EECS120: Signals and Systems Midterm 2
Write name and ID number on each page of your solutions
If you do more than the required number of parts, you get extra credit.
"Proofs by Pictures" are worth substantial partial credit if essentially correct,
but give some equations alongside for full credit.

Some useful information for you to use:

$$\text{CTFT } X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

$$\text{ICTFT } x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{+j\omega t}d\omega$$

$$\text{CTFS } X_k = \frac{1}{T}\int_{0}^{T} x(t)e^{-j\frac{2\pi k}{T}t}dt$$

$$\text{ICTFS } x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{+j\frac{2\pi k}{T}t}$$

$$\text{DTFT } X(\omega) = \sum_{t=-\infty}^{+\infty} x[t]e^{-j\omega t}$$

$$\text{IDTFT } x[t] = \frac{1}{2\pi}\int_{0}^{2\pi} X(\omega)e^{+j\omega t}d\omega$$

$$\text{DFT } X_k = \sum_{k=0}^{T-1} x[t]e^{-j\frac{2\pi k}{T}t}$$

$$\text{IDFT } x[t] = \frac{1}{T}\sum_{k=0}^{T-1} X_k e^{+j\frac{2\pi k}{T}t}$$

$$\text{Z-Transform } X(z) = \sum_{t=-\infty}^{+\infty} x[t]z^{-t}$$

$$\text{Laplace-Transform } X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

If $X(\omega) = 1$ for $\omega \in [-\frac{\pi}{T}, +\frac{\pi}{T}]$ and 0 otherwise, then $x(t) = \frac{1}{T}\text{sinc}(\frac{t}{T})$
If $x(t) = 1$ for $t \in [-\frac{T}{2}, +\frac{T}{2}]$ and 0 otherwise, then $X(\omega) = T\text{sinc}(\omega\frac{T}{2\pi})$.
If $y(t) = e^{j\omega_0 t}x(t)$, then $Y(\omega) = X(\omega - \omega_0)$.
If $x(t) = \sum_{k=-\infty}^{\infty}\delta(t - kT)$, then $X(\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - k\frac{2\pi}{T})$.

Problem 2.1 *Build A COM System*

Justify your answers for full credit: use any combination of time-domain and frequency-domain reasoning to show that your system works.

Depending on the problem, you are allowed to use some of the following ingredients in making your system. If you are allowed to use it, you can use as many of that item as you would like, including none at all. If it is parametric, you can set the parameters for it as you like and those parameters can vary among instantiations used.

Master Ingredients List:

1. *Adder: Takes as many inputs as you like and returns the sum of them on a time-by-time basis. e.g.* $x(t) = a(t) + b(t)$

2. *Scalar Multiply: Is parametrized by the (possibly complex) scalar gain* α*. e.g.* $y(t) = \alpha c(t)$

3. *Function Generator: Is parameterized by the (possibly complex) function of time* $f(t)$ *and just returns the signal* $f(t)$*. e.g. Use* $f(t) = \cos(t)$ *to get a signal* $z(t) = \cos(t)$*. NOTE: You are allowed to use Dirac delta functions within* $f(t)$ *if you would like.*

4. *Memoryless Function: Is parameterized by the (possibly complex) function* $g(w)$ *of a single complex variable* w *and returns the input signal having been run through* $g(w)$ *on a time-by-time basis. e.g. Use* $g(w) = |w|$ *and then get* $y(t) = g(d(t))$*.*

5. *Signal Multiplier: Returns the product of its input signals on a time-by-time basis. e.g.* $y(t) = a(t)b(t)$

6. *LTI filter: Parametrized by either the impulse response or the frequency response. Runs the input signal through an LTI system and outputs the result. e.g. Use* $h(t) = sinc(t)$ *and get* $y(t) = x(t) * h(t)$ *where* $*$ *denotes convolution.*

7. *Sampler: Parametrized by the inter-sample time* T_s*. Given input signal* $i(t)$*, it has as output the discrete-time signal* $i_d[k] = i(kT_s)$*.*

8. *Weighted Summer: Parametrized by a (possibly complex) pulse-shape* $p(t)$ *and an inter-pulse time* T_p*. Given a discrete-time signal* $i_d[k]$*, it has as output the continuous-time signal* $l(t) = \sum_{k=-\infty}^{+\infty} i_d[k]p(t - kT_p)$

a. *20pts* *You are given a real-valued speech signal* $x(t)$ *for which* $X(\omega) = 0$ *for* $|\omega| \leq 40\pi$ *and* $|\omega| \geq 8000\pi$. *Furthermore,* $|x(t)| < 1$ *for every* t.

