EECS120 - Fall 2003
Homework No. 6
Due: In the 120 Box On Oct 23

You are strongly urged to turn in a photocopy of your work so that you can check against the solutions.

Collaboration permitted and solutions to be written up by groups up to 3.
Be clear and precise in your answers
Questions can be asked in ucb.class.ee120 or in office hours

Problem 6.1 *BIG BONUS Continues:*

Go to www.scitoys.com or other similar sites to look up how to make simple crystal radios. Build one and report on how it works. Can you give an explanation in terms of the concepts taught in this class?

Can you simulate this radio in MATLAB using idealized circuit components and illustrate the operation?

This problem is good for the rest of the term.

Problem 6.2 *BONUS: Book Problems from Oppenheim, Willsky, and Nawab*

(You really ought to do these though, but the main problem below is quite challenging on its own and so these are officially bonus.)

Problems 7.41, 7.42, 7.44

Problem 6.3 *"Exotic" Sampling*

This problem is designed to complement the discussion in class by considering the sampling and reconstruction of signals that go beyond those presented in class. For all the cases below, you can assume that the signal $x(t)$ has a Fourier Transform $X(\omega)$ that is a continuous and appropriately smooth function of ω.

All of the below come up in real world communication and signal processing problems in various disguises.

a. *Consider a complex signal $x(t)$ for which $X(\omega) = 0$ for $\omega < -2\pi$ and also for $\omega > 6\pi$. Show that if you sample this signal at $f_s = 6$, then it can be reconstructed perfectly using the sinc-based interpolation showed in class.*

b. *Consider the complex signal $x(t)$ from part (a). Show that if we sample $x(t)$ at $f_s = 4$, it can still be reconstructed perfectly by giving the appropriate interpolation formula for it.*

   (HINT: First think about shifting it down in frequency first, then sampling, then reconstructing, and finally shifting back up in frequency. Then write out the final answer for $x(t) =$ and see if you can shift all the work to the interpolation formula.)

c. *Consider a bandlimited real signal $x(t)$ for which $X(\omega) = 0$ outside of $\omega \in [-4\pi, +4\pi]$. Instead of sampling it at $t = l\frac{1}{4}$ for l integers, we are offset in time and sample it regularly at $t = \frac{1}{6} + l\frac{1}{4}$. Show that we can still reconstruct $x(t)$ perfectly from its samples at those times using the appropriate interpolation formula.*

d. *Suppose that the signal $x(t)$ from part (c) had been modulated first. The modulated signal is given by:*

$$y(t) = x(t)\cos(8\pi t)$$

Instead of demodulating the signal and then sampling it, we just sample $y(t)$ directly at times $t = l\frac{1}{4}$.

Show that we can still reconstruct $x(t)$ perfectly from its samples at those times using the appropriate interpolation formula.

e. *Consider a real signal $x(t)$ for which $X(\omega) = 0$ outside of $\omega \in [-8\pi, -4\pi] \cup [+4\pi, +8\pi]$.*

Show that you can reconstruct $x(t)$ perfectly from samples taken at times $t = l\frac{1}{4}$ for integers l using the appropriate interpolation formula.

f. *Consider the real signal $x(t)$ from part (e) above. Interpret it as a signal that resulted from modulating two real signals $x_1(t)$ and $x_2(t)$ that were then modulated by $\cos(6\pi t)$ and $\sin(6\pi t)$ carriers respectively. What do the transforms $X_1(\omega)$ and $X_2(\omega)$ look like?*

Give a strategy that recovers $x_1(t)$ and $x_2(t)$ from the samples taken in part (e).

g. *Consider a real signal $x(t)$ for which $X(\omega) = 0$ outside of $\omega \in [-8\pi, +8\pi]$.*

Suppose that this represented some physical phenomena. Instead of sampling it at one place at the rate $f_s = 8$, suppose that we sampled it at two distinct physical locations at $f_s = 4$ each. This gave rise to two sets of samples:

$$\ldots, x(-\frac{4}{8}), x(-\frac{2}{8}), x(0), x(+\frac{2}{8}), x(+\frac{4}{8}), \ldots$$

and

$$\ldots, x(-\frac{3}{8} + \frac{1}{6}), x(-\frac{1}{8} + \frac{1}{6}), x(+\frac{1}{8} + \frac{1}{6}), x(+\frac{3}{8} + \frac{1}{6}), x(+\frac{5}{8} + \frac{1}{6}), \ldots$$

so we have $x(\frac{2l}{8})$ and $x(\frac{(2l-1)}{8} + \frac{1}{6})$ for all integer l.

Show that it is in principle possible to reconstruct $x(t)$ exactly from this set of non-uniform samples.

(HINT: Divide $X(\omega) = X_1(\omega) + X_2(\omega)$ where $X_1(\omega) = 0$ for $|\omega| > 4\pi$ and $X_2(\omega) = 0$ for $|\omega| \leq 4\pi$. Then see how both of these components show up in the two sets of samples. From there, you should be able to setup a system of linear equations at every ω that will give you the answer and let you solve for $X(\omega)$ based on the two discrete time signals.

Alternatively, look at OWN problem 7.37 for help)