Notes 13 largely plagiarized by %khc

1 Amplitude Modulation (AM)

There are three major amplitude modulation schemes that we will study:

- dual sideband, large carrier (AM-DSB-LC),
- dual sideband, suppressed carrier (AM-DSB or AM-DSB-SC),
- and single sideband (AM-SSB).

In this set of notes, we will only discuss the dual sideband schemes, leaving single sideband until next time.

2 AM Transmission

In AM-DSB-LC, the transmitted signal is:

$$\begin{aligned} x(t) &= (1 + \mu m(t)) \cos \omega_c t \\ &= \cos \omega_c t + \mu m(t) \cos \omega_c t \end{aligned}$$

In AM-DSB-SC, the transmitted signal is:

$$x(t) = \mu m(t) \cos \omega_c t$$

AM-DSB-LC is otherwise known as broadcast radio; a large DC component is added to the message before modulation occurs. AM-DSB-SC saves you the expense of having to broadcast the shifted DC component; you'll end up saving on the power, but your receiver is going to require some synchronization with the carrier in order to recover your message.

Block diagrams for the transmission schemes are shown in Figure 1.

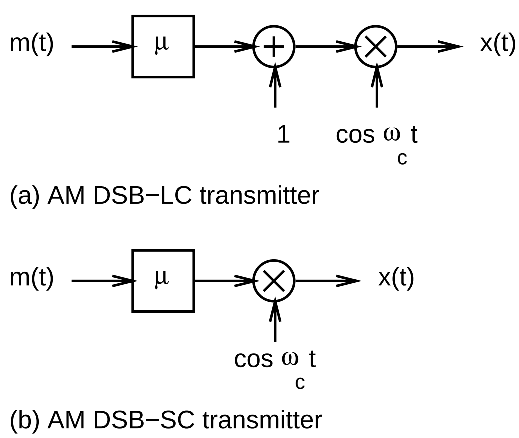


Figure 1: AM transmitters.

Exercise Verify that the block diagrams give the transmitted signals above.

Let's examine each of these transmitters in the frequency domain, by watching what happens to an input signal $m(t)$ with triangular Fourier transform $M(\omega)$.

Exercise Verify that $m(t)$ is going to have a sinc^2 shape.

With AM-DSB-LC, our $M(\omega)$ gets scaled by μ and then gets superposed with an impulse centered at $\omega = 0$ [from the constant 1 that gets added to the scaled message]. Multiplication with a cosine in time replicates the composite spectrum from the previous step [scaled by $\frac{1}{2}$, since the impulses in the cosine are scaled by π, but the convolution in frequency brings with it another factor of $\frac{1}{2\pi}$] in frequency, one copy at ω_c and the other at $-\omega_c$. This is illustrated in Figure 2.

In AM-DSB-SC, the spectrum is similar to that of AM-DSB-LC, except that the impulses at ω_c and $-\omega_c$ are not present. This is illustrated in Figure 3.

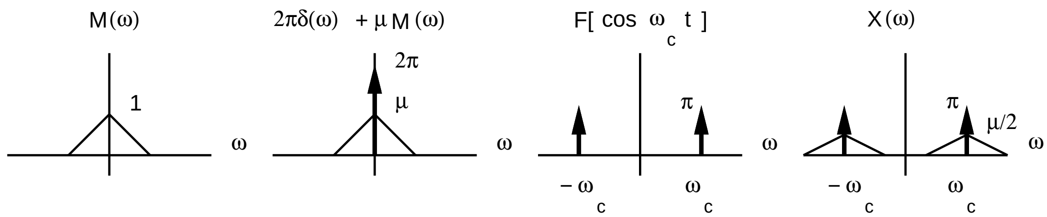


Figure 2: AM-DSB-LC in the frequency domain.

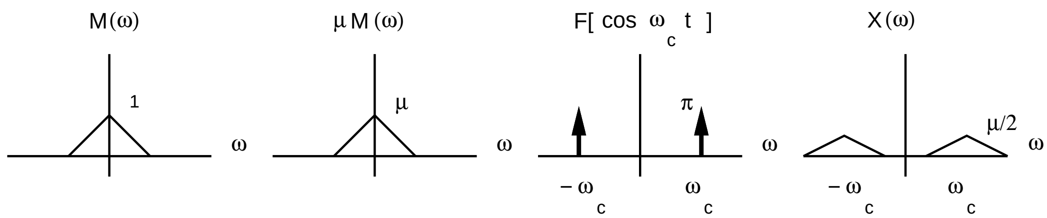


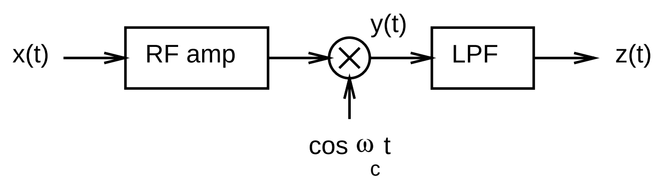
Figure 3: AM DSB-SC in the frequency domain.