Construct a transmitter that gives a real-valued continuous time signal $y(t)$ *that satisfies* $Y(\omega) = 0$ *for* $|\omega| \leq 200000\pi$ *and* $|\omega| \geq 216000\pi$.

Furthermore, construct a receiver that gets access to $y(t)$ *and must output* $x(t)$.

You are allowed to use anything from the ingredients list.

Do either (b) or (c) for full credit. Both is extra credit.

b. *20pts* *Repeat Part (a), except this time you are banned from using any of item (5) the Signal Multipliers.*

c. *20pts* *Repeat Part (a), except this time you are only allowed to use items (1) Adders, (5) Signal Multipliers and (6) LTI filters.*

In addition, you have access to instantiations of broken versions of (3) Function Generator. These broken versions only generates periodic pulses of the form: $a(t) = \sum_{k=-\infty}^{+\infty} g(t - kT_p)$*. You can adjust* T_p *as you wish, but* $g(t)$ *is stuck to be* 1 *if* $|t| \leq \frac{1}{1000000}$ *and* 0 *otherwise.*

Problem 2.2 *True/False. Do at least 2 of the following for full credit.*

If the bold statement is true, give a proof for it. If the statement is false, show a counterexample or proof that it is false.

a. 20pts **All continuous-time LTI systems having a real-valued impulse response $h(t)$ also have a well defined CTFT $H(\omega)$.**

b. 20pts **If we know that the real valued signal $x(t)$ can be reconstructed by some interpolation formula $x(t) = \sum_{k=-\infty}^{\infty} x[k]p(t - kT_s)$ from its samples $x[k] = x(kT_s)$ for some constant T_s and function $p(t)$, then $X(\omega)$ must be bandlimited in the sense that there exists some W for which $\omega > W$ implies that $X(\omega) = 0$.**

c. 20pts **If a real-valued continuous time signal $x(t)$ is such that $X(\omega) = 0$ unless $|\omega| \in [4\pi, 8\pi]$, then $x(t)$ can be reconstructed exactly from samples $x_d[k] = x(\frac{k}{4})$ using the appropriate interpolation formula.**

Problem 2.3 *Generation of Continuous-time Media from Discrete-time Signals*

For this problem, you are again allowed to use any of the ingredients from problem 1 as well as their discrete-time equivalents unless told otherwise. Justify your answers for full credit: use any combination of time-domain and frequency-domain reasoning to show that your system works.

a. 20pts *Given a real-valued discrete-time signal* $d[n]$ *which generates a new value every millisecond, please construct a continuous-time real-valued signal* $x(t)$ *for which* $X(\omega) = 0$ *if* $|\omega| > 1000\pi$.

Show how to recover $d[n]$ *from* $x(t)$.

Do either (b) or (c) for full credit. Both is extra credit.

b. *30pts* *Repeat part (a), except now you must make sure* $X(\omega) = 0$ *unless* $|\omega| \in [200000\pi, 201000\pi]$.

c. 30pts Repeat part (a), but now you do not have access to item (3) Function Generator and furthermore, your item number (8) Weighted Summer is broken. You are free to choose T_p, but can only use a first-order-hold

$$p(t) = \begin{cases} 0 & \textit{if } t < -T_p \\ \frac{t}{T_p} + 1 & \textit{if } -T_p \le t \le 0 \\ 1 - \frac{t}{T_p} & \textit{if } 0 < t < T_p \\ 0 & \textit{if } t \ge T_p \end{cases}$$