EECS120 - Fall 2003

Homework No. 5

Due: In the 120 Box On Oct 16 **You are strongly urged to turn in a photocopy of your work so that you can check against the solutions.**

Collaboration permitted and solutions to be written up by groups up to 3.

Be clear and precise in your answers

Questions can be asked in ucb.class.ee120 or in office hours

Problem 5.1 *BIG BONUS:*

Go to www.scitoys.com or other similar sites to look up how to make simple crystal radios. Build one and report on how it works. Can you give an explanation in terms of the concepts taught in this class?

Can you simulate this radio in MATLAB using idealized circuit components and illustrate the operation?

Problem 5.2 *Book Problems from Lee and Varaiya. Problem 10.16, 11.4, 11.6*

Problem 5.3 *Book Problems from Oppenheim, Willsky, and Nawab Problem 8.35, 8.37 (Hint: Try 8.34 first), 8.41*

Problem 5.4 *What is wrong with* $\sec(t)$

This problem is designed to explore the idea of trying to demodulate a signal by multiplying by $\sec(t)$*. This is meant to fill in and complement the discussion in class.*

Consider a bandlimited signal $x(t)$ *that has no power outside of* $\omega \in [-\frac{1}{4}, +\frac{1}{4}]$.

The modulated signal is given by:

$$y(t) = x(t)\cos(t)$$

We wish to demodulate this signal by multiplication by $\sec(t)$ *or some suitable approximation thereof.*

a. *First, we want to verify that without noise, this makes some sense. Suppose that* $x(t) = \cos(\frac{1}{4}t + \frac{\pi}{3})$*. Let* $z(t) = y(t)\sec(t)$ *for all* t *for which* $\cos(t) \neq 0$*. Show that the right and left limits of* $z(t)$ *at* $t = \frac{\pi}{2}$ *exist and that both equal* $x(\frac{\pi}{2})$*.*

b. *Verify that the average power in the signal* $\sec(t)$ *is infinite.*

c. *To see what might be happening in the frequency domain, we will attempt to calculate a formal Fourier Series for* $\sec(t)$*. Consider the period to be from* $[-\pi, \pi]$ *and do the integrals over the domain* $[-\pi, -\frac{\pi}{2} - \epsilon] \cup [-\frac{\pi}{2} + \epsilon, \frac{\pi}{2} - \epsilon] \cup [\frac{\pi}{2} + \epsilon, \pi]$ *and then let* $\epsilon \to 0$ *to get the integral we are interested in.*

   Begin with the DC term. Show that it is zero.

d. *Continue part (c) and calculate the Fourier Series terms at* $\omega = \pm 1$*.*

   (HINT: Write $e^{jt} = \cos(t) + j\sin(t)$ and then do separate integrals for the real and imaginary parts.)

e. *To continue part (c), write a recursion that relates the k-th Fourier Series coefficient to the $k-2$ one.*

(HINT: First prove a little lemma that $\cos(kt) = 2\cos(t)\cos((k-1)t) - \cos((k-2)t)$. This is very easy to see in the frequency domain...)

f. *Apply (c,d,e) to write the formal Fourier Series for* $\sec(t)$. *Use the relation between Fourier Series and the CTFT to write out and plot the Fourier Transform of* $\sec(t)$ *using Dirac delta functions.*

g. *Now, suppose we choose to approximate* $\sec(t)$ *by only the* $-2m+1 \leq k \leq 2m-1$ *terms of the Fourier Series when doing the multiplication for* $z(t)$. *Simplify the resulting expression for* $z(t)$ *and show what it looks like in the Frequency Domain. What is happening as we increase the order of approximation* m *?*

h. *BONUS: Use Matlab or other plotting software to graphically compare the time-domain realization of* $\sec(t)$ *with that of various orders of Fourier Series approximations to it. Does it look like it is converging in the normal sense?*

i. *What if the original signal was* $y(t) = x(t)\cos(t+\phi)$ *where* ϕ *is small. Repeat part (g) with this mismatch in the phase.*