Exercise Verify the frequency domain representations of the transmitted signal for both AM-DSB-LC and AM-DSB-SC.

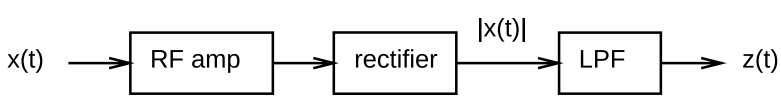
Note that we are wasting bandwidth. Our message has half the bandwidth of the transmitted signal.

3 AM Reception

Let's assume that we have stations transmitting $x(t)$ in either DSB-LC or DSB-SC format. Once the signal has been transmitted, we need some way of receiving that signal. The block diagrams of two of the simplest receivers are given in Figure 4.



(a) AM receiver with synchronous demodulator



(b) AM receiver with asynchronous demodulator

Figure 4: AM receivers.

Note that each receiver has an RF amplifier. The point of an RF amplifier is to make sure that you receive only one station at a time [i don't know about you, but i have a hard time listening to two things at the same time]. For now, let's assume that the RF amplifier is a tunable ideal bandpass filter with $H(\omega) = 1$ for $\omega_c - \omega_M < |\omega| < \omega_c + \omega_M$, where ω_M is the message bandwidth.

We analyzed the action of the synchronous demodulator in the time domain in ps3, problem 10. But now we can also look at it in the frequency domain.

Exercise Go back and check out ps3, problem 10. Make sure that you understand the solution set.

As in the transmitter, the multiplication with the cosine makes two more copies of the spectrum. So the output of the multiplier will have spectrum $Y(\omega)$ as in Figure 5. If our LPF has frequency response $H(\omega) = 1$ for $|\omega| < \omega_M$, we can then recover our original message, modulo a gain factor. Note that if the frequency at which we demodulate is slightly off, we are not going to get the message, but some random mess.

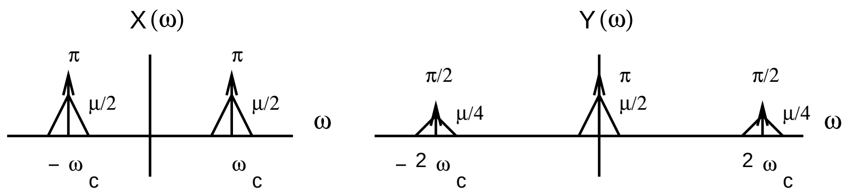


Figure 5: Synchronous demodulator in the frequency domain.

Exercise Verify that this synchronous demodulator can also demodulate AM-DSB-SC format signals.

The asynchronous demodulator is slightly more interesting. As previously discussed in lecture, with the same RF amplifier as that used above in the synchronous demodulator, the output of the rectifier is the absolute value of the input [assuming that the rectifier is a full-wave rectifier]. The LPF then acts as a peak detector, smoothing out the ripples in the absolute value of the received signal. This serves to pick off the envelope of the received signal. After running the envelope through a blocking capacitor to get rid of the DC term,[1] we can then drop the rest of the signal into a speaker to get our daily dose of Rush Limbaugh[2]. This process in the time domain is lamely illustrated in Figure 6.

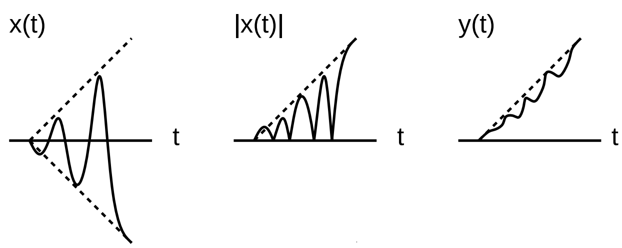


Figure 6: Asynchronous demodulation in the time domain.

