EECS120 - Fall 2003
Homework No. 3
Due: In the 120 Box or in lecture by the beginning of lecture on 9/18/2003
Collaboration permitted and solutions to be written up by groups up to 3.
Be clear and precise in your answers
Questions can be asked in ucb.class.ee120 or in office hours

Problem 3.1 *Book Problems from Lee and Varaiya, chapter 10.*
Problems: 2,3,5

Problem 3.2 *Linear Algebra Review*

a. *What is the determinant of aI where I is the $n \times n$ identity matrix? What is the trace?*

b. *What are the eigenvalues and eigenvectors of the following matrices:*

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 6 \\ 0 & 8 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

c. *Consider vectors in three-dimensional space. Let H be a system that takes an incoming vector, projects it into the plane that is perpendicular to the direction $[111]^T$, and proceeds to rotate the resulting vector 90 degrees around the Z axis. Is H linear? Invertible? Representable as a matrix? (If so, what is the matrix representation?)*

d. *Let the $n \times n$ real matrix M be such that it has n distinct eigenvalues λ_i. Show that there exists a coordinate system in which the operation of M can be represented by a diagonal matrix.*

Problem 3.3 *Finite domains without wrap-around*

Consider signals on the set $\{0, 1, 2, \ldots, n-1\}$. Suppose that we interpret delay/shift to mean that if a signal is shifted by $+i$, then the first i values will be set to zero, while the first $n-i$ values will become the last $n-i$ values. Similarly, if a signal is shifted by $-i$, then the last i values will be set to zero while the last $n-i$ values will become the first $n-i$ values.

a. *Prove that if L is an LTI system, then L is just a scalar gain.*

b. *(Bonus) Characterize the entire class of time-invariant systems.*

Problem 3.4 *Finite domains with wrap-around*

Consider the domain Z_n (the integers mod $n > 0$). Here we interpret delay to mean that $[D_\tau x](t) = x(t - \tau \bmod n)$.

Consider $n = 3$.

a. *Show that the set of real-valued signals on this domain is representable by vectors in 3-dimensional space using the standard basis vectors.*

b. *Show that all linear systems that map real-valued signals on this domain to real-valued signals on this domain are representable by real 3×3 matrices.*

c. *What is the class of matrices that correspond to LTI systems?*

d. *Show that there exists a complex coordinate system in which every LTI system is representable by a diagonal matrix.*

e. *Represent D_1 and D_2 in both the coordinate system of (c) and (d).*

f. *Show explicitly that the coordinate system of part (d) is orthogonal. (i.e. the basis vectors are all orthogonal to each other using the regular Euclidean inner product on complex spaces.)*

g. *Do all complex diagonal matrices in the coordinate system of (d) correspond to real LTI systems? If not, which subset of the complex diagonal matrices correspond to real LTI systems?*

h. *Give an LTI system that removes the DC offset of a signal, but otherwise leaves it unchanged. Express it both in terms of the impulse response and the "frequency response."*

i. *Repeat parts d,e,f,g,h for $n = 5, 6$*

Problem 3.5 *Suppose that $x(t)$ is a discrete-time signal that is periodic with period T (positive integer). If $y = Lx$ where L is an LTI system, is y guaranteed to be a periodic signal? Why or why not?*