate signal $v(t)$ defined by

$$v(t) = x(t - b)$$

The time shift has replaced t in $x(t)$ by $t - b$. Next, the time-scaling operation is performed on $v(t)$. This replaces t by at, resulting in the desired output

$$\begin{aligned} y(t) &= v(at) \\ &= x(at - b) \end{aligned}$$

To illustrate how the operation described in Eq. (1.25) can arise in a real-life situation, consider a voice signal recorded on a tape recorder. If the tape is played back at a rate faster than the original recording rate, we get compression (i.e., $a > 1$). If, on the

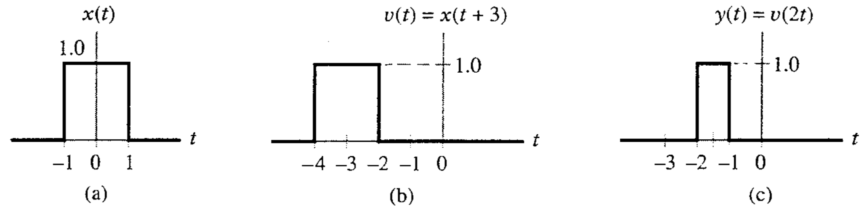

FIGURE 1.23 The proper order in which the operations of time scaling and time shifting should be applied for the case of a continuous-time signal. (a) Rectangular pulse $x(t)$ of amplitude 1.0 and duration 2.0, symmetric about the origin. (b) Intermediate pulse $v(t)$, representing time-shifted version of $x(t)$. (c) Desired signal $y(t)$, resulting from the compression of $v(t)$ by a factor of 2.

other hand, the tape is played back at a rate slower than the original recording rate, we get expansion (i.e., $a < 1$). The constant b, assumed to be positive, accounts for a delay in playing back the tape.

EXAMPLE 1.4 Consider the rectangular pulse $x(t)$ of unit amplitude and duration of 2 time units depicted in Fig. 1.23(a). Find $y(t) = x(2t + 3)$.

Solution: In this example, we have $a = 2$ and $b = -3$. Hence shifting the given pulse $x(t)$ to the left by 3 time units relative to the time axis gives the intermediate pulse $v(t)$ shown in Fig. 1.23(b). Finally, scaling the independent variable t in $v(t)$ by $a = 2$, we get the solution $y(t)$ shown in Fig. 1.23(c).

Note that the solution presented in Fig. 1.23(c) satisfies both of the conditions defined in Eqs. (1.26) and (1.27).

Suppose next that we purposely do not follow the precedence rule; that is, we first apply time scaling, followed by time shifting. For the given signal $x(t)$, shown in Fig. 1.24(a), the waveforms resulting from the application of these two operations are shown in Figs. 1.24(b) and (c), respectively. The signal $y(t)$ so obtained fails to satisfy the condition of Eq. (1.27).

This example clearly illustrates that if $y(t)$ is defined in terms of $x(t)$ by Eq. (1.25), then $y(t)$ can only be obtained from $x(t)$ correctly by adhering to the precedence rule for time shifting and time scaling.

Similar remarks apply to the case of discrete-time signals.

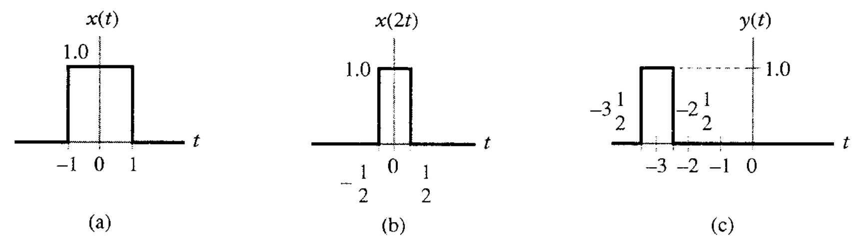

FIGURE 1.24 The incorrect way of applying the precedence rule. (a) Signal $x(t)$. (b) Time-scaled signal $x(2t)$. (c) Signal $y(t)$ obtained by shifting $x(2t)$ by 3 time units.

EXAMPLE 1.5 A discrete-time signal $x[n]$ is defined by

$$x[n] = \begin{cases} 1, & n = 1, 2 \\ -1, & n = -1, -2 \\ 0, & n = 0 \text{ and } |n| > 2 \end{cases}$$

Find $y[n] = x[2n + 3]$.

Solution: The signal $x[n]$ is displayed in Fig. 1.25(a). Time shifting $x[n]$ to the left by 3 yields the intermediate signal $v[n]$ shown in Fig. 1.25(b). Finally, scaling n in $v[n]$ by 2, we obtain the solution $y[n]$ shown in Fig. 1.25(c).

Note that as a result of the compression performed in going from $v[n]$ to $y[n] = v[2n]$, the samples of $v[n]$ at $n = -5$ and $n = -1$ (i.e., those contained in the original signal at $n = -2$ and $n = 2$) are lost.

► **Drill Problem 1.12** Consider a discrete-time signal $x[n]$ defined by

$$x[n] = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & |n| > 2 \end{cases}$$

Find $y[n] = x[3n - 2]$.

Answer: $y[n] = \begin{cases} 1, & n = 0, 1 \\ 0, & \text{otherwise} \end{cases}$ ◄

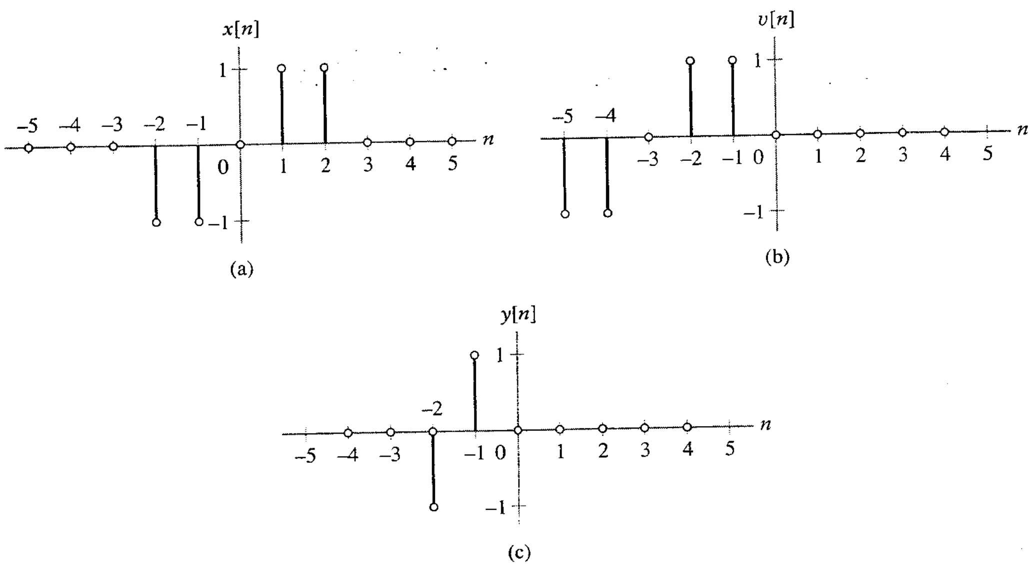

FIGURE 1.25 The proper order of applying the operations of time scaling and time shifting for the case of a discrete-time signal. (a) Discrete-time signal $x[n]$, antisymmetric about the origin. (b) Intermediate signal $v[n]$ obtained by shifting $x[n]$ to the left by 3 samples. (c) Discrete-time signal $y[n]$ resulting from the compression of $v[n]$ by a factor of 2, as a result of which two samples of the original $x[n]$ are lost.