A full wave rectifier takes the absolute value of its input. If the input is some modulated even signal $x(t)$ that periodically changes its sign, this is the same as multiplying the input by an even square wave $s(t)$ of ± 1 peak-to-peak amplitude with the same period as the sign change in $x(t)$, as in Figure 7. This trick then permits us to examine what happens in the frequency domain. $s(t)$ is the sum of twice the standard even square wave less a constant of 1. An even

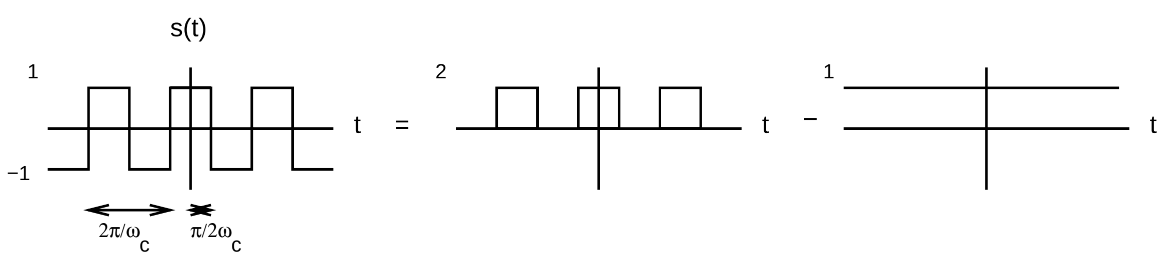


Figure 7: The decomposition of $s(t)$.

square wave has FS coefficients $a_k = \frac{1}{k\pi} \sin k2\pi \frac{T_1}{T}$. With our particular choice of period and duty cycle, $T_1 = \frac{\pi}{2\omega_c}$

[1] Sound is variation in pressure, we can't hear DC at all. If we didn't run the DC through a blocking capacitor, all we would end up doing is heating up the speaker, making it more nonlinear, messing up the frequency response, and wasting power.

[2] Nothing like Rush to give that digestive tract a good reaming

and $T = \frac{2\pi}{\omega_c}$. Twice an even square wave then has FS coefficients $a_k = \frac{2}{k\pi} \sin k\frac{\pi}{2}$. Subtracting the constant just zeros a_0. So the FS of $s(t)$ has coefficients:

$$a_k = \begin{cases} 0 & \text{if } k = 0 \\ \frac{2}{k\pi} \sin k\frac{\pi}{2} & \text{otherwise} \end{cases}$$

Note that the even coefficients of $s(t)$ are zero, since $s(t)$ exhibits half wave odd symmetry.

The FT of $s(t)$ is then

$$\begin{aligned} S(\omega) &= 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0) \\ &= 2\pi \sum_{k\,odd} (\frac{2}{k\pi} \sin k\frac{\pi}{2}) \delta(\omega - k\omega_c) \end{aligned}$$

Note that $S(\omega)$ only has nonzero impulses for odd k.

Now we multiply $x(t)$ by $s(t)$ to get $y(t) = |x(t)|$. This means we have to convolve $X(\omega)$ with $S(\omega)$ in frequency [with a factor of $\frac{1}{2\pi}$] to get $Y(\omega)$. This convolution replicates the spectrum of $X(\omega)$ quite a bit, as partially illustrated in Figure 8. If our LPF has frequency response $H(\omega) = 1$ for $|\omega| < \omega_M$, we can then recover our original message,

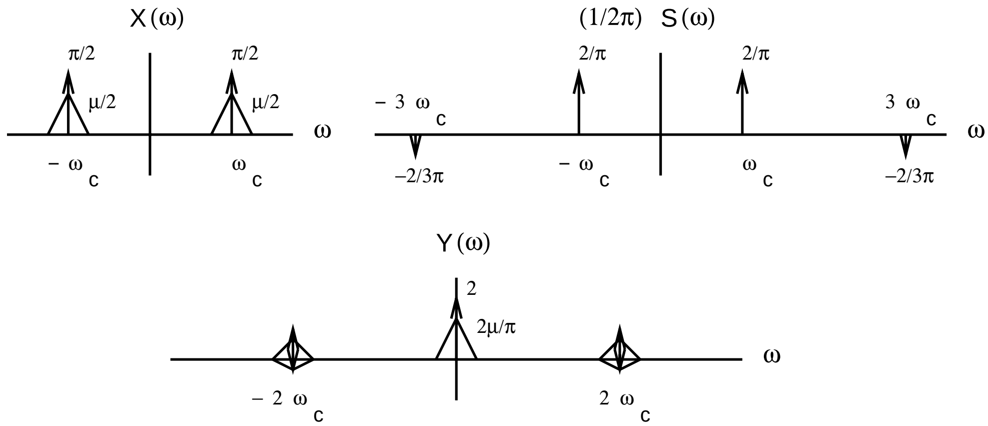


Figure 8: Asynchronous demodulation in the frequency domain.

modulo a gain factor.

The important thing to note here is that this is a mathematical trick that lets us perform the analysis of this system in the frequency domain. Contrary to popular belief, *there is no square wave generator anywhere in the circuit.*

Exercise Redo this frequency domain analysis for a half wave rectifier. Note that you will have to use a square wave with an amplitude that varies from 0 to 1.